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FUZZY QUOTIENT-3 CORDIAL LABELING ON SOME CYCLE RELATED GRAPHS - PAPER I

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Abstract

Let $G = (V, E)$ be a simple, finite and planar graph of order p and size q. Let $\sigma : V(G) \to [0, 1]$ be a function defined by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$. For each edge uv define $\mu : E(G) \to [0, 1]$ by $\mu(uv) = \frac{1}{10}\left\lceil \frac{3\sigma(u)}{\sigma(v)} \right\rceil$ where $\sigma(u) \le \sigma(v)$. The function σ is called fuzzy quotient-3 cordial labeling of G if the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, the number of edges labeled with i and the number of edges labeled with j differ by at most 1 where $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \ne j$. The number of vertices having label i denotes $v_\sigma(i)$ and the number of edges having label i denotes $e_\mu(i)$. Here it is proved that some cycle related graphs are Fuzzy quotient-3 cordial.

1. Introduction

Graphs considered here are finite and simple. Graph labeling is used in several areas of science and technology like coding theory, astronomy, circuit design etc. The cordial labeling concept was first introduced by Cahit [2]. we

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introduced fuzzy quotient-3 cordial labeling of graphs and proved some star related graphs are fuzzy quotient-3 cordial [3], [4]. In this paper we proved that n-Sunlet graph, flower graph, Sun flower graph, helm graph, and shell graph are fuzzy quotient-3 cordial.

2. Definitions

Let $\sigma : V(G) \to [0, 1]$ be a function defined by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.

For each edge uv define $\mu : E(G) \to [0, 1]$ by $\mu(uv) = \frac{1}{10}\left\lceil \frac{3\sigma(u)}{\sigma(v)} \right\rceil$ where $\sigma(u) \leq \sigma(v)$. The function σ is called fuzzy quotient-3 cordial labeling of G if the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, the number of edges labeled with i and the number of edges labeled with j differ by at most 1 where $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$. The number of vertices having label i denotes $v_\sigma(i)$ and the number of edges having label i denotes $e_\mu(i)$.

Definition 2.1. n-Sunlet graph The n-Sunlet graph is a graph on $2n$ vertices obtained by attaching n pendant edges to a cycle graph C_n.

Definition 2.2. Flower graph A flower graph Fl_n is a graph obtained from a helm by joing each pendant vertex to the central vertex of the helm.

Definition 2.3. Sun flower graph Sun flower graph Sf_n is the resultant graph obtained from the flower graph by adding n pendant edges to the central vertex.

Definition 2.4. Helm graph The Helm H_n is a graph obtained from a wheel graph $W_{n,1}$ by attaching a pendant edge at each vertex of the n-cycle.

Definition 2.5. Shell graph A Shell graph Sl_n is defined as a cycle C_n with $n - 3$ chords sharing a common end point called the apex.

3. Main Result

Theorem 3.1. *The n-Sunlet graph, $n \geq 6$ is fuzzy quotient-3 cordial.*

Proof. Let G be a n-Sunlet graph. $V(G) = \{x_k : 1 \leq k \leq n\} \cup \{y_k : 1 \leq k \leq n\}$ and $E(G) = \{x_k x_{k+1} : 1 \leq k \leq n-1\} \cup \{x_1 x_n\} \cup \{x_k y_k : 1 \leq k \leq n\}$. $|V| = 2n, |E| = 2n$ we define $\sigma : V(G) \to [0, 1]$ as $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$. For $1 \leq k \leq n$, the values of $\sigma(x_k)$ and $\sigma(y_k)$ are tabulated below.

Table 3.1.1.

