Advances and Applications in Mathematical Sciences
Volume 20, Issue 4, February 2021, Pages 549-554

ALPHA, BETA AND GAMMA STRONG VERTICES

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Abstract

In this paper, boundary nodes are easily found with the help of α, β and γ strong nodes from fuzzy vertex order colouring. Some of the properties are discussed.

1. Introduction

The concepts of boundary nodes and interior nodes using sum distance are introduced by Tom and Sunitha [6]. The fuzzy vertex order colouring using α, β and γ-strong nodes are introduced by A. Nagoor Gani and B. Fathima Kani [7]. In this paper we connect these three α, β and γ-strong nodes to boundary nodes. In Section 2 we discussed the basic definitions. Definitions of Boundary nodes, eccentricity nodes, fuzzy radius and fuzzy diameter are given in section 3 and also properties related to boundary nodes and α, β and γ strong nodes are also discussed.

2. Basic Definitions

A fuzzy graph $G : (\sigma, \mu)$ is said to be complete fuzzy graph if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. Let $G^* : (V, E)$ be a graph. Length of $u - v$ path P as the sum of the weights of the arcs in P, $L(P) = \sum_{i=1}^{n} \mu(u_{i-1}, u_i)$. The sum distance between u and v denoted by

2010 Mathematics Subject Classification: 03E72, 05C72.
Keywords: α, β and γ strong nodes, complete fuzzy graph, boundary nodes.
Received January 11, 2020; Accepted May 13, 2020

$d_s(u, v) = \min \{L(P_i) : P_i \in P, i = 1, 2, \ldots\}$. $d_s(u, v)$ is a metric on V. A node v is strictly α-strong if $d(v) > d(u), \forall u \in A(v)$. A γ-strong node is strict always. A β-strong vertex is strong as well as weak.

3. Relation among Boundary Nodes and α, β and γ Strong Vertices

Definition 3.1 [6]**.** A boundary node in a connected fuzzy graph satisfies $d_s(u, v) \geq d_s(u, w)$ for each neighbour w of v. Collection of all boundary nodes denoted by u^b.

Example 3.2. Here $d(u) = 1.5, d(v) = 0.9, d(w) = 1.7, d(x) = 1.3$. $S_\alpha(V) = \{w\}, \beta(V) = \{u\}, W_\gamma(V) = \{v, x\}$. In example 3.2 $u^b = \{x\}, v^b = \{x\}$, $w^b = \{v, x\}, x^b = \{v\}$. The boundary nodes of G are $\{v, x\}$.

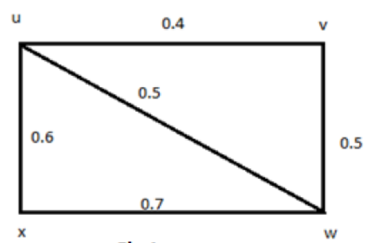

Fig.1

Property 3.3. In a fuzzy-graph all α, γ- strong nodes are the boundary nodes.

Proof. If $x \in u^b$, (i.e.) $u^b = \{x\}$, then $d_s(u, x) \geq d_s(u, w)$ for each neighbour w of x. That is $d_s(u, x) \geq d_s(u, A(x))$. A vertex is a weak or γ-strong vertex if $d(x) < d(A(x)) \forall x \in A(x)$. Combining these two equations we get $d(A(x)) > d(x) > d_s(u_i, x) \geq d_s(u_i, A(x))$ or $d(A(x)) > d_s(u_i, x) > d(x)$ $\geq d_s(u_i, A(x))$ for every vertices $u_i \in V$. We know that if a node v is α-strong if $\{\forall v \in V / d(v) \geq d(A(v))\}$. It is clear that $d(v) \geq d(A(v)) \geq d_s(u_i, v)$ $\geq d_s(u_i, A(v))$. Thus α-strong nodes are also serve as a boundary nodes.

Example 3.4. Consider fuzzy graph G. Here $u^b = \{x\}, v^b = \{u, x\}$, $w^b = \{u, x\}, x^b = \{u, y\}, y^b = \{u, x\}$. Boundary nodes of G are $\{u, x, y\}$.

