Advances and Applications in Mathematical Sciences
Volume 21, Issue 3, January 2022, Pages 1343-1358

ON CONTRA λ_g^α-MAPS IN TOPOLOGICAL SPACES

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Abstract

We aim to put forth the concept of contra λ_g^α-continuous maps and contra λ_g^α-irresolute maps as an extension of the definitions-λ_g^α-continuous maps and λ_g^α-irresolute maps respectively. We also derive the dependency and independency relations among the maps. Further, we derive the properties of the defined maps and investigate the composition of mappings among them.

1. Introduction

In the advent of the topological study there were a wide range of research done by many researchers and gained vast knowledge about it. In its order, recently Subhalakshmi and Balamani [11] introduced and analysed the new class of sets called λ_g^α-closed sets in topological spaces. They also have derived the definitions of λ_g^α-closed (λ_g^α-open) maps, λ_g^α-continuous and λ_g^α-irresolute maps in topological spaces as mentioned in [13] and [12] respectively. Later as an extended theory, the study of contra mappings gave

2020 Mathematics Subject Classification: 54C10, 54C20, 54C08, 54C05.

Keywords: topological spaces, λ-closed set, α-closed set, contra λ_g^α-continuous map, contra λ_g^α-irresolute map.

Received October 25, 2021; Accepted November 10, 2021

many interesting results in topology. Initially, Baker [1] introduced the concepts of contra-open maps and contra-closed maps in topological spaces and paved way to derive applications with the respective closed sets. Following this article, Dontchev [7] defined and studied the contra continuous maps which are independent of continuous maps. Further, Caldas et al., [2] introduced the concept of contra λ-continuous maps and studied its properties. Since then, many variations of contra continuity have been investigated by many researchers which gave futuristic approaches in the area of study. In this article, contra λ_g^α-continuous maps and contra λ_g^α-irresolute maps in topological spaces are defined as a special type of λ_g^α-continuous maps and λ_g^α-irresolute maps. Moreover, their properties, dependencies and independencies are also investigated.

2. Preliminaries

Definition "Let (M, μ) be a topological space. A subset A of (M, μ) is called

(i) α-open if $A \subseteq \operatorname{int}(\operatorname{cl}(\operatorname{int}(A)))$ [10]

(ii) λ-closed if $A = L \cap D$, where L is a Λ-set and D is a closed set [8]

(iii) λ_g^α-closed if $cl_\lambda(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (M, μ) [11]

(iv) λ_g^α-clopen if it is both λ_g^α-open and λ_g^α-closed in (M, μ) [14]"

Definition "Let (M, μ) and (N, ν) be the topological spaces. A map $u : (M, \mu) \to (N, \nu)$ is called

(i) Continuous if $u^{-1}(V)$ is closed in (M, μ) for every closed set V of (N, ν). [9]

(ii) Irresolute if $u^{-1}(V)$ is semi-closed in (M, μ) for every semi-closed set V of (N, ν). [6]

(iii) λ-closed if $u(V)$ is λ-closed in (N, ν) for every closed set V in (M, μ). [4]

(iv) λ-continuous if $u^{-1}(V)$ is λ-closed in (M, μ) for every closed set V in (N, ν). [8]

(v) λ-irresolute if $u^{-1}(V)$ is λ-closed in (M, μ) for every λ-closed set V in (N, ν). [3]

(vi) λ_g^α-closed if $u(V)$ is λ_g^α-closed in (N, ν) for every closed set V in (M, μ). [13]

(vii) λ_g^α-open if $u(V)$ is λ_g^α-open in (N, ν) for every open set V in (M, μ). [13]

(viii) λ_g^α-continuous if $u^{-1}(V)$ is λ_g^α-closed in (M, μ) for every closed set V in (N, ν). [12]

(ix) λ_g^α-irresolute if $u^{-1}(V)$ is λ_g^α-closed in (M, μ) for every λ_g^α-closed set V in (N, ν). [12]

(x) Strongly λ_g^α-continuous if $u^{-1}(S)$ is λ_g^α-clopen in (M, μ) for every subset S in (N, ν). [14]

(xi) Perfectly λ_g^α-continuous if $u^{-1}(S)$ is clopen in (M, μ) for every λ_g^α-closed set S in (N, ν). [14]

(xii) Totally λ_g^α-continuous if $u^{-1}(T)$ is λ_g^α-clopen in (M, μ) for every open set T in (N, ν). [14]

