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# EDGE ISOLATED DOMINATION FOR JAHANGIR GRAPHS

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## Abstract

Let $G = (V, E)$ be a non trivial finite graph with vertex set $V$ and edge set $E$. A subset $S$ of $V$ in a graph $G$ is said to be an edge-dominating set if every edges in $G$ is incident with a vertex in $S$. A edge dominating set $S$ such that $\langle S \rangle$ has an isolated vertex or $\langle V - S \rangle$ is a single vertex is called edge isolated dominating set [3]. The minimum cardinality number of a edge isolated dominating set is called the edge isolated dominating number of a Graph. Jahangir graphs $J_{s,m}$ for $s \geq 2, m \geq 2,$ is a graph on $sm + 1$ vertices consisting of a cycle $C_{sm}$ with one additional vertex which is adjacent to $m$ vertices of $C_{sm}$ at distance s to each other on $C_{sm}$. The definition of Jahangir graph $J_{2,m}$ was introduced by Surahmat and Tomescu in [1]. In this paper we obtained the edge isolated dominating number of a Jahangir graphs $J_{s,m}$ with different combinations on their adjacency and distances and some results based on it.

## 1. Introduction

Jahangir graphs $J_{s,m}$ for $s \geq 2, m \geq 2,$ is a graph on $sm+1$ vertices consisting of a cycle $C_{sm}$ with one additional vertex which is adjacent to $m$ vertices of $C_{sm}$ at distance $s$ to each other on $C_{sm}$. Figure below shows

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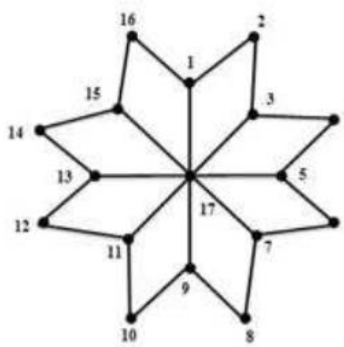
Jahangir graph $J_{2,8}$ [2]. This graph $J_{2,8}$ appears on the Jahangir's tomb in his mausoleum. It lies in 5 kilometer north-west of Lahore, Pakistan across the River Ravi. His tomb was built around 1637 A. D. by his Queen Noor Jehan and his son Shah-Jehan, the emperor who constructed one of the wonders of the world Taj Mahal in India. It has a majestic structure made of red sand-stone and marble. The definition of Jahangir graph $J_{2,m}$ was introduced by Surahmat and Tomescu in 2006 [1].

**Definition 1.1.** An edge dominating set $S \subset V$ is called an edge isolated dominating set if $S$ has an isolated vertex or $\langle V - S \rangle$ is a single vertex [4]. The minimum cardinality of an edge isolated dominating set is called edge isolated domination number of $G$, and is denoted by $\gamma_{ei}(G)$ [5].

**Definition 1.2.** Jahangir graphs $J_{s,m}$ for $s \geq 2$, $m \geq 2$, is a graph on $sm + 1$ vertices consisting of a cycle $C_{sm}$ with one additional vertex which is adjacent to $m$ vertices of $C_{sm}$ at distance $s$ to each other on $C_{sm}$. The definition of Jahangir graph $J_{2,m}$ was introduced by Surahmat and Tomescu in [1].

**Example 1.3.** The following figure shows Jahangir graphs $J_{2,8}$.




$J_{2,8}$.

**Theorem 1.4.** *Let G be a Jahangir graph* $J_{s,m}$. *Then* $\gamma_{ei}(G) = \left\lfloor \frac{|V|}{2} \right\rfloor$, *if s is even and* $m \geq 2$.