Nature of n	$\sigma(x_k) = 0.1$	$\sigma(x_k) = 0.2$	$\sigma(x_k) = 0.3$
$n \equiv 0, 1, 4 (\text{mod } 6)$	$k \equiv 1, 2 (\text{mod } 6)$	$k \equiv 4, 5 (\text{mod } 6)$	$k \equiv 0, 3 (\text{mod } 6)$
$n \equiv 2 (\text{mod } 6)$	$k \equiv 1, 2 (\text{mod } 6)$	$k \equiv 4, 5 (\text{mod } 6)$ and $k = n$	$k \equiv 0, 3 (\text{mod } 6)$
$n \equiv 3 (\text{mod } 6)$	$k \equiv 4, 5 (\text{mod } 6)$ and $k = n - 2$	$k \equiv 1, 2 (\text{mod } 6)$ and $k = n$	$k \equiv 0, 3 (\text{mod } 6)$ and $k = n - 1$
$n \equiv 5 (\text{mod } 6)$	$k \equiv 1, 2 (\text{mod } 6)$	$k \equiv 4, 5 (\text{mod } 6)$	$k \equiv 0, 3 (\text{mod } 6)$ and $k = n$

Table 3.1.2.

Nature of n	$\sigma(y_k) = 0.1$	$\sigma(x) = 0.2$	$\sigma(y_k) = 0.3$
$n \equiv 0, 1, 4, 5 (\text{mod } 6)$	$k \equiv 0, 1 (\text{mod } 6)$	$k \equiv 3, 4 (\text{mod } 6)$	$k \equiv 2, 5 (\text{mod } 6)$
$n \equiv 2 (\text{mod } 6)$	$k \equiv 0, 1 (\text{mod } 6)$	$k \equiv 3, 4 (\text{mod } 6)$ and $k = n$	$k \equiv 2, 5 (\text{mod } 6)$ and $k = n - 1$
$n \equiv 3 (\text{mod } 6)$	$k \equiv 3, 4 (\text{mod } 6)$ and $k = n - 2$	$k \equiv 0, 1 (\text{mod } 6)$ and $k = n - 1$	$k \equiv 2, 5 (\text{mod } 6)$ and $k = n$

For $n \equiv l (\text{mod } 6)$, where $0 \leq l \leq 6$, the following table shows that the number of vertices labeled with i and the number of vertices labeled with j

differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1, where $i,\ j \in \left\{\frac{r}{10},\ r \in Z_4 - \{0\}\right\}$.

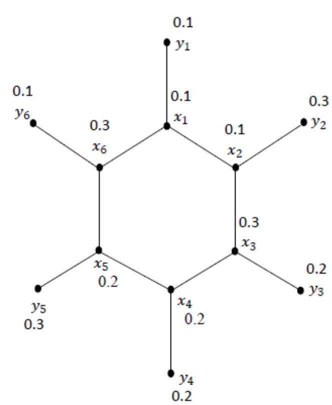
Table 3.1.3.

l	$v_\sigma(0.1)$	$v_\sigma(0.3)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
0	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
1	$\frac{2n+1}{3}$	$\frac{2n+1}{3}-1$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}-1$	$\frac{2n+1}{3}$
2	$\frac{2n-1}{3}$	$\frac{2n-1}{3}+1$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}+1$	$\frac{2n-1}{3}$
3	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
4	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}-1$	$\frac{2n+1}{3}-1$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
5	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}+1$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}+1$

From the above table 3.1.3, it is concluded that the n-Sunlet graph is fuzzy quotient-3 cordial.

Example 3.1.4. The 6-Sunlet graphis fuzzy quotient-3 cordial.

x_5



Theorem 3.2. *The flower graph* $Fl_n n \geq 6$ *is fuzzy quotient-*3 *cordial.*

Proof. Let G be a flower graph Fl_n.

$$V(G) = \{x_k : 1 \leq k \leq n\} \cup \{y_k : 1 \leq k \leq n\} \cup \{x\}$$

and

$$E(G) = \{x_k x_{k+1} : 1 \leq k \leq n-1\} \cup \{x_1 x_n\} \cup \{x_k y_k : 1 \leq k \leq n\}$$

$$\cup \{x, x_k : 1 \leq k \leq n\} \cup \{x\, y_k : 1 \leq k \leq n\}$$

$$|V| = 2n+1, |E| = 4n,$$

we define $\sigma : V(G) \to [0, 1]$ as $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$ $\sigma(x) = 0.1$ for $n \equiv 0, 1, 3, 4, 5 \pmod 6$ and $\sigma(x) = 0.3$ for $n \equiv 2 \pmod 6$. For $1 \leq k \leq n$, the values of $\sigma(x_k)$ and $\sigma(y_k)$ are tabulated below.

