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CRACKED AND NON CRACKED PLATE ANALYSIS USING EXTENDED FINITE ELEMENT AND FINITE ELEMENT FORMULATION

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Abstract

This paper presented , the force of extended finite element method which allow us to modelize discontinuity thinks that can not be done by finite element method, through the example of the addition plate element to freeware FEM-Object, and Object-Oriented OpenXFEM++, an Object-Oriented (C++)finite element program, and extended finite element respectively, how Object-Oriented approaches, as opposed to procedural approaches, make extended and finite element codes more compact, more modular and versatile but mainly more easily expandable. The fundamental traits of Object-Oriented programming are first briefly reviewed, and it is shown how such an approach simplifies the coding process. Then, the isotropic and orthotropic cracked and non-cracked plate formulations used are given and the discretized equations developed. Numerical examples using the newly created plate element are shown.

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1. Introduction

In recent years, easy maintenance, the extensibility and flexibility of the finite element code, encouraged many researchers to change from procedural to object oriented programming in order to make the finite element software more flexible. The Object-Oriented programming approach appeared in the seventies and found its first convivial concretisation with the Small Talk language at the beginning of the eighties. Applications in the finite element field appeared around 1990 [9, 10, 11, 12], and since then its use has spread. Object-Oriented approaches yield more modular programs than the standard procedural approach. In this article, an Object-Oriented approach of the finite element method for the study of bending of isotropic and orthotropic plates in dynamic is presented; freeware FEM Object is used as a basis for this development [13] and Open XFEM++ [3].

2. Object Oriented Programming

Procedural programming tends to show its limits especially for large finite-element programs, wich contain hundreds of thousands of lines of code.

Such procedural programs are generally written in FORTRAN. This code contains a significant number of data structures which are accessible throughout the program, creating a loss of flexibility. It thus becomes difficult to modify the existing code and extend it to new uses. Loss of flexibility shows up in different manners:

- detailed knowledge of the program is required to work on a small part of the code;
- the re-use of the code is difficult;
- a small change in the data structures can influence the whole system;
- many interdependences between the components of the design are dissimulated and difficult to establish;
- the integrity of the data structures is not protected.

3. Recall of extended finite element and finite element plate theory

Let's begin with finite element plate formulation. The orthotropic plate formulation for finite elements in bending-shear is based on the theory of Reissner-Mindlin. Their conformity requires only C^0 continuity of w, β_x and β_y [1, 5]. We consider isoparametric quadrilateral elements.

$$\beta_x = N^T(\xi, \eta)\widehat{\beta}_x \;\; \beta_x = N^T(\xi, \eta)\widehat{\beta}_x \tag{1}$$

The strain vector can be expressed:

$$\{\varepsilon\} = \{\{\varepsilon_f\}^t, \{\varepsilon_f\}^t\} = \{Z\{\chi\}^t, \{\gamma\}^t\}^t \tag{2}$$

Contribution of bending and shear effect respectively is

$$\{\varepsilon_f\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = z\{\chi\} = z\begin{Bmatrix} \dfrac{\partial\beta_x}{\partial x} \\ \dfrac{\partial\beta_y}{\partial y} \\ \dfrac{\partial\beta_x}{\partial y} + \dfrac{\partial\beta_y}{\partial x} \end{Bmatrix} \{\varepsilon_c\} = \{\gamma\} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \beta_x + \dfrac{\partial w}{\partial x} \\ \beta_y + \dfrac{\partial w}{\partial y} \end{Bmatrix} \tag{3}$$

The standard interpolation functions used are the usual interpolation functions of isoparametric quadrilaterals. In the case of the linear quadrilateral, one gets:

$$N^T = [N_1 \quad N_2 \quad N_3 \quad N_4] \text{ with } N_i(\xi, \eta) = \frac{1}{4}(1 + \xi\,\xi_i)(1 + \eta\eta_i) \tag{4}$$

