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A NEW PERCEPTION ON NANO CONTINUITY AND NANO IRRESOLUTES

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Abstract

This article's main goal is to investigate a new type of Náno-continuity and Náno-irresolutes for the Náno-near open sets, namely Náno-JD* open sets, which rely primarily on Náno-g interior and Náno-g closure operators. Fundamental properties of these functions are studied and compared to the previous ones. It turns out that every Náno continuous, Náno semi continuous, Náno semi* continuous and Náno continuous functions are Náno JD* continuous functions. In addition, Náno JD* irresolutes are defined, as well as some results involving their characterization are deduced.

1. Introduction

One of the fundamental ideas of Náno topology is the Náno continuity. M. Lellis [4] introduced the concept of Náno topology in the year 2013. In 2014, Recently, S. Jackson and T. Gnanapoo [3] established a new form of Náno generalised open sets namely Náno JD* open sets in Náno topological spaces.

This article bears the main aim of recognising the characterisations of Náno JD* continuity and Náno JD* irresolutes in Náno topological spaces

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and its analogies with other existing concepts. Throughout this paper N_n int and $N_n cl$ represents Nano interior and Nano closure operators.

2. Preliminaries

Prior to proceeding, let me briefly review some basic notions in Náno topological spaces.

Definition 2.1 [4]. If $(M, \tau(P))$ is a Náno topological space and $J \subseteq M$. We claim J to be

(i) Náno semi-open if $J \subseteq N_n cl(N_n \operatorname{int}(J))$.

(ii) Náno pre-open if $J \subseteq N_n \operatorname{int}(N_n cl(J))$.

(iii) Náno α-open if $J \subseteq N_n \operatorname{int}(N_n cl(N_n \operatorname{int}(J)))$.

Definition 2.2 [3]. For a Náno topological space $(M, \tau(P))$ and $J \subseteq M$, we say J to be

(i) Náno semi-open if $J \subseteq N_n cl^*(N_n \operatorname{int}(J))$.

(ii) Náno semi*-closed if $M \setminus J$ is Náno Semi*-open.

(iii) Náno pre*-open if $J \subseteq N_n \operatorname{int}^*(N_n cl(J))$.

(iv) Náno pre*-closed if $M \setminus J$ is nano Pre*-open.

(v) Náno α^*-open if $J \subseteq N_n \operatorname{int}^*(N_n cl(N_n \operatorname{int}^*(J)))$.

(vi) Náno JD* open if $J \subseteq N_n \operatorname{int}^*(N_n cl^*(J)) \cup N_n cl^*(N_n \operatorname{int}(J))$.

(vii) Náno JD* closed if $N_n \operatorname{int}^*(N_n cl^*(J)) \cap N_n cl^*(N_n \operatorname{int}(J)) \subseteq J$.

3. Nano JD* Continuous Functions

In this segment we define Nano JD*-continuous functions, a new notion of Nano continuous functions and have studied its remarkable characterizations in Nano topological spaces.

Definition 3.1. Let $(M, \tau(P))$ and $(N, \tau(Q))$ be two Nano topological spaces. Then a mapping $h : (M, \tau(P)) \to (N, \tau(Q))$ is claimed to be Nano JD* continuous if $h^{-1}(J)$ is Nano JD* open in M for every Nano open set J in N.

Example 3.2. Here we have given an example for Náno JD* continuous.

Let $M = \{e, r, w, v, b\}$ with $M \setminus R = \{\{e, r\}, \{w, v\}, \{b\}\}$ and $X = \{r, b\}$.

Then the Náno topology $\tau_R(P) = \{M, \phi, \{b\}, \{e, r\}, \{e, r, b\}\}$

Let $N = \{z, s, r, t, k\}$ with $N/R' = \{\{z\}, \{k, s\}, \{r, t\}\}$ and $Q = \{z, s, k\} \subseteq N$. Then $\tau_R(Q) = \{N, \varphi, \{z, s, k\}\}$. Define $h : M \to N$ as $h(e) = k; h(r) = z; h(w) = s; h(v) = r; h(b) = t$. Then $h^{-1}(z, s, k) = \{r, w, e\}$, which is Náno JD* open. Therefore f is Náno JD* continuous.