Table 3.2.1.

Nature of n	$\sigma(x_k) = 0.1$	$\sigma(x_k) = 0.2$	$\sigma(x_k) = 0.3$
$n \equiv 0, 4 \pmod 6$	$k \equiv 1, 2 \pmod 6$	$k \equiv 4, 5 \pmod 6$	$k \equiv 0, 3 \pmod 6$
$n \equiv 1 \pmod 6$	$k \equiv 2, 3 \pmod 6$	$k \equiv 0, 5 \pmod 6$	$k \equiv 1, 4 \pmod 6$
$n \equiv 2, 3 \pmod 6$	$k \equiv 1, 2 \pmod 6$	$k \equiv 4, 5 \pmod 6$ and $k = n$	$k \equiv 0, 3 \pmod 6$
$n \equiv 5 \pmod 6$	$k \equiv 1, 2 \pmod 6$ and $k = n$	$k \equiv 4, 5 \pmod 6$	$k \equiv 0, 3 \pmod 6$

Table 3.2.2.

Nature of n	$\sigma(y_k) = 0.1$	$\sigma(x) = 0.2$	$\sigma(y_k) = 0.3$
$n \equiv 0, 3, 4, 5 \pmod 6$	$k \equiv 0, 1 \pmod 6$	$k \equiv 3, 4 \pmod 6$	$k \equiv 2, 5 \pmod 6$
$n \equiv 1 \pmod 6$	$k \equiv 1, 2 \pmod 6$	$k \equiv 4, 5 \pmod 6$ and $k = n$	$k \equiv 0, 3 \pmod 6$
$n \equiv 2 \pmod 6$	$k \equiv 0, 1 \pmod 6$	$k \equiv 3, 4 \pmod 6$ and $k = n$	$k \equiv 2, 5 \pmod 6$ and $k = n - 1$

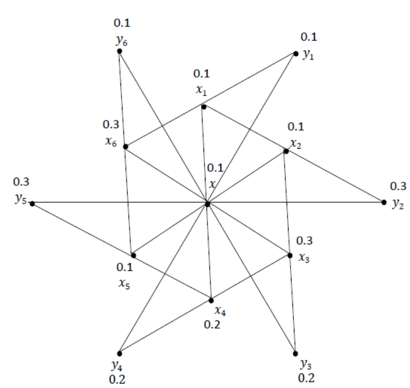
For $n \equiv l(\text{mod } 6)$, where $0 \leq l \leq 6$, the following table shows that the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1, where $i, j \in \left\{\frac{r}{10}, Z_4 \in Z_4 - \{0\}\right\}$.

Table 3.2.3.

l	$v_\sigma(0.1)$	$v_\sigma(0.3)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
0	$\frac{2n+1-1}{3}+1$	$\frac{2n+1-1}{3}$	$\frac{2n+1-1}{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$
1	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{4n-1}{3}$	$\frac{4n-1}{3}+1$	$\frac{4n-1}{3}$
2	$\frac{2n+1+1}{3}$	$\frac{2n+1+1}{3}-1$	$\frac{2n+1+1}{3}$	$\frac{4n-2}{3}+1$	$\frac{4n-1}{3}$	$\frac{4n-2}{3}+1$
3	$\frac{2n+1-1}{3}+1$	$\frac{2n+1-1}{3}$	$\frac{2n+1-1}{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$
4	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{4n-1}{3}$	$\frac{4n-1}{3}+1$	$\frac{4n-1}{3}$
5	$\frac{2n+1+1}{3}$	$\frac{2n+1+1}{3}$	$\frac{2n+1+1}{3}-1$	$\frac{4n-1}{3}-1$	$\frac{4n-2}{3}$	$\frac{4n-2}{3}-1$

From the above table 3.2.3 it is concluded that the flower graph Fl_n is fuzzy quotient-3 cordial.

Example 3.2.4. The flower graph Fl_6 is fuzzy quotient-3 cordial.