(xiii) Quasi λ_g^α-continuous if $u^{-1}(T)$ is closed in (M, μ) for every λ_g^α-closed set T in (N, ν). [14]"

Definition "Let (M, μ) and (N, ν) be the topological spaces. A map $u : (M, \mu) \to (N, \nu)$ is called

(i) Contra continuous if $u^{-1}(V)$ is open (closed) in (M, μ) for every closed (open) set V in (N, ν). [7]

(ii) Contra open if $u(V)$ is closed in (N, ν) for every open set V in (M, μ). [1]

(iii) Contra λ-continuous if $u^{-1}(V)$ is λ-open (λ-closed) in (M, μ) for every closed (open) set V in (N, ν). [2]"

Results

(i) "Every λ-closed (λ-open) set is a λ_g^α-closed (λ_g^α-open) set.[11]"

(ii) "Every closed and open set is a λ_g^α-closed (λ_g^α-open) set.[11]"

(iii) "In a $T_{1/2}$ -space, every subset is a λ_g^α-closed set.[11]"

3. Contra λ_g^α-continuous maps

Hereafter, u represents the mapping from the domain (M, μ) to the co-domain (N, ν) i.e., $u : (M, \mu) \to (N, \nu)$ and v represents the mapping from the domain (N, ν) to the co-domain (K, κ) i.e., $v : (N, \nu) \to (K, \kappa)$.

Definition 1. A map u is called contra λ_g^α-continuous if $u^{-1}(T)$ is λ_g^α-open in (M, μ) for every closed set T of (N, ν).

Example 1. Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i, j\}, M\}$ and $\nu = \{\phi, \{i, j, k\}, N\}$. Then u is a contra λ_g^α-continuous map when labelled as $u(i) = j$; $u(j) = k$; $u(k) = l$ and $u(l) = i$.

Proposition 1. *Every contra λ-continuous map is contra λ_g^α-continuous but not conversely.*

Proof. Let T be any closed set in (N, ν) and let$u$be contra λ-continuous. Then $u^{-1}(T)$ is λ-open in (M, μ). Now using Result (i), $u^{-1}(T)$ is also λ_g^α-

open set in (M, μ). Hence u is contra λ_g^α-continuous.

Example 2. Consider M, N, μ, ν and u as in Example 1. Then u is a contra λ_g^α-continuous map but not a contra λ-continuous map, since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{k\}$ is not λ-open in (M, μ).

Proposition 2. *Every continuous map is contra λ_g^α-continuous*, b*ut not conversely*.

Proof. Let S be an open set in (N, ν) and let u be a continuous map. Then $u^{-1}(S)$ is open in (M, μ). Now using Result (ii), $u^{-1}(S)$ is λ_g^α-closed in (M, μ). Thus u is contra λ_g^α-continuous.

Example 3. Consider M, N, μ, ν and u as in Example 1. Then u is a contra λ_g^α-continuous map and since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{k\}$ is not closed in (M, μ) it is not a continuous map.

Proposition 3. *Every contra continuous map is contra λ_g^α-continuous, but not conversely*.

Proof. Evident from Result (ii).

Example 4. Consider M, N, μ, ν and u as in Example 1. Then u is a contra λ_g^α-continuous map and since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{k\}$ is not open in (M, μ) it is not a contra continuous map,

Remark 1. The following examples show that λ-continuous and contra λ_g^α-continuous maps are independent generally.

Example 5. Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i, j, k\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, N\}$. Then u is a contra λ_g^α-continuous map when labelled as $u(i) = j$; $u(j) = i$; $u(k) = l$ and $u(l) = k$, but not a λ-continuous map, since for the closed set $\{k, l\}$ in (N, ν), $u^{-1}(\{k, l\}) = \{k, l\}$ is not λ-closed in (M, μ).

Example 6. Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i, j, k\}, N\}$. Then u is a λ-continuous map when labelled as $u(i) = k$; $u(j) = l$; $u(k) = i$ and $u(l) = j$, but not a contra λ_g^α-continuous map, since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{j\}$ is not λ_g^α-closed in (M, μ).

Proposition 4. *If the domain* (M, μ) *is a* $T_{1/2}$ *space, then any map u is contra* λ_g^α*-continuous.*

Proof. Evident from Result (iii).

Proposition 5. *Every totally* λ_g^α*-continuous map is contra* λ_g^α*-continuous but not conversely.*

Proof. Let u be a totally λ_g^α-continuous map and let S be any open set of (N, ν). Then $u^{-1}(S)$ is λ_g^α-clopen in (M, μ). Hence u is contra λ_g^α-continuous.