**Proof.** Let $G$ be a Jahangir graph $J_{s,m}$ with $sm+1$ vertices and $sm+m$ edges. Let $C_{sm}$ be the cycle with $sm$ vertices and $w_1$ be the middle vertex of $J_{s,m}$. Let $\{v_{s1}, v_{s2}, v_{s3}, \ldots, v_{si}, \ldots, v_{sm-1}, v_{sm}, w_1\} \in V(G)$, where $w_1$ is adjacent to $m$ vertices on the cycle $C_{sm}$ with distance $s$. Since $s$ is even $sm$ is even and hence $C_{sm}$ is an even cycle with $v_{s1} = v_{sm}$. $d(v_{si}) = 2$ for $i = 1, 2, 3, \ldots, m$ and since $w_1$ is adjacent to $m$ vertices on the cycle $C_{sm}$ there exist exactly $m$ vertices of degree 3.

Choose $v_{s1} \in \gamma_{ri}(G)$, such that $d(v_{s1}) = 3$. Where $v_{s1}$ is adjacent to $v_{s1}v_{s2}, v_{s1}w_1, v_{s1}v_{sm-1}$.

To cover the remaining edges which are not incident with $v_{s1}$,

Omitting the next vertex $v_{s2}$ such that $d(v_{s2}) = 2$ since $s \geq 2$.

Without loss of generality, choose $v_{s3} \in \gamma_{ei}(G)$, such that $d(v_{s3}) = 3$, $v_{s3}$ is at the even distance from $v_{s1}$.

$v_{s3}$ is adjacent to $v_{s3}v_{s2}, v_{s3}w_1, v_{s3}v_{s4}$, based on the distance $s$ on the cycle $C_{sm}$.

To cover the remaining edges which are not incident with $v_{s3}$.

Omitting the next vertex $v_{si}$, where $d(v_{si}) = 2$, since $v_{si}$ is in the odd distance from $v_{s1}$. $v_{si}$ is adjacent to $v_{si}v_{si-1}, v_{si}v_{si+1}$.

Choose $v_{si+1} \in \gamma_{ei}(G)$, and $d(v_{si+1}) = 3$, since $v_{si+1}$ is at the even distance from $v_{s1}v_{si+1}$ is adjacent to $v_{si+1}v_{si}, v_{si+1}w_1, v_{si+1}v_{si+2}$, based on the distance $s$ on the cycle $C_{sm}$.

Proceeding in the same way, choose $v_{sm-4} \in \gamma_{ei}(G)$, since $v_{sm-4}$ is at the even distance from $v_{s1}v_{sm-4}$ is adjacent to $v_{sm-4}v_{sm-5}, v_{sm-4}w_1$, $v_{sm-4}v_{sm-3}$, based on the distance $s$ on the cycle $C_{sm}$. Similarly, choose $v_{sm-2} \in \gamma_{ei}(G)$, since $v_{sm-2}$ is at the even distance from $v_{s1}v_{sm-2}$ is adjacent to $v_{sm-2}v_{sm-3}, v_{sm-2}w_1, v_{sm-2}v_{sm-1}$, based on the distance $s$ on the cycle $C_{sm}$. By the above selection procedure. If $v_{s1}$ is selected in $\gamma_{ei}(G)$ set, then

$v_{s3}, v_{s5}, \ldots, v_{sm-4}, v_{sm-2}$ has to be selected in $\gamma_{ei}(G)$ set. Hence $\{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sm-4}, v_{sm-2}\}$ is a set in which all $v_{sj}$'s are at even distance from $v_{s1}$ and since $w_1$ is adjacent to $m$ vertices at even distance , thus $w_1$ is adjacent to either of the vertices in this set.

$\therefore \{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sm-4}, v_{sm-2}\}$ is an *ei*-set which covers all the $sm + m$ edges of $G$.

Let $I = \{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sm-4}, v_{sm-2}\}$ is a *ei*-set, and $I' = \{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sm-4}, v_{sm-2}, w_1\}$ is also a *ei* set. But $|I| < |I'|$ and there exist no $v_{sk}$, $k = 1, 3, 5, \ldots, sm-4, sm-2$, such that $\{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sk-1}, v_{sk+1}, \ldots, v_{sm-4}, v_{sm-2}\}$ is an *ei*-set.