Theorem 3.3. *The Sunflower graph* Sf_n, $n \geq 6$ *is fuzzy quotient*-3 *cordial.*

Proof. Let G be a Sunflower graph Sf_n.

$$V(G) = \{x_k : 1 \leq k \leq n\} \cup \{y_k : 1 \leq k \leq n\} \cup \{z_k : 1 \leq k \leq n\} \cup \{x\}$$

and

$$E(G) = \{x_k, x_{k+1} : 1 \leq k \leq n-1\} \cup \{x_1 x_n\} \cup \{x_k, y_k : 1 \leq k \leq n\}$$

$$\cup \{x, x_k : 1 \leq k \leq n\} \cup \{x, z_k : 1 \leq k \leq n\} \cup \{x, y_k : 1 \leq k \leq n\}.$$

Here $|V| = 3n + 1$, $|E| = 5n$.

$\sigma(x) = 0.1$ for $n \equiv 0, 1 (\text{mod } 6)$ and $\sigma(x) = 0.3$ for $n \equiv 2, 3, 4, 5 (\text{mod } 6)$. For $1 \leq k \leq n$, the values of $\sigma(x_k)$ and $\sigma(y_k)$ are tabulated below.

Table 3.3.1.

Nature of n	$\sigma(y_k) = 0.1$	$\sigma(x) = 0.2$	$\sigma(y_k) = 0.3$
$n \equiv 0, 3, 4 (\text{mod } 6)$	$k \equiv 1, 2 (\text{mod } 6)$	$k \equiv 4, 5 (\text{mod } 6)$	$k \equiv 0, 3 (\text{mod } 6)$
$n \equiv 1 (\text{mod } 6)$	$k \equiv 2, 3 (\text{mod } 6)$	$k \equiv 0, 5 (\text{mod } 6)$	$k \equiv 1, 4 (\text{mod } 6)$
$n \equiv 2 (\text{mod } 6)$	$k \equiv 1, 2 (\text{mod } 6)$	$k \equiv 4, 5 (\text{mod } 6)$ and $k = n$	$k \equiv 0, 3 (\text{mod } 6)$
$n \equiv 5 (\text{mod } 6)$	$k \equiv 1, 2 (\text{mod } 6)$ and $k = n$	$k \equiv 4, 5 (\text{mod } 6)$	$k \equiv 0, 3 (\text{mod } 6)$

Table 3.3.2.

Nature of n	$\sigma(y_k) = 0.1$	$\sigma(y_k) = 0.2$	$\sigma(y_k) = 0.3$
$n \equiv 0, 3, 4, 5 (\text{mod } 6)$	$k \equiv 0, 1 (\text{mod } 6)$	$k \equiv 3, 4 (\text{mod } 6)$	$k \equiv 2, 5 (\text{mod } 6)$
$n \equiv 1 (\text{mod } 6)$	$k \equiv 1, 2 (\text{mod } 6)$	$k \equiv 4, 5 (\text{mod } 6)$ and $k = n$	$k \equiv 0, 3 (\text{mod } 6)$
$n \equiv 2 (\text{mod } 6)$	$k \equiv 0, 1 (\text{mod } 6)$	$k \equiv 3, 4 (\text{mod } 6)$ and $k = n$	$k \equiv 2, 5 (\text{mod } 6)$ and $k = n - 1$

Table 3.3.3.

Nature of n	$\sigma(z_k) = 0.1$	$\sigma(z_k) = 0.2$	$\sigma(z_k) = 0.3$
$n \equiv 0, 1(\text{mod } 6)$	$k \equiv 1, 2(\text{mod } 6)$	$k \equiv 4, 5(\text{mod } 6)$	$k \equiv 0, 3(\text{mod } 6)$
$n \equiv 2(\text{mod } 6)$	$k \equiv 4, 5(\text{mod } 6)$	$k \equiv 1, 2(\text{mod } 6)$	$k \equiv 0, 3(\text{mod } 6)$
$n \equiv 3(\text{mod } 6)$	$k \equiv 1, 4(\text{mod } 6)$	$k \equiv 2, 5(\text{mod } 6)$	$k \equiv 0, 3(\text{mod } 6)$
$n \equiv 4(\text{mod } 6)$	$k \equiv 0, 1(\text{mod } 6)$	$k \equiv 3, 4(\text{mod } 6)$	$k \equiv 2, 5(\text{mod } 6)$
$n \equiv 5(\text{mod } 6)$	$k \equiv 0, 5(\text{mod } 6)$	$k \equiv 2, 3(\text{mod } 6)$	$k \equiv 1, 4(\text{mod } 6)$