Theorem 3.3. *Let $(M, \tau(P))$ and $(N, \tau(Q))$ be Náno topological spaces and $h : (M, \tau(P)) \to (N, \tau(Q))$ be a mapping, then*

(i) *Náno continuous function is Náno JD* continuous function*

(ii) *Náno semi continuous function is Náno JD* continuous function*

(iii) *Náno semi* continuous function is Náno JD* continuous function*

(iv) *Náno α continuous function is Náno JD* continuous function.*

Proof. (i) Assume $h : (M, \tau(P)) \to (N, \tau(Q))$ to be Náno continuous and J be Náno open in N. Then $h^{-1}(J)$ is Náno open in M. Since any Náno open set is Náno JD* open [3], $h^{-1}(J)$ is Náno JD*-open in M, h is Náno JD* continuous.

(ii) Assume $h : (M, \tau(P)) \to (N, \tau(Q))$ to be Náno semi continuous and J be Náno open in N. Then $h^{-1}(J)$ is Náno semi open in M. Since any Náno semi open set is Náno JD* open [3], $h^{-1}(J)$ is Náno JD*-open in M, h is Náno JD*continuous.

(iii) Assume $h : (M, \tau(P)) \to (N, \tau(Q))$ to be Náno semi* continuous and B be Náno open in N. Then $h^{-1}(B)$ is Náno semi* open in M. Since any Náno semi* open set is Náno JD* open [3], $h^{-1}(B)$ is Náno JD*-open in M, h is

Náno JD* continuous.

(iv) Let $h : (M, \tau(P)) \to (N, \tau(Q))$ be Náno α continuous and J be Náno open in N. Then $h^{-1}(J)$ is Náno α open in M. Since any Náno α open set is Náno JD* open [3], $h^{-1}(J)$ is Náno JD*-open in M, h is Náno JD* continuous.

Remark 3.4. The reverse implications isn't true as evidenced by the illustration below.

Example 3.5. $M = \{l, r, w, v, i\}$ with $M/R = \{\{l, r, w\}, \{v\}, \{i\}\}$ and $P = \{r, w, v\}$. Then the topology $\tau_R(X) = \{U, \varphi, \{l, r, w, v\}, \{l, r, w\}, \{v\}\}$. Then $N = \{z, s, r, t, k\}$ with $N/R' = \{\{y\}, \{z\}, \{v\}, \{u, x\}\}$ and $Q = \{u, y, z\} \subseteq N$. Then $\tau_R(Q) = \{N, \varphi, \{z, r\}, \{t, k\}, \{z, r, t, k\}\}$. Define $h : M \to N$ as $h(l) = z, h(r) = s, h(w) = r, h(v) = t; h(i) = k$. Then $h^{-1}(\{t, k\}) = \{v, i\}$, $h^{-1}(\{z, r\}) = \{l, w\}, h^{-1}(z, r, t, k) = \{l, w, v, i\}$. Here h is Náno JD* continuous but not Náno continuous, Náno semi continuous, Náno semi* continuous and Náno α continuous.

From the above observations we have the following implications.

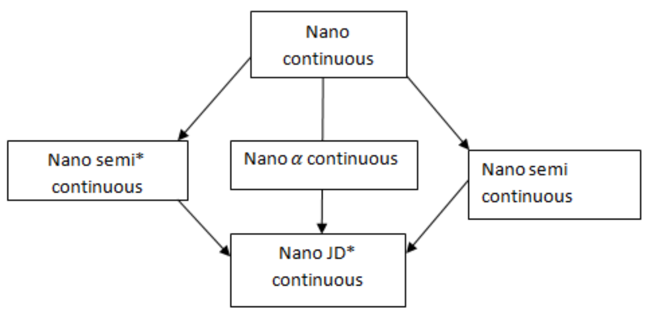

Figure 1. Nano JD* Continuity-An Outline view.

We present the characterization of Náno JD* continuous in terms of Náno JD* closure in the following theorems.

Theorem 3.6. *A mapping* $k : (M, \tau(P)) \to (N, \tau(Q))$ *is Náno JD* continuous iff* $k(NJD^{*}cl(J)) \subseteq Ncl(k(J))$ *for every subset* J *of* M.

Proof. Let k be Náno JD*-continuous and $J \subseteq M$. Then $k(J) \subseteq N$. Since k is Náno JD* continuous and $N_n clk(J)$ is also Náno closed in N, $k^{-1}(N_n clk(J))$ is also Náno JD* closed in M. Since $k(J) \subseteq N_n clkj(J)$, $k^{-1}(k(J)) \subseteq k^{-1}(N_n clk(J))$, then $NJD^* cl(J) \subseteq NJD^* cl[k^{-1}(N_n clk(J))]$ $k^{-1}(N_n clk(J))$.