Bending and shearing deformations matrices interpolation are:

$$\{\chi\} = \{\overline{\varepsilon}_f\} = \begin{bmatrix} 0 & \dfrac{\partial N^T}{\partial x} & 0 \\ 0 & 0 & \dfrac{\partial N^T}{\partial y} \\ 0 & \dfrac{\partial N^T}{\partial y} & \dfrac{\partial N^T}{\partial x} \end{bmatrix} \begin{Bmatrix} W \\ \widehat{\beta}_x \\ \widehat{\beta}_y \end{Bmatrix} \{\varepsilon_c\} = \{\gamma\} = \begin{bmatrix} \dfrac{\partial N^T}{\partial x} & N^T & 0 \\ \dfrac{\partial N^T}{\partial y} & 0 & N^T \end{bmatrix} \begin{Bmatrix} W \\ \hat{\beta}_x \\ \hat{\beta}_y \end{Bmatrix}$$

$$\{\varepsilon_c\} = \{\gamma\} = \begin{bmatrix} \dfrac{\partial N^T}{\partial x} & N^T & 0 \\ \dfrac{\partial N^T}{\partial y} & 0 & N^T \end{bmatrix} \begin{Bmatrix} W \\ \hat{\beta}_x \\ \hat{\beta}_y \end{Bmatrix} \tag{5}$$

Deformation energy makes allow us to calculate the stiffness matrix [3, 4], we have: $U = U_F + U_C$

$$U = \frac{1}{2}\int_{S^e}\{\chi\}^T[D_f]\{\chi\}\,dx\,dy + \frac{1}{2}\int_{S^e}\{\gamma\}^T[D_c]\{\gamma\}dx\,dy = \frac{1}{2}\{q\}^T[K]\{q\} \quad (6)$$

where

$$[D_f] = \frac{h^3}{12}\begin{bmatrix} \frac{E_1}{1-\nu_{21}\nu_{12}} & \frac{E_2\nu_{12}}{1-\nu_{21}\nu_{12}} & 0 \\ \frac{E_2\,\nu_{12}}{1-\nu_{21}\nu_{12}} & \frac{E_2}{1-\nu_{21}\nu_{12}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} [D_c] = hk\begin{bmatrix} G_{13} & 0 \\ 0 & G_{23} \end{bmatrix} \quad (7)$$

$[D]$ is the constitutive matrix, E_1, E_2 the young moduli in two directions $(x)(y)$ ν_{12}, ν_{21} the Poisson's ratios, G_{12}, G_{13}, G_{23} shear moduli, k coefficient of correction of transverse shear. Minimization of expression (6), leads to:

$$[K] = \int_{-1}^{+1}\int_{-1}^{+1}[\overline{\beta}_f]^T[D_f][\overline{\beta}_f]\det[J]\,d\xi\,d\eta + \int_{-1}^{+1}\int_{-1}^{+1}[\beta_\gamma]^T[D_c][\beta_\gamma]\det[J]d\xi d\eta \quad (8)$$

For the case of discontinues structure Extended finite element method made it possible to locate the discontinuity (crack, inclusion,...) independently of the mesh, it is able to reproduce the state of singular stress at the crack tip [2, 4, 6, 7, 8].

The approximate field of displacement for modeling in 2D is given by the following expression:

$$u^h(x) = \sum_{I\in N} N_I(x)u_I + \sum_{J\in N_{er}} \tilde{N}_J(x)(H(x) - H(x_J))a_J + \sum_{K\in N_{tip}} \tilde{N}_K(x)$$

$$\sum_{\alpha=1}^{4}(B_\alpha(x) - B_\alpha(x_K))b_{\alpha K}$$

N: is the set of nodes constituting the mesh. N_{er} : is the set of nodes enriched by the discontinuity (the set of nodes belonging to the elements divided by the crack). N_{tip} : is the set of nodes enriched by the branch function (the set of nodes belonging to the element containing the crack tip). $N_I(x),\ \tilde{N}_j(x),\ \tilde{N}_k(x)$: the standard FE shape function.

u_I : the classical degree of freedom for node i. a_J : the additional degree of freedom for the discontinuity function. $b_{\alpha K}$: the additional degree of freedom for the branch function.