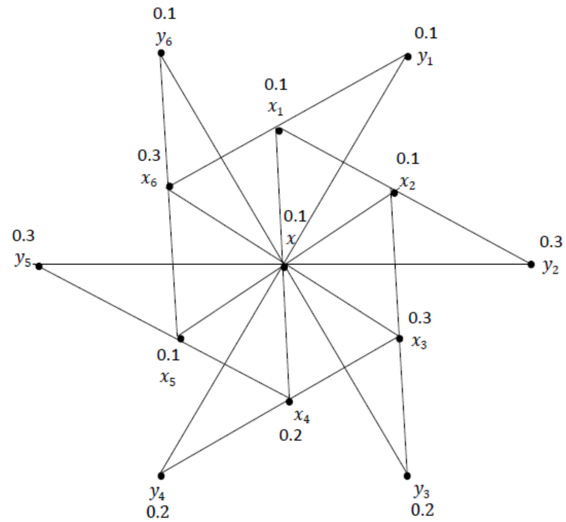
For $n \equiv l(\text{mod } 6)$, where $0 \leq l \leq 6$, the following table shows that the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1, where $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$.

Table 3.3.4.

l	$v_\sigma(0.1)$	$v_\sigma(0.4)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
0	$\frac{3n}{3}+1$	$\frac{3n}{3}$	$\frac{3n}{3}$	$\frac{5n}{3}$	$\frac{5n}{3}$	$\frac{5n}{3}$
1	$\frac{3n}{3}+1$	$\frac{3n}{3}$	$\frac{3n}{3}$	$\frac{5n+1}{3}-1$	$\frac{5n+1}{3}$	$\frac{5n+1}{3}$
2	$\frac{3n}{3}$	$\frac{3n}{3}+1$	$\frac{3n}{3}$	$\frac{5n-1}{3}$	$\frac{5n-1}{3}+1$	$\frac{5n-1}{3}$
3	$\frac{3n}{3}+1$	$\frac{3n}{3}$	$\frac{3n}{3}$	$\frac{5n}{3}$	$\frac{5n}{3}$	$\frac{5n}{3}$
4	$\frac{3n}{3}+1$	$\frac{3n}{3}$	$\frac{3n}{3}$	$\frac{5n+1}{3}$	$\frac{5n+1}{3}$	$\frac{5n+1}{3}-1$
5	$\frac{3n}{3}$	$\frac{3n}{3}+1$	$\frac{3n}{3}$	$\frac{5n-1}{3}$	$\frac{5n-1}{3}$	$\frac{5n-1}{3}+1$

From the above table 3.3.4 it is concluded that the Sunflower graph Sf_n is fuzzy quotient-3 cordial.

Example 3.3.5. The Sunflower graph Sf_6 is fuzzy quotient-3 cordial.



Theorem 3.4. *The Helm graph $H_n, n \geq 6$ is fuzzy quotient-3 cordial.*

Proof. Let G be a Helm graph H_n

$$V(G) = \{x_k : 1 \leq k \leq n\} \cup \{y_k : 1 \leq k \leq n\} \cup \{x\}$$

and

$$E(G) = \{x_k, x_{k+1} : 1 \leq k \leq n-1\} \cup \{x_1, x_n\} \cup \{x_k, y_k : 1 \leq k \leq n\}$$

$$\cup \{x, x_k : 1 \leq k \leq n\}$$

$$|V(G)| = 2n + 1, |E(G)| = 3n.$$

$\sigma(x) = 0.1$ for $n \equiv 0, 3 (\text{mod } 6)$ and $\sigma(x) = 0.3$ for $n \equiv 1, 2, 4, 5 (\text{mod } 6)$. For $1 \leq k \leq n$, the values of $\sigma(x_k)$ and $\sigma(y_k)$ are tabulated below.

