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DEGREE BOUNDARIES AND MODEL OF COIN SPLITTING SYSTEM IN ANTI FUZZY GRAPH

P. KOUSALYA[1], V. GANESAN[2] and N. SATHYA SEELAN[3]

[1]Ph.D Research Scholar
[2, 3] Assistant Professor
Department of Mathematics
Thiru Kolanjiappar Government
Arts College (Grade - I)
Vridhachalam-606001, Tamilnadu, India
E-mail: ramya8848@gmail.com
vganesanmath@gmail.com
nseelan83@gmail.com

Abstract

The objective of this paper is to analyze the nature of effective edges in anti fuzzy graph and its complement. This investigation computes the boundaries of minimum and maximum effective degree of vertices in anti fuzzy graph. A necessary and sufficient condition is proved for a vertex in anti fuzzy graph is incident with effective edges are to be an isolated vertex in its complement graph for $(n \geq 1)$. An application of effective edges in coin splitting system is evaluated through complete anti fuzzy graph. Moreover this paper compares the order and size of anti fuzzy graph with its complementary graph to attain the inequality relationship.

1. Introduction

A mathematical model helps to discharge the problem in a complex situation. The best possible solution is to convert the problem into graph. Fuzzy graph theory has used to model many decision making problems in uncertain environment. It have numerous applications in computer science, modern science in technology, especially in the field of information theory, neural network, cluster analysis, diagnosis and control theory etc., The main

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part of the problem is considered as vertices and their relationship between theses vertices are considered as edges. These vertices and edges are assigned with fuzzy value to solve the vagueness. Some time, vagueness exists in a relation that attains maximum value. This type of model is known as Anti Fuzzy Graph which is the recent development area and an extension of fuzzy graphs.

In 1965, [1] L. A. Zadeh introduced the fuzzy sets to deal the real life decision problems, which are often uncertain. [2] Rosenfeld (1975) introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. [3] Mordeson introduced the concept of complement of fuzzy graphs. [4] M. S sunitha and A. Vijayakumar gave a modified definition of complement of fuzzy graph and proved a necessary and sufficient condition for a fuzzy graph to be self complementary. [5] and [6] R. Muthuraj and A. Sasireka introduced the concept of an Anti fuzzy graph and its degree of vertices. Moreover the definition of effective and weak edges are also studied. [7] A. Nagoorgani and J. Malarvizhi gave the result for size of fuzzy graph and its complement. [8] A. Nagoorgani and S. Shajitha Begum introduced the degree, order and size of intuitionistic fuzzy graphs. In which, they have categorized the minimum and maximum effective degree of IFG. [9] P. Kousalya and Dr. V. Ganesan investigate the properties of self complementary and complete fuzzy graphs with examples.

Based on this result, we describe the method to finding a minimum, maximum and effective degree of a vertex, order and size in AFG. This paper investigates the effective edges of an AFG and its characteristic. Minimum and maximum effective degree of a vertex and its boundaries in AFG's are identified. Order and size of a complement of AFG is also computes and compared with its underlying graph. A necessary and sufficient condition is proved for complete AFG $(K_n, n \geq 1)$ is to be complementary null graph. This relation is described by the model of coin splitting system.

2. Preliminaries

This part provides some important definitions which are used to originate the results of this paper. Throughout this paper we consider an undirected simple and connected graph. $G(\sigma, \mu)$ is a fuzzy graph $G_A(\sigma, \mu)$ is an anti fuzzy graph with underlying set S. That is, S is a fuzzy subset of non empty set is a mapping $\sigma : S \to [0, 1]$, and a fuzzy relation μ on fuzzy subset σ, is a fuzzy subset of $S \times S$. $\overline{G}_A(\overline{\sigma}, \overline{\mu})$ is the complement of AFG. The following are the required definitions to attain the main result of this paper.

1. The underlying crisp graph of the fuzzy graph $G(\sigma, \mu)$ is denoted as $G^*(\sigma^*, \mu^*)$ where $\sigma^* \operatorname{supp}(\sigma) = \{a \in S / \sigma(a) > 0\}$ and $\mu^* \operatorname{supp}(\mu) = \{(a, b) \in S \times S / \mu(a, b) > 0\}$.

2. A Fuzzy Graph $G(\sigma, \mu)$ with $\sigma : S \to [0, 1]$ and $\mu : S \times S \to [0, 1]$ such that $\mu(x, y) \leq \min(\sigma(x), \sigma(y)) \forall\, x, y$ in S.

3. A fuzzy graph $G_A(\sigma, \mu)$ with underlying set S, the order of G is defined and denoted as $p = \sum_{x \in S} \sigma(x)$ and size of fuzzy graph G is defined and denoted as $q = \sum_{y \in S} \mu(x, y)$.