Thus, $N_n JD^* cl(J) \subseteq k^{-1}(N_n clk(J))$. Therefore, $k(NJD^* cl(J)) \subseteq N_n clk(J)$ for every nano subset J of M. Conversely, $k(N_n cl(J)) \subseteq N_n clk(J)$ for every subset J of M. If K is Náno closed in N, since $k^{-1}(K) \subseteq k(NJD^* cl(k^{-1}(K)))$ $\subseteq N_n cl(k(k^{-1}(K))) = N_n cl(K)$. That is, $NJD^* cl(k^{-1}(K)) \subseteq k^{-1}(N_n cl(K))$ $= k^{-1}(K)$, since K is Náno closed. Thus $NJD^* cl(k^{-1}(K)) \subseteq k^{-1}(K)$. Therefore, $k^{-1}(K)$ is Náno JD* closed in M for every Náno closed set K in N. That is, k is Náno JD*continuous.

Remark 3.7. If $h : (M, \tau(P)) \to (N, \tau(Q))$ is Náno JD* Continuous, then $h(NJD^* cl(J))$ need not be necessariy equal to $N_n cl(h(J))$.

Example 3.8. $M = \{l, r, w, v, i\}$, $M/R = \{\{l, w\}, \{r, i\}, \{v\}\}$ $P = \{r, v\}$. Then $\tau_R(P) = \{\varphi, M, \{v\}, \{r, i\}, \{r, v, i\}\}$. Let $N = \{z, s, r, t, k\}$ with $N/R' = \{\{z\}, \{k, s\}, \{r, t\}\}$ and $Q = \{z, s, k\} \subseteq N$. Then $\tau_R(Q)$ $= \{N, \varphi, \{k, s, z\}\}$. Define $h : M \to N$ as $h(l) = z; h(r) = k; h(w) = s; h(v) = r;$ $h(i) = t;$ $h^{-1}(M) = N; h^{-1}(\varphi) = \varphi; h^{-1}(\{z, s, k\}) = \{l, r, w\}$ which is Náno JD* open in M. Therefore h is Náno JD* continuous in M, Let $J = \{l, r, w\} \subseteq N$. Then $h(NJD^* cl(J)) = h(\{l, r, w\}) = \{z, s, k\}$. But $N_n clh(J) = Ncl\{z, s, k\}$ $= N$. Therefore $h(NJD^* cl(J)) \neq Ncl(h(J))$ even though h is Náno JD* continuous.

Theorem 3.9. *A function* $k : (M, \tau(P)) \to (N, \tau(Q))$ *is Náno JD* continuous iff* $N_n JD^*(k^{-1}(D)) \subseteq k^{-1}(N_n cl(D))$.

Proof. If k is Náno JD* continuous and $D \subseteq N$, then $N_n cl(D)$ is Náno closed in N and hence $k^{-1}(N\, n\, cl(D))$ is Náno JD* closed in M. Therefore, $NJD^{*}cl[k^{-1}(N_n cl(D))] = k^{-1}N_n cl(D)$. Since, $D \subseteq NJD^{*}cl(D)$, $k^{-1}(D) \subseteq k^{-1}N_n cl(D)$. $NJD^{*}cl(k^{-1}(D)) \subseteq NJD^{*}cl(k^{-1}N_n cl(D)) = k^{-1}N_n cl(D))$. That is $NJD^{*}c(k^{-1}(D)) \subseteq k^{-1}N_n cl(D))$. Conversely, let for every subset D of N, if D is Náno closed in N, then $N_n cl(D) = D$. By assumption $NJD^{*}cl(k^{-1}(D)) \subseteq k^{-1}N_n cl(D) = k^{-1}(D)$. But $k^{-1}(D) \subseteq NJD^{*}cl(k^{-1}(D))$. Therefore, $NJD^{*}cl(k^{-1}(D)) = k^{-1}(D)$. That is $k^{-1}(D)$ is Náno JD* closed in M for every Náno closed set D in N. Therefore k is Náno JD*-continuous on M.

The theorems that follow define Náno JD* continuous functions in terms of the inverse image of Náno JD* interior.