Table 3.4.1.

Nature of n	$\sigma(x_k) = 0.1$	$\sigma(x_k) = 0.2$	$\sigma(x_k) = 0.3$
$n \equiv 1, 2, 4, 5 (\text{mod } 6)$	$k \equiv 1, 2 (\text{mod } 6)$	$k \equiv 4, 5 (\text{mod } 6)$	$k \equiv 0, 3 (\text{mod } 6)$
$n \equiv 3 (\text{mod } 6)$	$k \equiv 4, 5 (\text{mod } 6)$ and $k = n - 2$	$k \equiv 1, 2 (\text{mod } 6)$ and $k = n$	$k \equiv 0, 3 (\text{mod } 6)$ and $k = n - 1$

Table 3.4.2

Nature of n	$\sigma(z_k) = 0.1$	$\sigma(z_k) = 0.2$	$\sigma(z_k) = 0.3$
$n \equiv 0, 4 \pmod 6$	$k \equiv 0, 1 \pmod 6$	$k \equiv 3, 4 \pmod 6$	$k \equiv 2, 5 \pmod 6$
$n \equiv 1 \pmod 6$	$k \equiv 0, 1 \pmod 6$	$k \equiv 3, 4 \pmod 6$ and $k = n$	$k \equiv 2, 5 \pmod 6$
$n \equiv 2 \pmod 6$	$k \equiv 0, 1 \pmod 6$	$k \equiv 3, 4 \pmod 6$ and $k = 1, n$	$k \equiv 2, 5 \pmod 6$
$n \equiv 3 \pmod 6$	$k \equiv 3, 4 \pmod 6$ and $k = n - 2$	$k \equiv 0, 1 \pmod 6$ and $k = n - 1$	$k \equiv 2, 5 \pmod 6$ and $k = n$
$n \equiv 5 \pmod 6$	$k \equiv 0, 1 \pmod 6$ and $k = n - 2$	$k \equiv 3, 4 \pmod 6$ and $k = n - 1$	$k \equiv 2, 5 \pmod 6$ and $k = n$

For $n \equiv l(\text{mod}\, 6)$, where $0 \leq l \leq 6$, the following table shows that the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1, where $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$.

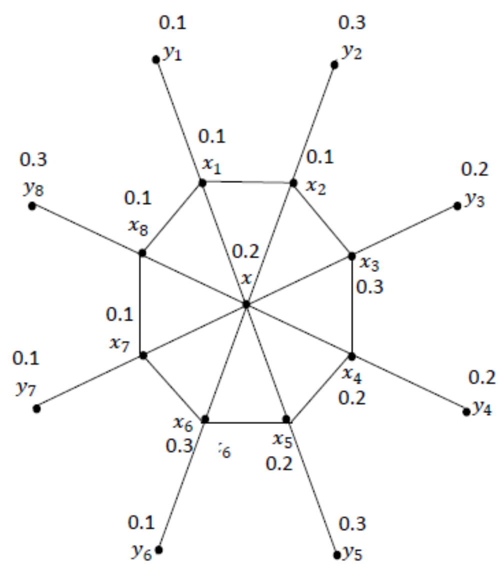
Table 3.4.3.

l	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
0	$\frac{2n+1-1}{3}+1$	$\frac{2n+1-1}{3}$	$\frac{2n+1-1}{3}$	$\frac{3n}{3}$	$\frac{3n}{3}$	$\frac{3n}{3}$
1	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{3n}{3}$	$\frac{3n}{3}$	$\frac{3n}{3}$
2	$\frac{2n+1+1}{3}$	$\frac{2n+1+1}{3}$	$\frac{2n+1+1}{3}-1$	$\frac{3n}{3}$	$\frac{3n}{3}$	$\frac{3n}{3}$
3	$\frac{2n+1-1}{3}+1$	$\frac{2n+1-1}{3}$	$\frac{2n+1-1}{3}$	$\frac{3n}{3}$	$\frac{3n}{3}$	$\frac{3n}{3}$

4	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{3n}{3}$	$\frac{3n}{3}$	$\frac{3n}{3}$
5	$\frac{2n+1-2}{3}+1$	$\frac{2n+1-2}{3}$	$\frac{2n+1-2}{3}+1$	$\frac{3n}{3}$	$\frac{3n}{3}$	$\frac{3n}{3}$

From the above table 3.4.3 it is concluded that the Helm graph H_n is fuzzy quotient-3 cordial.