$\therefore I$ is a minimum $\gamma_{ei}$ set and $|I| = \frac{sm}{2}$.

Hence,

$$\gamma_{ei}(J_{s,m}) = \gamma_{ei}(C_{sm})$$

$$\gamma_{ei}(J_{s,m}) = \left\lfloor \frac{|v|}{2} \right\rfloor.$$

**Theorem 1.5.** *Let G be a Jahangir graph* $J_{s,m}$. *Then* $\gamma_{ei}(J_{s,m}) = \left\lfloor \frac{|v|}{2} \right\rfloor$, *if s is odd and m is even.*

**Proof.** Let $G$ be a Jahangir graph $J_{s,m}$ with $sm+1$ vertices and $sm+m$ edges. Let $C_{sm}$ be the cycle with $sm$ vertices and $w_1$ be the middle vertex of $J_{s,m}$. Let $\{v_{s1}, v_{s2}, v_{s3}, \ldots, v_{si}, \ldots, v_{sm-1}, v_{sm}, w_1\} \in V(G)$, where $w_1$ is adjacent to $m$ vertices on the cycle $C_{sm}$ with distance $s$. Since $s$ is odd and $m$ is even $sm$ is even and hence $C_{sm}$ is an even cycle with $v_{s1} = v_{sm}$, $d(v_{si}) = 2$ for $i = 1, 2, 3, \ldots, m$ and since $w_1$ is adjacent to $m$ vertices on the cycle, $C_{sm}$ there exist exactly $m$ vertices of degree 3.

Choose $v_{s1} \in \gamma_{ei}(G)$, such that $d(v_{s1}) = 3. v_{s1}$ is adjacent to $v_{s1}v_{s2}$, $v_{s1}w_1$, $v_{s1}v_{sm-1}$.

To cover the remaining edges which are not incident with $v_{s1}$, omitting the next vertex $v_{s2}$ such that $d(v_{s2}) = 2$ since $s \geq 2$.

Without loss of generality, choose $v_{s3} \in \gamma_{ei}(G)$, such that $d(v_{s3}) = 2$, since $v_{s3}$ is at the even distance from $v_{s1}$, $v_{s3}$ is adjacent to $v_{s3}v_{s2}$, $v_{s3}v_{s4}$. To cover the remaining edges which are not incident with $v_{s3}$. Omitting the next vertex $v_{si}$, where $v_{si}$ is in the odd distance from $v_{s1}$ and $w_1$ is adjacent to $v_{si}$, based on the distance $s$ on $C_{sm}$. $v_{si}$ is adjacent to $v_{si}\,v_{si-1}$, $v_{si}w_1$, $v_{si}\,v_{si+1}$.

Choose $v_{si+1} \in \gamma_{ei}(G)$, and $d(v_{si+1}) = 2$, since $v_{si+1}$ is at the even distance from $v_{s1}v_{si+1}$ is adjacent to $v_{si+1}v_{si}$, $v_{si+1}v_{si+2}$. By choosing the vertices $v_{s3}$ and $v_{si+1}$, the edges $v_{si}v_{si-1}$ and $v_{si}v_{si+1}$ of $v_{si}$ are covered for $\gamma_{ei}(G)$, but the edge $v_{si}w_1$ of $v_{si}$ is not covered. Proceeding in the same way, choose $v_{sm-4} \in \gamma_{ei}(G)$, such that $d(v_{sm-4}) = 2$, since $v_{sm-4}$ is at the even distance from $v_{s1}$, $v_{sm-4}$ is adjacent to $v_{sm-4}v_{sm-5}$, $v_{sm-4}v_{sm-3}$.

Similarly, choose $v_{sm-2} \in \gamma_{ei}(G)$, such that $d(v_{sm-2}) = 2$, since $v_{sm-2}$ is at the even distance from $v_{s1}$. $v_{sm-2}$ is adjacent to $v_{sm-2}v_{sm-3}$, $v_{sm-2}v_{sm-1}$.