Theorem 3.10. *A function* $k : (M, \tau_R(P)) \to (N, \tau_R(Q))$ *is Náno JD* continuous iff* $k^{-1}(N_n \operatorname{int}(D)) \subseteq NJD^{*} \operatorname{int}(k^{-1}(D))$ *for every subset D of N.*

Proof. If k is Náno JD* continuous and $D \subseteq N$, then $N_n \operatorname{int}(D)$ is Náno open in N and $k^{-1}(N_n \operatorname{int}(D))$ is Náno JD* open in M. Therefore, $NJD^{*} \operatorname{int}[k^{-1}(N_n \operatorname{int}(D))] = k^{-1}(N_n \operatorname{int}(D))$. Also $N_n \operatorname{int}(D) \subseteq D$ implies that $k^{-1}N_n \operatorname{int}(D) \subseteq k^{-1}(D)$. Therefore, $NJD^{*} \operatorname{int}(k^{-1}(N_n \operatorname{int}(D))) \subseteq NJD^{*} \operatorname{int}(k^{-1}(D))$. That is, $k^{-1}(N_n \operatorname{int}(D)) \subseteq N_n JD^{*} \operatorname{int}(k^{-1}(D))$. Conversely, let $k^{-1}(N_n \operatorname{int}(D)) \subseteq NJD^{*} \operatorname{int}(D)$ for every $D \subseteq N$. If D is Náno open in N, then $N \operatorname{int}(D) = D$. By assumption $k^{-1}(N_n \operatorname{int}(D)) \subseteq NJD^{*} \operatorname{int}(k^{-1}(D))$. Thus $k^{-1}(D) \subseteq N_n JD^{*} \operatorname{int}(k^{-1}D)$. But $NJD^{*} \operatorname{int}(k^{-1}(D)) \subseteq k^{-1}(D)$. Therefore, $k^{-1}(D) = NJD^{*} \operatorname{int}(k^{-1}(D))$. That is, $k^{-1}(D)$ is Náno JD* open in M for every Náno open set in N. Therefore, k is Náno JD* continuous on M.

Remark 3.11. Reverse implications of the above theorems 3.10 and 3.11 isn't true as evidenced by the illustrations below.

Example 3.12. $M = \{l, r, w, v\}$ with $M \setminus R = \{\{l, v\}, \{r\}, \{w\}\}$. Let $P = \{l, w\} \subseteq M$. Then $\tau_R(P) = \{M, \phi, \{w\}, \{l, v\}, \{l, w, v\}\}$. $N = \{r, w, v, l\}$

with $N \setminus R' = \{\{r\}, \{w\}, \{v\}, \{l\}\}; Q = \{r, l\} \subseteq N$. Then $\tau_R(Q) = \{N, \phi, \{l, r\}\}$. Define $h : M \to N$ as $h(l) = r; h(r) = w; h(w) = v; h(v) = l$. Here h is Náno JD* continuous on M.

1. $D = \{l, v\} \subseteq N$. Then $h^{-1}(N_n cl(D)) = h^{-1}(N) = M$ and $N_n JD^* cl(f^{-1}(D)) = N_n JD^* cl(\{r, w\}) = \{r, w, v\}$. Therefore $N_n JD^* cl(h^{-1}(D)) \neq h^{-1}(N_n cl(D))$.

2. $B = \{r, w\} \subseteq N$. Then $h^{-1}(N_n \operatorname{int}(\{r, w\})) = h^{-1}(\{r, w\}) = \{l, r\}$, $N_n JD^* cl(h^{-1}(D)) = N_n JD^* \operatorname{int}(h^{-1}\{r, w\}) = N_n JD^* \operatorname{int}(\{l, r\}) = \{l\}$. Therefore $h^{-1}(N_n \operatorname{int}(D)) \neq N_n JD^* \operatorname{int}(h^{-1}(D))$.

Theorem 3.13. *If $(M, \tau_R(P))$ and $(N, \tau_R(Q))$ are Náno topological spaces with respect to $P \subseteq M$ and $Q \subseteq N$ respectively, then for any function $h : M \to N$ respectively, the following conditions are equivalent*:

a. $h : (M, \tau(P)) \to (N, \tau(Q))$ *is Náno JD* continuous*

b. $h(NJD^* cl(J)) \subseteq N_n cl(h(J))$ *for every subset J of M.*

c. $NJD^* cl(h^{-1}(J)) \subseteq h^{-1}(N_n cl(J))$ *for every subset J of N.*

Proof. (a) $\Rightarrow$ (b) Let h be Náno JD* continuous and $J \subseteq M$. Then $h(J) \subseteq N$. Since h is Náno JD* continuous and $N_n cl(h(J))$ is Náno closed in N, $h^{-1}(N_n cl(h(J)))$ is Náno JD* closed in M. Since $h(J) \subseteq N_n cl(h(J))$, $h^{-1}h(J) \subseteq h^{-1}(N_n cl(h(J)))$ then $NJD^* cl(h(J)) \subseteq NJD^* cl[h^{-1}(N_n cl(h(J))]$ $= h^{-1}(N_n cl(h(J)))$. Thus $NJD^* cl(J) \subseteq h^{-1}(N_n cl(h(J)))$. Therefore, $h(NJD^* cl(J)) \subseteq N_n cl(h(J))$ for every subset J of M.