Example 7. Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{l\}, \{i, j\}, \{i, j, l\}, N\}$. Then u is a contra λ_g^α-continuous map when labelled as $u(i) = k$; $u(j) = l$; $u(k) = i$ and $u(l) = j$, but not a totally λ_g^α-continuous map, since for the open set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{j\}$ is not λ_g^α-clopen in (M, μ).

Proposition 6. *Every strongly* λ_g^α*-continuous map is contra* λ_g^α*-continuous but not conversely.*

Proof. Evident from the Definitions of strongly λ_g^α-continuous and contra λ_g^α-continuous maps.

Example 8. Consider M, N, μ, ν and u as in Example 7. Then u is a contra λ_g^α-continuous map but not a strongly λ_g^α-continuous map, since for the subset $\{i, j, k\}$ in (N, ν), $u^{-1}(\{i, j, k\}) = \{i, k, l\}$ is not a λ_g^α-clopen set in

(M, μ).

Proposition 7. *Every perfectly* λ_g^α*-continuous map is contra* λ_g^α*-continuous but not conversely*.

Proof. Let u be a perfectly λ_g^α-continuous map and let T be a closed set in (N, ν). Now using Result (ii), T is λ_g^α-closed in (N, ν). As u is perfectly λ_g^α-continuous, $u^{-1}(T)$ is clopen in (M, μ). Again, using Result (ii), $u^{-1}(T)$ is λ_g^α-open in (M, μ). Hence u is contra λ_g^α-continuous.

Example 9. Consider M, N, μ, ν and u as in Example 7. Then u is a contra λ_g^α-continuous map but not a perfectly λ_g^α-continuous map, since for the λ_g^α-closed set $\{j, l\}$ in (N, ν), $u^{-1}(\{j, l\}) = \{i, l\}$ is not clopen set in (M, μ).

Remark 2. The following examples show that contra λ_g^α-continuous map and λ_g^α-continuous map are independent generally.

Example 10. Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, \{i, k\}, N\}$. Then u is a contra λ_g^α-continuous map when labelled as $u(i) = j$; $u(j) = i$ and $u(k) = k$, but not a λ_g^α-continuous map, since for the closed set $\{i, k\}$ in (N, ν), $u^{-1}(\{j, k\}) = \{i, k\}$ is not λ_g^α-closed in (M, μ).

Example 11. Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i, j, k\}, N\}$. Then u is a λ_g^α-continuous map when labelled as $u(i) = i$; $u(j) = l$; $u(k) = k$ and $u(l) = j$, but not a contra λ_g^α-continuous map, since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{j\}$ is not λ_g^α-open in (M, μ).

Theorem 1. *A map u is contra* λ_g^α*-continuous if* $u^{-1}(S)$ *is* λ_g^α*-closed in* (M, μ) *for every open set S in* (N, ν) *and vice versa.*

Proof. Let S be any open set in (N, ν), then $N \setminus S$ is closed in (N, ν).

Since u is contra λ_g^α-continuous, $u^{-1}(N \setminus S) = M \setminus u^{-1}(S)$ is λ_g^α-open in $(M, \mu) \Rightarrow u^{-1}(S)$ is λ_g^α-closed in (M, μ).

On the other side, let T be any closed set in (N, ν). Then $N \setminus T$ is open in (N, ν). By assumption, $u^{-1}(N \setminus T) = M \setminus u^{-1}(T)$ is λ_g^α-closed in $(M, \mu) \Rightarrow u^{-1}(T)$ is λ_g^α-open in (M, μ). Hence u is contra λ_g^α-continuous.

Theorem 2. *If a map u is a contra λ-continuous and A is an open set of (M, μ), then the restriction $u_A : A \to N$ is contra λ_g^α-continuous.*

Proof. Let S be an open set of (N, ν) and A be an open set of (M, μ). Using the fact that 'Every open and closed set is λ-closed'. A is λ-closed in (M, μ). Since u is contra λ-continuous, $u^{-1}(S)$ is λ-closed in (M, μ). Hence, we have $u^{-1}(S) \cap A$ is λ-closed in (M, μ), which is also λ_g^α-closed in (M, μ) using Result (i). Also, $[u^{-1}(S) \cap A] \subseteq A \subseteq M$ and therefore using Result (ii), $u^{-1}(S) \cap A = u^{-1}(S)$ is λ_g^α-closed in A. Thus, the restriction $u_A : A \to N$ is a contra λ_g^α-continuous map.