Example 3.4.3. The Helm graph H_8 is fuzzy quotient-3 cordial.



Theorem 3.5. *The Shell graph is fuzzy quotient-*3 *cordial*, for $n \geq 6$.

Proof. Let G be a Shell graph.

$$V(G) = \{x_k : 1 \leq k \leq n\}$$

and

$$E(G) = \{x_k, x_{k+1} : 1 \leq k \leq n-1\} \cup \{x_1, x_n\} \cup \{x_1, x_{k+2} : 1 \leq k \leq n-3\}$$

$$|V(G)| = n, |E(G)| = 2n-3.$$

For $1 \leq k \leq n,$ the values of $\sigma(x_k)$ are tabulated below.

Table 3.5.1.

Nature of n	$\sigma(x_k) = 0.1$	$\sigma(x_k) = 0.2$	$\sigma(x_k) = 0.3$
$n \equiv 0, 1, 2 (\text{mod } 6)$	$k \equiv 2, 3 (\text{mod } 6)$	$k \equiv 0, 5 (\text{mod } 6)$	$k \equiv 1, 4 (\text{mod } 6)$
$n \equiv 3 (\text{mod } 6)$	$k \equiv 0, 5 (\text{mod } 6)$	$k \equiv 2, 3 (\text{mod } 6)$	$k \equiv 1, 4 (\text{mod } 6)$
$n \equiv 4, 5 (\text{mod } 6)$	$k \equiv 1, 2 (\text{mod } 6)$	$k \equiv 4, 5 (\text{mod } 6)$	$k \equiv 0, 3 (\text{mod } 6)$

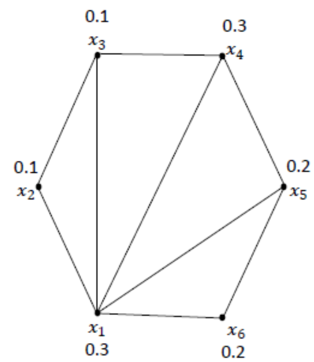
For $n \equiv l (\text{mod } 6)$, where $0 \le l \le 6$, the following table shows that the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1, where $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$.

Table 3.5.2.

l	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
0	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$
1	$\frac{n-1}{3}$	$\frac{n-1}{3}$	$\frac{n-1}{3}+1$	$\frac{2n-3+1}{3}-1$	$\frac{2n-3+1}{3}$	$\frac{2n-3+1}{3}$
2	$\frac{n+1}{3}$	$\frac{n+1}{3}-1$	$\frac{n+1}{3}$	$\frac{2n-3-1}{3}+1$	$\frac{2n-3-1}{3}$	$\frac{2n-3-1}{3}$
3	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$
4	$\frac{n-1}{3}+1$	$\frac{n-1}{3}$	$\frac{n-1}{3}$	$\frac{2n-3+1}{3}$	$\frac{2n-3+1}{3}$	$\frac{2n-3+1}{3}-1$
5	$\frac{n+1}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3}-1$	$\frac{2n-3-1}{3}$	$\frac{2n-3-1}{3}+1$	$\frac{2n-3-1}{3}$

From the above two table 3.5.2 it is concluded that the Shell graph is fuzzy quotient-3 cordial.

Example 3.5.3. The Shell graph Sl_6 is fuzzy quotient-3 cordial.



4. Conclusion

In this paper we proved that n-Sunlet graph, flower graph, Sun flower graph, helm graph, and shell graph are fuzzy quotient -3 cordial. The existence of Fuzzy quotient-3 cordial labeling of different families of graphs will be the future work.

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