By the above selection procedure. If $v_{s1}$ is selected in $\gamma_{ei}(G)$ set, then $v_{s3}, v_{s5}, \ldots, v_{sm-4}, v_{sm-2}$ has to be selected in $\gamma_{ei}(G)$ set. In this selection process, exactly $\frac{m}{2}$ edges of $w_1$ is not covered by any of the vertices in the set $\{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sm-4}, v_{sm-2}\}$, hence to cover the $\frac{m}{2}$ edges of $w_1$. Choose $w_1 \in \gamma_{ei}(G)$, such that $d(w_1) = m$, $w_1$ is adjacent to $w_1v_{s1}, \ldots, w_1v_{sj}$, $j =$ odd distance from $v_{s1}$.

$\therefore \{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sm-4}, v_{sm-2}, w_1\}$ is a *ei*-set with $\frac{sm}{2} + 1$ vertices.

Let $I = \{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sm-4}, v_{sm-2}, w_1\}$ is a *ei*-set, and $I' = \{v_{s1}, v_{s2}, v_{s3}, \ldots, v_{sm-2}, v_{sm-1}, w_1\}$ is also a *ei*-set.

But $|I| < |I'|$ and there exist no $v_{sk}$, $k = 1, 3, 5, \ldots, sm-4, sm-2$, such that $\{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sk-1}, v_{sk+1}, \ldots, v_{sm-4}, v_{sm-2}, w_1\}$ is an *ei*-set.

$\therefore I$ is a minimum $\gamma_{ei}$ set and $|I| = \frac{sm}{2} + 1$.

Therefore $\gamma_{ei}(J_{s,m}) = \gamma_{ei}(C_{sm}) + 1$

$$\gamma_{ei}(J_{s,m}) = \left\lceil \frac{|v|}{2} \right\rceil.$$

**Theorem 1.6.** *Let G be a Jahangir graph* $J_{s,m}$. *Then* $\gamma_{ei}(G) = \frac{|v|}{2} + 1$, *if both s and m are odd.*

**Proof.** Let $G$ be a Jahangir graph $J_{s,m}$ with $sm + 1$ vertices and $sm + m$ edges. Let $C_{sm}$ be the cycle with $sm$ vertices and $w_1$ be the middle vertex of $J_{s,m}$.

Let $\{v_{s1}, v_{s2}, v_{s3}, \ldots, v_{si}, \ldots, v_{sm-1}, v_{sm}, w_1\} \in V(G)$, where $w_1$ is adjacent to $m$ vertices on the cycle $C_{sm}$ with distance $s$. Since $s$ is odd and $m$ is odd $sm$ is odd and hence $C_{sm}$ is an odd cycle with $v_{s1} = v_{sm}$, $d(v_{si}) = 2$ for $i = 1, 2, 3, \ldots, m$ and since $w_1$ is adjacent to $m$ vertices on the cycle $C_{sm}$ there exist exactly $m$ vertices of degree 3.

Choose $v_{s1} \in \gamma_{ei}(G)$, such that $d(v_{s1}) = 3$. $v_{s1}$ is adjacent to $v_{s1}v_{s2}, v_{s1}w_1, v_{s1}v_{sm-1}$. To cover the remaining edges which are not incident with $v_{s1}$. Omitting the next vertex $v_{s2}$ such that $d(v_{s2}) = 2$ since $s \geq 2$. Without loss of generality, choose $v_{s3} \in \gamma_{ei}(G)$, such that $d(v_{s3}) = 2$ since $v_{s3}$ is at the even distance from $v_{s1}$, $v_{s3}$ is adjacent to $v_{s3}v_{s2}, v_{s3}v_{s4}$.