Proposition 8. *The composition of two contra continuous maps is contra λ_g^α-continuous.*

Proof. Let u and v be the contra continuous maps and let S be an open set of (K, κ). Then $v^{-1}(S)$ is a closed set in (N, ν) as v is a contra continuous map. And $u^{-1}(v^{-1}(S)) = (v \circ u)^{-1}(S)$ is an open set in (M, μ), as u is a contra continuous map. Now using Result (ii), $(v \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M, μ). Thus $(v \circ u)$ is a contra λ_g^α-continuous map.

Proposition 9. *The composition of two continuous maps is contra λ_g^α-continuous.*

Proof. Evident from Definition of continuity and contra λ_g^α-continuity.

Proposition 10. *If u is a contra λ_g^α-continuous map and v is a continuous map, then $(v \circ u)$ is a contra λ_g^α-continuous map.*

Proof. Let T be a closed set in (K, κ). Then $v^{-1}(T)$ is closed in (N, ν) as v is continuous. Since u is contra λ_g^α-continuous, $(v \circ u)^{-1}(T) = u^{-1}(v^{-1}(T))$ is λ_g^α-open in (M, μ). Hence $(v \circ u)$ is contra λ_g^α-continuous.

Proposition 11. *If u is a continuous map and v is a contra continuous map, then $(v \circ u)$ is a contra λ_g^α-continuous map.*

Proof. Let S be an open set of (K, κ). Then $v^{-1}(S)$ is a closed set in (N, ν) as v is contra continuous map and $u^{-1}(v^{-1}(S)) = (v \circ u)^{-1}(S)$ is a closed set in (M, μ) as u is a continuous map. Since every closed set is λ_g^α-closed, $(v \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M, μ). Thus $(v \circ u)$ is a contra λ_g^α-continuous map.

Proposition 12. *If u is a contra continuous map and v is a continuous map, then $(v \circ u)$ is a contra λ_g^α-continuous map.*

Proof. Let S be an open set of (K, κ). Then $v^{-1}(S)$ is an open set in (N, ν) as v is a continuous map and $u^{-1}(v^{-1}(S)) = (v \circ u)^{-1}(S)$ is a closed set in (M, μ) as u is a contra continuous map. Now using Result (ii), $(v \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M, μ). Thus $(v \circ u)$ is a contra λ_g^α-continuous map.

Proposition 13. *If u is λ_g^α-continuous and v is contra continuous, then $(v \circ u)$ is contra λ_g^α-continuous.*

Proof. Let S be an open set of (K, κ). Then $v^{-1}(S)$ is a closed set in (N, ν) as v is contra continuous and $u^{-1}(v^{-1}(S)) = (v \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M, μ) as u is a λ_g^α-continuous map. Thus $(v \circ u)$ is a contra λ_g^α-continuous map.

Proposition 14. *If u is contra λ-continuous and v is continuous, then $(v \circ u)$ is a contra λ_g^α-continuous map.*

Proof. Let S be an open set of (K, κ). Then $v^{-1}(S)$ is an open set in (N, ν) as v is a continuous map and $u^{-1}(v^{-1}(S)) = (v \circ u)^{-1}(S)$ is a λ-closed set in (M, μ) as u is a contra λ-continuous map. Using Result (i), $(v \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M, μ). Thus $(v \circ u)$ is a contra λ_g^α-continuous map.

Proposition 15. *The composition of two λ-irresolute maps is a contra λ_g^α-continuous map.*

Proof. Evident from Result (i).

Proposition 16. *Let u and v be the bijective maps. Then the following are true:*

(i) *If $(v \circ u)$ is contra continuous and u is λ_g^α-closed then v is contra λ_g^α-continuous.*

(ii) *If $(v \circ u)$ is contra λ-continuous and u is λ-closed then v is contra λ_g^α-continuous.*

(iii) *If $(v \circ u)$ is λ-irresolute and u is λ-closed then v is contra λ_g^α-continuous.*

(iv) *If $(v \circ u)$ is contra continuous and u is λ-closed then v is contra λ_g^α-continuous.*

Proof. (i) Let S be an open set in (K, κ). Since $(v \circ u)$ is a contra continuous map $(v \circ u)^{-1}(S) = u^{-1}(v^{-1}(S))$ is closed in (M, μ). Now since u is a λ_g^α-closed map $u(u^{-1}(v^{-1}(S))) = v^{-1}(S)$ is λ_g^α-closed set in (N, ν). Thus v is a contra λ_g^α-continuous map.