(b) $\Rightarrow$ (c) Let $h(NJD^* cl(J)) \subseteq N_n cl(h(J))$ and $J = h^{-1}(D) \subseteq M$ for every subset $D \subseteq N$. Since $h(NJD^* cl(J)) \subseteq N_n cl(h(J))$ we have, $h(NJD^* cl(h^{-1}(D))) \subseteq N_n cl(h(h^{-1}(D))) \subseteq N_n cl(D)$. That is, $h(NJD^* cl(h^{-1}(D)))$ $\subseteq N_n cl(D)$ which implies that $NJD^* cl(h^{-1}(D)) \subseteq h^{-1}(N_n cl(D))$ for every subset $D \subseteq N$.

(c) $\Rightarrow$ (a) Let $NJD^{*}cl(h^{-1}(D)) \subseteq h^{-1}(N_n cl(D))$ for every subset D of N. If D is Náno closed in N, then $N_n cl(D) = D$. By assumption, $NJD^{*}cl(h^{-1}(D)) \subseteq h^{-1}(N_n cl(D)) = h^{-1}(D)$. Thus $NJD^{*}cl(h^{-1}(D)) \subseteq h^{-1}(D)$. But $h^{-1}(D) \subseteq NJD^{*}cl(h^{-1}(D))$. Therefore $NJD^{*}cl(h^{-1}(D)) = h^{-1}(D)$. That is, $h^{-1}(D)$ is Náno JD* closed M for every Náno closed set D in N. Therefore h is Náno JD* continuous on M.

4. Nano JD* Irresolutes

The goal of this section is to discuss and analyse the remarkable characterizations of Náno JD* irresolutes.

Definition 4.1. A map $h : (M, \tau(P)) \to (N, \tau(Q))$ is called Náno JD* irresolute if $h^{-1}(O)$ is Náno JD* open set in $(M, \tau(P))$ for every Náno JD* open set J in $(N, \tau(Q))$.

Theorem 4.2. *A mapping $h : (M, \tau(P)) \to (N, \tau(Q))$ is Náno JD* irresolute then it is Náno JD* continuous.*

Proof. Let $f : (M, \tau(P)) \to (N, \tau(Q))$ be a Náno JD* irresolute map. Let us consider J to be an Náno open set in N. Then h is Náno JD* open set in N. Since h is Náno JD* irresolute map, $h^{-1}(O)$ is Náno JD* open in M. Therefore, h is Náno JD* continuous.

Remark 4.3. Reverse implications need not be true which can be demonstrated as follows.

Example 4.4. Let $M = \{l, r, w, v\}$ with $M/R = \{\{l, v\}, \{r\}, \{w\}\}$ and $P = \{l, w\}$. Then the Náno topology $\tau_R(P) = \{M, \phi, \{w\}, \{l, v\}, \{l, w, v\}\}$. Let $N = \{l, r, w, v\}$ with $M/R' = \{\{r\}, \{w\}, \{v\}, \{l\}\}$ and $Q = \{r, v\} \subseteq N$. Then $\tau_R(Q) = \{N, \phi, \{l, r\}\}$. Define $k : M \to N$ as $f(l) = v; f(r) = w; f(w) = l; f(v) = r$. Here k is Náno JD* continuous but not Náno JD* irresolute.

Theorem 4.5. *Assume $k : (M, \tau(P)) \to (N, \tau(Q))$ to be a Náno JD* Náno* *map from M into N and N be a Náno JD* -* $T_{\frac{1}{2}}$ *space. Then k is Náno JD* irresolute.*

Proof. Assume J to be Náno JD* open set in N. Since N is Náno JD*- $T_{\frac{1}{2}}$ space, J is a Náno open set in N. Since k is Náno JD* continuous, $k^{-1}(J)$ is Náno JD* open in M. Hence k is a Náno JD* irresolute function.