Here $d(u) = 1.4$, $d(v) = 2.0$, $d(w) = 1.8$, $d(x) = 2.2$, $d(y) = 1.0$, $S_{\alpha}(V) = \{x\}$, $\beta(V) = \{v, w\}$, $W_{\gamma}(V) = \{u, y\}$.

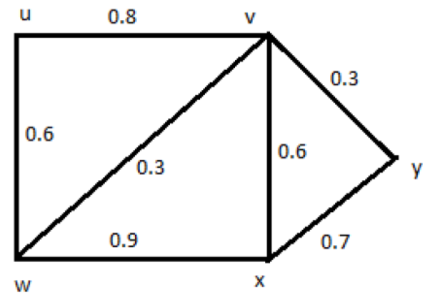

Fig. 2

Thus we clear that α-strong node $\{x\}$ and γ-strong nodes $\{u, y\}$ are boundary nodes.

Definition 3.5 [5]. The fuzzy eccentricity of a vertex u is $e_w(u) = \max_{v \in V} d_s(u, v)$.

Definition 3.6 [5]. The min of the fuzzy eccentricities of all vertices is the fuzzy radius say, $\gamma_s(G) = \min_{u \in V} e_w(u)$.

Definition 3.7 [5]. The max of the fuzzy eccentricities of all the vertices is called the fuzzy diameter say, $d_s(G) = \max_{u \in V} e_w(u)$.

Property 3.8. In a fuzzy-graph β-strong node is a fuzzy radius and γ-strong nodes are fuzzy diameter of a fuzzy graph.

Proof. Property 3.3 states that, fuzzy graph G have boundary nodes which are α and γ-strong nodes. Thus the sum distance of γ-strong nodes are greater than or equal to its sum distance of their neighbours. We know that fuzzy diameter is the max of all eccentricities. Thus γ-strong nodes are fuzzy diameter. Only β-strong nodes have the min eccentricity. Hence β-strong nodes are fuzzy radius.

Example 3.9. Here $(u) = 1.2$, $d(v) = 1.7$, $d(w) = 1.1$, $d(x) = 1.8$. $S_{\alpha}(V) = \{x\}$, $\beta(V) = \{v\}$, $W_{\gamma}(V) = \{u, w\}$.

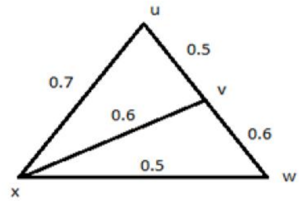

Fig.3

In this example $u^b = \{w\}$, $v^b = \{w, x\}$, $w^b = \{u\}$, $x^b = \{u\}$. Hence the boundary nodes of G are $\{u, x, w\}$. It is clear that α-strong node and γ-strong nodes are the boundary nodes of G.

Here $e_w(u) = 1.1$, $e_w(v) = 0.6$, $e_w(w) = 1.1$, $e_w = 0.7$.

We know that $\gamma_s(G) = \min_{u \in V} e_w(u) = e_w(v) = 0.6$. Thus β-strong node '$v$' is fuzzy radius. And $d_s(G) = \max_{u \in V} e_w(u) = e_w(u) = e_w(u) = 1.1$. Hence γ-strong nodes are fuzzy diameter of G.

Property 3.10. Let G be a fuzzy graph and its underlying crisp graph is complete. Then α-strong and β-strong nodes are the boundary nodes.

Example 3.11. Here $d(u) = 2.0$, $d(v) = 1.5$, $d(w) = 1.8$, $d(x) = 1.9$. $S_\alpha(V) = \{u\}$, $\beta(V) = \{x, w\}$, $W_\gamma(V) = \{v\}$. In this example 3.11, $u^b = \{w\}$, $v^b = \{x\}$, $w^b = \{u\}$, $x^b = \{u\}$. Hence the boundary nodes are $\{u, x, w\}$. It is clear that Then α-strong and β-strong nodes are the boundary nodes.