(ii), (iii) and (iv) can be proved similarly.

Proposition 17. *Let u be perfectly λ_g^α-continuous and v be contra λ_g^α-*

continuous then $(v \circ u)$ *is totally* λ_g^α*-continuous.*

Proof. Evident from the Result (ii).

Remark 3. The composition of two contra λ_g^α-continuous maps need not be a contra λ_g^α-continuous map as seen from the following example.

Example 12. Let $M = N = K = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$, $\nu = \{\phi, \{i, j\}, N\}$ and $\kappa = \{\phi, \{i, j, k\}, K\}$. Consider the map u to be the identity map and the map v be defined by $v(i) = i$; $v(j) = l$; $v(k) = k$ and $v(l) = j$. Here u and v are both contra λ_g^α-continuous map but their composition is not a contra λ_g^α-continuous map, since for the closed set $\{l\}$ in (K, κ), $(v \circ u)^{-1}(\{l\}) = u^{-1}(v^{-1}(\{l\}) = \{j\}$ is not λ_g^α-open in (M, μ).

Proposition 18. *Let u be a λ_g^α-irresolute map and v be a contra λ_g^α-continuous map, then $(v \circ u)$ is a contra λ_g^α-continuous map.*

Proof. Let T be a closed set in (K, κ). Since v is contra λ_g^α-continuous, $v^{-1}(T)$ is λ_g^α-open in (N, ν). Since u is λ_g^α-irresolute, $(v \circ u)^{-1}(T) = u^{-1}(v^{-1}(T))$ is λ_g^α-open in (M, μ). Hence $(v \circ u)$ is a contra λ_g^α-continuous map.

4. Contra λ_g^α-irresolute maps

Definition 2. A map u is called contra λ_g^α-irresolute if $u^{-1}(S)$ is λ_g^α-closed in (M, μ) for every λ_g^α-open set S in (N, ν).

Example 13. Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, \{i, k\}, N\}$. Consider the map u defined by $u(i) = j$; $u(j) = i$ and $u(k) = k$, then u is a contra λ_g^α-irresolute map.

Theorem 3. *Let u be a map. Then the following statements are equivalent*

(i) *u is a contra* λ_g^α*-irresolute map.*

(ii) *The inverse image of every* λ_g^α*-closed* (λ_g^α*-open*) *set in* (N, ν) *is* λ_g^α*-open* (λ_g^α*-closed*) *in* (M, μ).

Proof. Obvious from the Definition 2.

Proposition 19. *Every* λ_g^α*-irresolute map is contra* λ_g^α-con*tinuous but not conversely*.

Proof. Consider u to be λ_g^α-irresolute and S to be an open set in (N, ν). Then by Result (i), S is also λ_g^α-closed in (N, ν). Since u is a λ_g^α-irresolute map, $u^{-1}(S)$ is λ_g^α-closed in (M, μ). Thus u is a contra λ_g^α-continuous map.

Example 14. Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i, j\}, M\}$ and $\nu = \{\phi, , \{i, j, k\}, N\}$. Then the map u is a contra λ_g^α-continuous when labelled as $u(i) = k$; $u(j) = l$; $u(k) = i$ and $u(l) = j$, but not a λ_g^α-irresolute map, since for the λ_g^α-closed set $\{i, k\}$ in (N, ν), $u^{-1}(\{i, k\}) = \{i, k\}$ is not a λ_g^α-closed set in (M, μ).

Proposition 20.

(i) *Every contra* λ_g^α*-irresolute map is contra* λ_g^α*-continuous but not conversely*.

(ii) *Every contra* λ_g^α*-irresolute map is* λ_g^α*-continuous but not conversely*.

Proof. (i)Assume that u is a contra λ_g^α-irresolute map. Let S be any open set in (N, ν) using Result (ii), S is λ_g^α-open in (N, ν). Since u is contra λ_g^α-irresolute, $u^{-1}(S)$ is λ_g^α-closed in (M, μ). Hence u is a contra λ_g^α-continuous map.

(ii) similar to (i).

Example 15. Consider M, N, μ, ν and u as in Example 14. Then u is a contra λ_g^α-continuous but not a contra λ_g^α-irresolute map, since for the λ_g^α-closed set $\{i, k\}$ in (N, ν), $u^{-1}(\{i, k\}) = \{i, k\}$ is not λ_g^α-open in (M, μ).