Remark 4.6. The Náno JD* irresolute and Náno g irresolute maps are independent of each other which is demonstrated below.

Example 4.7. Let $M = \{l, r, w, v\}$ with $M \setminus R = \{\{l, v\}, \{r\}, \{w\}\}$ and $P = \{l, w\}$. Then the Náno topology $\tau_R(P) = \{U, \phi, \{w\}, \{l, v\}, \{l, w, v\}\}$. Let $N = \{l, r, w, v\}$ with $N \setminus R' = \{\{l, v\}, \{r\}, \{w\}\}$ and $Q = \{r, v\} \subseteq N$. Then $\tau_R(Q) = \{N, \phi, \{r\}, \{l, v\}, \{l, r, v\}\}$. Define $h : M \to N$ as $h(l) = l; h(r) = r; h(w) = v; h(v) = w$. Here h is Náno g irresolute but not Náno JD* irresolute.

Example 4.8. Let $M = \{l, r, w, v\}$ with $M \setminus R = \{\{l\}, \{v\}, \{r, w\}\}$ and $P = \{l, w\}$. Then the Náno topology $\tau_R(P) = \{U, \phi, \{l, r, w\}, \{l\}, \{r, w\}\}$. Let $N = \{l, r, w, v\}$ with $N \setminus R' = \{\{l, v\}, \{r\}, \{w\}\}$ and $Q = \{a, c\} \subseteq V$. Then $\tau_R(Q) = \{V, \phi, \{w\}, \{l, v\}, \{l, w, v\}\}$. Define $s : M \to N$ as $s(l) = l; s(r) = r; s(w) = w; s(v) = v$. Here s is Náno JD* irresolute but not Náno g irresolute.

Theorem 4.9. *Let* $(M, \tau_R(P)), (N, \tau_R(Q)), (O, \tau(T))$ *be any Náno topological spaces. For any Náno JD* irresolute map* $s : (M, \tau(P)) \to (N, \tau(Q))$ *and* $d : (N, \tau(Q)) \to (O, \tau(T))$ *be any Náno JD* continuous map. Then their composition* $(d \circ s) : (M, \tau(P)) \to (O, \tau(T))$ *is Náno JD* continuous.*

Proof. Let J be any Náno open set in O. Since d is Náno JD* continuous, $d^{-1}(J)$ is Náno JD* open in N and s is Náno JD* irresolute, $s^{-1}(d^{-1}(J))$ is Náno JD* open in M. But $s^{-1}(d^{-1}(J)) = (d \circ s)^{-1}(J)$. Therefore, $(s \circ d) : (M, \tau(P)) \to (O, \tau(T))$ is Náno JD* continuous.

Theorem 4.11. *Let* $s : (M, \tau(P)) \to (N, \tau(Q))$ *and*

$d:(N, \tau(Q)) \to (O, \tau(T))$ *be any two functions, then*

(i) $(s \circ d):(M, \tau(P)) \to (O, \tau(T))$ *is Náno JD* continuous if d is Náno continuous and s is Náno JD* continuous.*

(ii) $(d \circ s):(M, \tau(P)) \to (O, \tau(T))$ *is Náno JD* irresolute if both s and d is Náno JD* irresolute.*

Proof.

(i) Let J be a Náno open set in $(O, \tau(T))$. Since d is Náno continuous, we get $d^{-1}(J)$ to be nano open in $(N, \tau(Q))$, Also s is Náno JD* continuous, $s^{-1}(d^{-1}(0)) = (d \circ s)^{-1}(0)$ is Náno JD* open in $(M, \tau(P))$. Hence $(d \circ s)$ is Náno JD* continuous.

(ii) Let J be a Náno JD* open set in $(O, \tau(T))$. Since d is Náno JD* irresolute, we get $d^{-1}(J)$ to be Náno JD* open in $(N, \tau(Q))$. Also f is Náno JD* irresolute, this implies, $s^{-1}(d^{-1}(0)) = (dos)^{-1}(J)$ is Náno JD* open in $(M, \tau(Q))$. Hence $(d \circ s)$ is Náno JD* irresolute.

5. Conclusion

A new horizon for Náno continuous functions and Náno irresolutes were studied and analysed in this paper. We were able to obtain some important aspects and primary properties that were related to these functions. Subsequently, the interconnection between the weaker forms of Náno continuous functions and Nano JD* continuous functions are depicted. Furthermore, the concept of Náno irresolutes were extended to Náno JD* irresolutes and their remarkable characterizations are discussed.

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