$H(x)$: modified heaviside step function which takes on the value: $H(x) = \begin{cases} +1 & \text{above the crack} \\ -1 & \text{bellow the crack} \end{cases}$ $B_\alpha(x)$: the branch function.

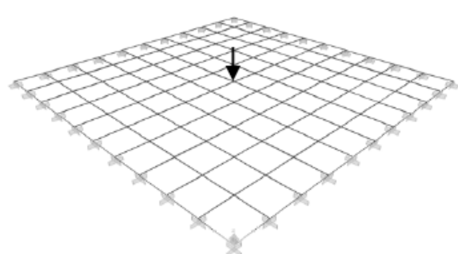
The enrichment functions at the bottom of the crack for an elastic isotopic material are given by:

$$B \equiv [B_1,\ B_2,\ B_3,\ B_4] = \left[\sqrt{r}\sin\frac{\theta}{2},\ \sqrt{r}\cos f\,\frac{\theta}{2},\ \sqrt{r}\sin\frac{\theta}{2}\cos\theta,\ \sqrt{r}\cos\frac{\theta}{2}\cos\theta\right] \tag{12}$$

Where r and θ are polar coordinates in the system of local crack tip coordinates.

4. Application

The first numerical validation examples are an orthotropic square plate, embedded on its four sides, subjected to a concentrated center load. The geometrical and mechanical data structure are given in figure 1, as well as plate model. In order to study convergence, figure 2 show the evolution in time of orthotropic plate response which is simply supported on its four sides. Our numerical results are compared with SAP2000 software as we can see on figure 2 the FEMOBJ and SAP2000 results are very close.



Data: $E1 = 2.068 \text{ X } 10^{11}$ N/m^2

$E2 = E1/15$

$G_{12} = G_{23} = G_{13} = 6.055 \text{ X} 10^{8}$ N/m^2

$a = 2$m $h = 0.01$m $\rho = 7800$ kg/m^3

$\gamma = 0.5$ $\beta = 0.25$ (newmark) $\nu = 0.3$

$DT = 0.0025$ s $P = 800$ N

Figure 1. Orthotropic square plate embedded on its four sides subjected to a concentrate load F.

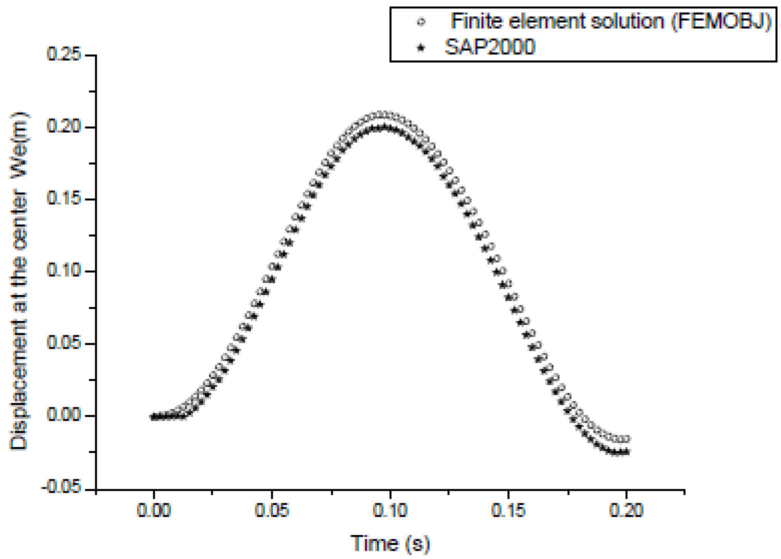


Figure 2. Dynamic response of finite element solution for orthotropic square plate embedded on its four sides.

The second numerical validation example is a homogeneous isotopic rectangular cracked plate, with $W = 10\,\text{cm}$ as width and height $2H = 20\,\text{cm}$, stressed by a distributed load $s = 100\,\text{KN}/m$, whose elastic properties are as follows: modulus of elasticity $E = 7 \times 106$ MPA, Poisson's ratio $\upsilon = 0.3$. The plate has a right crack on the left side of a length $a = 3\,\text{cm}$. The goal is the study position crack edge influence on stress intensity factor.