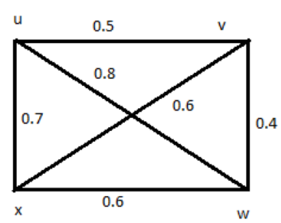

Fig.4

Property 3.12. Let G be a fuzzy-graph with its underlying crisp graph is complete. γ-strong nodes are the fuzzy radius and α-strong and β-strong nodes are the fuzzy diameter.

In example 3.11 $e_w(u) = 0.8$, $e_w(v) = 0.6$, $e_w(w) = 0.8$, $e_w(x) = 0.7$.

We know that $\gamma_s(G) = \min_{u \in V} e_w(u) = e_w(v) = 0.6$. Thus γ-strong node '$v$' is fuzzy radius. And $d_s(G) = \max_{u \in V} e_w(u) = e_w(u) = e_w(w)$. Hence α-strong and β-strong nodes are fuzzy diameter of G.

Property 3.13. If a fuzzy-graph G have more than one β-strong nodes then the highest degree β-strong node is fuzzy radius.

Example 3.14. Here $e_w = 1.4$, $e_w(v) = 0.8$, $e_w(w) = 1.6$, $e_w(x) = 1.4$, $e_w(y) = 1.6$. The vertex 'v' has the min fuzzy eccentricity. Thus the fuzzy radius is 'v', (i.e.) β-strong node. But in this example we have two β-strong nodes, say $\{w, v\}$. Degree of these nodes are $d(w) = 1.8$, $d(v) = 2.0$. Thus the highest degree node belong to fuzzy radius.

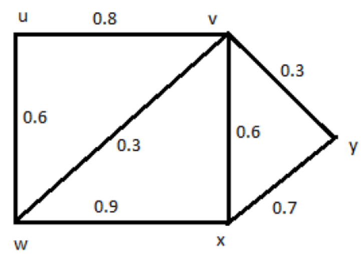

Fig.5

4. Conclusion

The graphs without vertices are meaningless. In the similar way in a fuzzy graph nodes are the most important for finding many applications in more areas like networking. Finding the nodes on the network boundary is the need for correct operation in wireless applications. Focusing on these applications a research under boundary nodes in a fuzzy graph are carried out. But finding boundary nodes in small order fuzzy graphs is very tedious way. For simplifying this process of finding boundary nodes, we can use the three strong nodes α-strong and β-strong, γ-strong. We can find these three nodes very easily as per the definitions. It is observed and verified that α-strong and γ-strong nodes are served as a boundary nodes. But if we consider a fuzzy graph G with its underlying crisp graph is complete then α-strong and β-strong nodes are served as a boundary nodes.

References

[1] K. R. Bhutani and A. Rosenfeld, Fuzzy end nodes in fuzzy graphs, Information Sciences 152 (2003), 323-326.

[2] G. Chartr and P. Zang, Introduction to Graph Theory, Tata McGraw Hill Edition, 2006.

[3] F. Harary, Graph Theory, Addition-Wesley Publishing Company, Inc., 1969.

[4] J. N. Mordeson and P. S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica-Verlag, 2000.

[5] J. K. Mathew and Sunil Mathew, A characterization of fuzzy self centered graphs, Annals of Fuzzy Mathematics and Informatics 10(2) (2015), 249-260.

[6] Mini Tom and M. S. Sunitha, Boundary and interior nodes in a fuzzy graph using sum distance, Fuzzy Inf. Eng. 8 (2016), 75-85.

[7] A. Nagoor Gani and B. Fathima Kani, Fuzzy vertex order colouring, International Journal of Pure and Applied Mathematics 107(3) (2016), 601-614.

[8] A. Rosenfeld, Fuzzy Graphs, In: L. A. Zadeh, K. S. Fu, M. Shimura (Eds.), Fuzzy Sets and Their Application to Cognitive and Decision Processes, Academic Press, New York, 1975, pp77-95.

[9] R. T. Yeh and S. Y. Bang, Fuzzy Relations, Fuzzy Graphs and Their Application to Clustering Analysis, Academic Press, New York (1975), 125-149.

[10] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965), 338-353.