4. Anti fuzzy graph $G_A(\sigma, \mu)$ consist of $\sigma : S \to [0, 1]$ and $\mu : S \times S \to [0, 1]$ such that $\mu(x, y) \geq \max(\sigma(x), \sigma(y)) \forall\, x, y$ in S.

5. The degree of a vertex $\sigma(x)$ of an anti fuzzy graph $G_A(\sigma, \mu)$ is denoted and by $d_{G_A}(\sigma(x)) = \sum_{x \neq y} \mu(x, y)$.

6. $G_A(\sigma, \mu)$ is called strong if $\mu(x, y) = \max(\sigma(x), \sigma(y))$ for all (x, y) in μ.

7. $G_A(\sigma, \mu)$ is called complete if $\mu(x, y) = \max(\sigma(x), \sigma(y))$ for all x, y in σ. Every complete graph has $\frac{a(a-1)}{2}$ edges for 'a' vertices.

8. The complement of Anti fuzzy graph $\overline{G}_A(\overline{\sigma}, \overline{\mu})$ is derived from an underlying Anti fuzzy graph $G_A(\sigma, \mu)$ where $\sigma = \overline{\sigma}$ and $\overline{\mu}(x, y) = 0$ if $\mu(x, y) > 0$. and $\overline{\mu}(x, y) = \max(\sigma(x), \sigma(y))$.

9. The modified definition of complement of anti fuzzy graph is denoted by $\overline{G}_A(\overline{\sigma}, \overline{\mu})$. Where $\sigma = \overline{\sigma}$ and $\overline{\mu}(x, y) = \mu(x, y) - \max(\sigma(x), \sigma(y))$.

10. If $G_A = \overline{G_A}$ then G_A is said to be a self complementary anti fuzzy graph.

11. The degree of a vertex $\sigma(u)$ of an anti fuzzy graph is sum of degree of membership of all those edges which are incident on vertex $\sigma(u)$ and is denoted by $d(\sigma(u)) = \sum_{u \neq v} \mu(u, v) = \sum_{uv \in vE} \mu(u, v)$.

3. Main Results

3. (A) Anti Fuzzy Graph (AFG) and its Complement

This section provides the methods to evaluate the minimum, maximum degree of a vertex in AFG. Order and size of AFG is defined. Some results are identified by comparing these results of AFG with its complement.

Definition 3A1. The minimum degree of a vertex in a AFG $G_A(\sigma, \mu)$ is denoted by $\delta(G_A)$ and it is defined by $\delta(G_A) = \delta(\sigma(G_A)) = \min\{d\,\sigma(x)/x \in \sigma\}$.

Definition 3A2. The maximum degree of a vertex in a AFG $G_A(\sigma, \mu)$ is denoted by $\Delta(G_A)$ and it is defined by $\Delta(G_A) = \Delta(\sigma(G_A)) = \max\{d\sigma(x)/x \in \sigma\}$.

Definition 3A3. The order of $G_A(\sigma, \mu)$ is defined by $p = o(G_A) = \sum_{x \in \sigma} \sigma(x)$.

Definition 3A4. The size of $G_A(\sigma, \mu)$ is defined by $q = \text{size}(G_A) = S(\mu(G_A)) = \sum_{xy \in \mu} \mu(x, y)$.

Example 1. Consider $G_A(\sigma, \mu)$ with $|\sigma| = 4$ and $|\mu| = 4$.

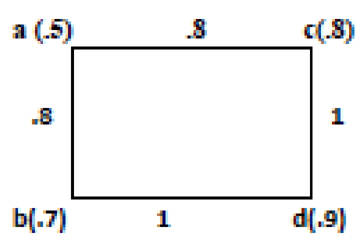

Figure 1. Anti Fuzzy Graph $G_A(\sigma, \mu)$.

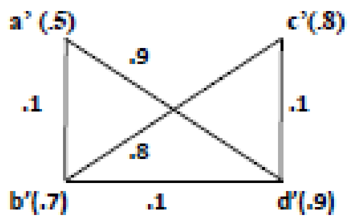

Figure 2. Complement of anti fuzzy graph $G_A(\sigma, \mu)$.

Result 3A5. If any vertex in AFG is isolated iff $\delta(G_A)$ is zero.

Result 3A6. If $\delta(G_A) = 0,$ iff the given graph is disconnected.