To cover the remaining edges which are not incident with $v_{s3}$, Omitting the next vertex $v_{si}$, where $v_{si}$ is in the odd distance from $v_{s1}$ and $w_1$ is adjacent to $v_{si}$ based on the distance $s$ on $C_{sm}$. $v_{si}$ is adjacent to $v_{si}v_{si-1}, v_{si}w_1, v_{si}v_{si+1}$. Choose $v_{si+1} \in \gamma_{ei}(G)$, and $d(v_{si+1}) = 2$, since $v_{si+1}$ is at the even distance from $v_{s1}$, $v_{si+1}$ is adjacent to $v_{si+1}v_{si}, v_{si+1}v_{si+2}$.

By choosing the vertices $v_{s3}$ and $v_{si+1}$, the edges $v_{si}v_{si-1}$ and $v_{si}v_{si+1}$ of $v_{si}$ are covered for $\gamma_{ei}(G)$, but the edge $v_{si}w_1$ of $v_{si}$ is not covered.

Proceeding in the same way, choose $v_{sm-3} \in \gamma_{ei}(G)$, such that $d(v_{sm-3}) = 3$, since $v_{sm-3}$ is at the even distance from $v_{s1}v_{sm-3}$ is adjacent to $v_{sm-3}v_{sm-2}$, $v_{sm-3}w_1$, $v_{sm-3}v_{sm-4}$ based on the distance $s$ on $C_{sm}$.

Similarly, choose $v_{sm-1} \in \gamma_{ei}(G)$, such that $d(v_{sm-1}) = 2$, since $v_{sm-1}$ is at the even distance from $v_{s1}$, $v_{sm-1}$ is adjacent to $v_{sm-1}v_{sm-2}$, $v_{sm-1}v_{sm}$.

By the above selection procedure, if $v_{s1}$ is selected in $\gamma_{ei}(G)$ set, then $v_{s3}, v_{s5}, \ldots, v_{sm-3}, v_{sm-1}$ has to be selected in $\gamma_{ei}(G)$ set. In this selection process, exactly $\frac{m-1}{2}$ edges of $w_1$ is not covered by any of the vertices in the set $\{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sm-3}, v_{sm-1}\}$, hence to cover the $\frac{m-1}{2}$ edges of $w_1$, Choose $w_1 \in \gamma_{ei}(G)$, such that $d(w_1) = m$, $w_1$ is adjacent to $w_1v_{s1}, \ldots, w_1v_{sj}$, $j =$ odd distance from $v_{s1}$.

$\therefore \{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sm-3}, v_{sm-1}, w_1\}$ is an *ei*-set with $\frac{(sm+1)}{2} + 1$ vertices.

Let $I = \{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sm-3}, v_{sm-1}, w_1\}$ is a *ei*-set, and $I' = \{v_{s1}, v_{s2}, v_{s3}, \ldots, v_{sm-2}, v_{sm-1}, w_1\}$ is also a *ei*-set.

But $|I| < |I'|$ and there exist no $v_{sk}$, $k = 1, 3, 5, \ldots, sm-3, sm-1$, such that $\{v_{s1}, v_{s3}, v_{s5}, \ldots, v_{sk-1}, v_{sk+1}, \ldots, v_{sm-3}, v_{sm-1}, w_1\}$ is an *ei*-set.

$\therefore I$ is a minimum $\gamma_{ei}$ set and $|I| = \frac{sm+1}{2} + 1$.

Hence $\gamma_{ei}(J_{s,m}) = \gamma_{ei}(C_{sm}) + 1$

$\therefore \gamma_{ei}(J_{s,m}) = \frac{|v|}{2} + 1$.

**Theorem 1.7.** *Let G be a Jahangir graph* $J_{s,m}$, $s \geq 2$, $m \geq 2$, *then* $\gamma_{ei}(S(G)) = sm + 1$.

**Proof.** Let $G$ be a Jahangir graph $J_{s,m}$, with $sm+1$ vertices and $sm + m$ edges.

Let $S(G)$ be the subdivision graph of $G$.

$$\gamma_{ei}(S(J_{sm})) = \gamma_{ei}(C_{2sm}) + 1$$

$$\therefore \gamma_{ei}(S(G)) = sm + 1.$$

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