The theoretical solution of stress intensity factor of cracked plate is

$$K_I = \sigma_{app}\sqrt{\pi a} f\left(\frac{a}{w}\right) \tag{13}$$

$$f\left(\frac{a}{w}\right) = 1.12 - 0.231\left(\frac{a}{w}\right) + 10.55\left(\frac{a}{w}\right)^2 - 21.72\left(\frac{a}{w}\right)^3 + 30.39\left(\frac{a}{w}\right)^4 \tag{14}$$

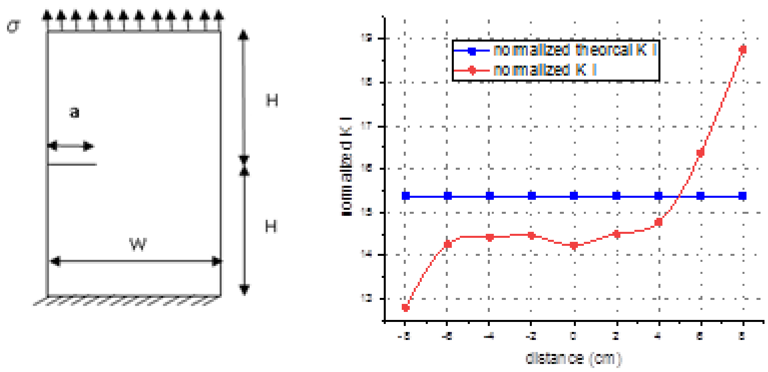
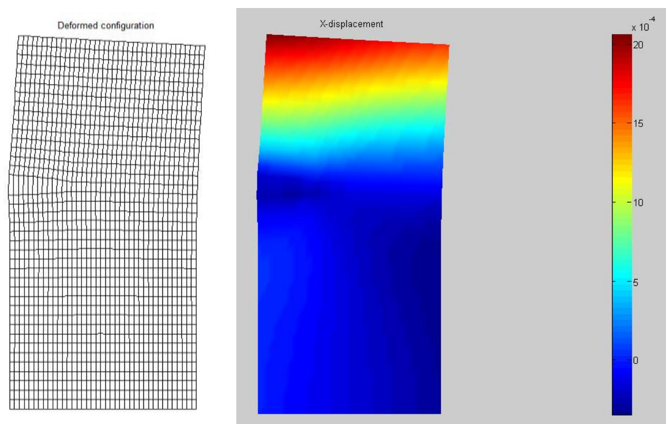


Figure 3. Stress intensity factor value in mode I (KI) in relation to the crack location.



(a) (b)

Figure 4. (a) Plate deformation, (b) Plate displacement according to the y-axis.

Results Discussion

Figure 2 show a clear convergence between finite element object oriented approach with sap2000 software. Figure 3, illustrates the approximation of stress intensity factor value to the theoretical or exact value of a crack assumed at mid-height of the plate, we notice that crack position has a big influence on the results. Concerning figure 4, it shows cracked plate deformation, and displacement value.

6. Conclusion

Through the example of the addition of a plate element in existing C++ code, FEMObject, this paper show as the degree of accuracy in the calculation in the field of fracture and solid mechanics; by the two studied examples in which the results are very satisfying. Through this paper we would like to shows also how Object-Oriented programming makes it possible to easily develop new features in finite element codes, showing such Object-Oriented programs to be more modular, reusable and more easily expandable.

The Object-Oriented approach is shown to offer undeniable advantages compared to earlier programming structures. It offers more generic structures, making it possible to overcome a number of difficulties of the traditional implementations of the Finite Element Method. Object-Oriented Programming also supports the upgrading needs of the finite element method implementation by enhancing modularity and flexibility of evolution. Thanks to the derivation of classes and a high level of abstraction, Object-Oriented Programming can be an effective tool to keep control of the codes of tomorrow. The late binding capability (polymorphism) facilitates adaptability.

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