Example 16. Consider M, N, μ and ν as in Example 14. Then the map u is a λ_g^α-continuous map when labelled as $u(i) = l$; $u(j) = j$; $u(k) = k$ and $u(l) = i$, but not a contra λ_g^α-irresolute map, since for the λ_g^α-closed set $\{i, l\}$ in (N, ν), $u^{-1}(\{i, l\}) = \{i, l\}$ is not λ_g^α-open in (M, μ).

Proposition 21. *Every perfectly λ_g^α-continuous map is a contra λ_g^α-irresolute map but not conversely*.

Proof. Let S be a λ_g^α-open set in (N, ν). As u is perfectly λ_g^α-continuous, $u^{-1}(S)$ is clopen in (M, μ) and using Result (ii), $u^{-1}(S)$ is λ_g^α-closed in (M, μ). Hence u is a contra λ_g^α-irresolute map.

Example 17. Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i, j\}, M\}$ and $\nu = \{\phi, , \{i\}, \{i, j\}, N\}$. Then the identity map u is a contra λ_g^α-irresolute map but not a perfectly λ_g^α-continuous map, since for the λ_g^α-closed set $\{j, k, l\}$ in (N, ν), $u^{-1}(\{j, k, l\}) = \{j, k, l\}$ is not clopen in (M, μ).

Proposition 22. *Every strongly λ_g^α-continuous map is a contra λ_g^α-irresolute map but not conversely*.

Proof. Let S be a λ_g^α-open set in (N, ν). As u is strongly λ_g^α-continuous, $u^{-1}(S)$ is λ_g^α-clopen in (M, μ). Therefore $u^{-1}(S)$ is λ_g^α-closed in (M, μ). Hence u is a contra λ_g^α-irresolute map.

Example 18. Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i, j\}, M\}$ and $\nu = \{\phi, , \{i\}, \{i, j\}, N\}$. Then the identity map u is a contra λ_g^α-irresolute map, but not a strongly λ_g^α-continuous map, since for the subset $\{k\}$ in (N, ν), $u^{-1}(\{k\})$

$= \{k\}$ is not λ_g^α-clopen in (M, μ).

Proposition 23. *Let u and v be two maps, then $(v \circ u)$ is a contra λ_g^α-irresolute if*

(i) *u is λ_g^α-irresolute and v is contra λ_g^α-irresolute*

(ii) *u is contra λ_g^α-irresolute and v is λ_g^α-irresolute*

(iii) *u is contra λ_g^α-continuous and v is quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous)*

Proof. (i) Let S be any λ_g^α-open set in (K, κ). Since v is contra λ_g^α-irresolute, $v^{-1}(S)$ is λ_g^α-closed in (N, ν). Since u is λ_g^α-irresolute, $(v \circ u)^{-1}(S) = u^{-1}(v^{-1}(S))$ is λ_g^α-closed in (M, μ). Hence $(v \circ u)$ is a contra λ_g^α-irresolute map.

Proof of (ii) and (iii) are similar to the proof of (i).

Proposition 24. *The composition of two contra λ_g^α-irresolute maps is λ_g^α-continuous.*

Proof. Let S be an open set of (K, κ). Then S is a λ_g^α-open set in (K, κ), using Result (ii). Now as v is a contra λ_g^α-irresolute map, $v^{-1}(S)$ is a λ_g^α-closed set in (N, ν). Since u is a contra λ_g^α-irresolute map, $(v \circ u)^{-1}(S) = u^{-1}(v^{-1}(S))$ is a λ_g^α-open set in (M, μ). Thus $(v \circ u)$ is a λ_g^α-continuous map.

Proposition 25. *The composition of two λ_g^α-irresolute maps is contra λ_g^α-continuous.*

Proof. Let S be a closed set of (K, κ). Then S is a λ_g^α-closed set in

(K, κ), using Result (ii). Now as v is a λ_g^α-irresolute map, $v^{-1}(S)$ is a λ_g^α-closed set in (N, ν). Since u is a λ_g^α-irresolute map, $(v \circ u)^{-1}(S)$ $= u^{-1}(v^{-1}(S))$ is a λ_g^α-open set in (M, μ). Thus $(v \circ u)$ is a contra λ_g^α-continuous map.

Proposition 26. *The composition of two contra λ_g^α-irresolute maps is contra λ_g^α-continuous.*

Proof. Evident from Definition 2 and Definition 1.

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