Theorem 3A7. *Every AFG and its complement satisfy the following inequality*

$$\text{Order}\,(G_A) = \text{Order}\,(\overline{G}_A) \text{ and } Size\,(G_A) + Size\,(\overline{G}_A) \geq \sum_{u \neq v} \sigma(x) \vee \sigma(v).$$

Proof. i. From the clarity of complement of AFG, $\sigma = \overline{\sigma}$ and $\overline{\mu}(x, y) = \mu(x, y) - \max(\sigma(x), \sigma(y)).$ This implies that, order $\text{Order}\,(G_A)$ $= \text{Order}\,(\overline{G}_A).$

ii. From the description of AFG,

$$\mu(x, y) \geq \max(\sigma(x), \sigma(y)) \tag{1}$$

$$\overline{\mu}(x, y) + \mu(x, y) = \max(\sigma(x), \sigma(y)) \tag{2}$$

$$\overline{\mu}(x, y) \geq \max(\sigma(x), \sigma(y)) \tag{3}$$

From all the above equations,

$$\max(\sigma(x), \sigma(y)) = \overline{\mu}(x, y) + \mu(x, y) \geq 2\max(\sigma(x), \sigma(y))$$

Therefore, $\text{Size}\,(G_A) + \text{Size}\,(\overline{G}_A) \geq 2\sum_{u \neq v} \sigma(x) \vee \sigma(y).$

From the above example (1),

$$\text{Size}\,(G_A) = 3.6; \text{Size}\,(\overline{G}_A) = 2;$$

$$\sum_{u \neq v} \sigma(x) \vee \sigma(y) = .8 + .9 + .9 + .7 = 3.3$$

$$\text{Size}(G_A) + \text{Size}(\overline{G}_A) \geq 2\sum_{u \neq v} \sigma(x) \vee \sigma(y). \leftrightarrow 5.6 \geq 6.6.$$

Remark. If the size is increased by the number of edges, at the same time the equality of RHS will also increased. This inequality is suitable for all kind of AFG's.

Theorem 3A8. *For* $K_n (n > 1)$ *AFG, its maximum degree cannot exceed* $n - 1$. That is, $\Delta(G_A) \leq n - 1$.

Proof. Consider the given AFG $G_A(\sigma, \mu)$ is complete with $|n|$ vertices. All the edges are adjacent to each other vertices with the membership values which are obtained by the maximum value of its corresponding vertices. Every complete graph has $\frac{n(n-1)}{2}$ edges.

By the Nature of AFG, the membership value of (σ, μ) is not exceed 1. The given graph is complete then, every $\mu(x, y) > 0$ and $\mu(x, y) = \max(\sigma(x), \sigma(y))$ for all x, y in σ.

Let $|n| = 4$, then the degree of every vertex is less than the sum of membership values of edges incident with it. Obviously, the degree of a vertex cannot exceed 3.

Example 2. K_4 Anti fuzzy graph.

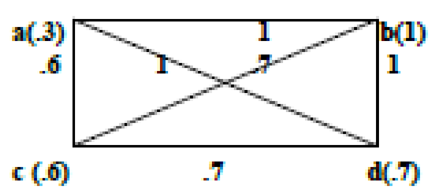

Figure 3. K_4 Anti Fuzzy Graph $G_A(\sigma, \mu)$.

$\Delta(G_A) = 3$. It cannot exceed $n - 1$.

Theorem 3A9. *For a given anti fuzzy graph* $G_A(\sigma, \mu)$ *with* $n(\sigma) = 1, 2$ *and* 3 *The following relation holds.*

i. *Order* $(G_A) =$ *Order* $(\overline{G}_A)$. ii. *Size* $(G_A) >$ *Size* $(\overline{G}_A)$.

Proof. Given AFG with $n = 3$ then its maximum degree cannot exceed 2[by theorem (3A.8)]. By the complement, $\sigma = \overline{\sigma}$ and $\overline{\mu}(x, y) = \mu(x, y) - \max(\sigma(x), \sigma(y))$.

From this we have concluded the order of both graphs is same.

If $\mu(x, y) = 0$, $\overline{\mu}(x, y) = \max(\sigma(x), \sigma(y))$.

If $\mu(x, y) \neq 0$, $\mu(x, y) \geq \max(\sigma(x), \sigma(y))$.

This implies, $\overline{\mu}(x, y) \leq \min(\sigma(x), \sigma(y))$.

Hence, $\text{Size}(G_A) > \text{Size}(\overline{G}_A)$ for $n = 1, 2, 3$.

Remark. The above result may vary for $n > 3$ according to the edge membership value of underlying graph.

Example 3.

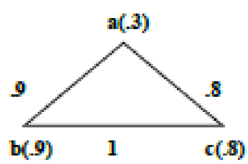

Figure 4. Anti Fuzzy Graph $G_A(\sigma, \mu)$.

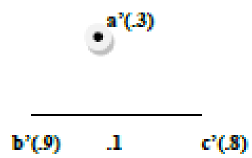

Figure 5. complement Anti Fuzzy Graph $\overline{G}_A(\sigma, \mu)$.

3 (B). Effective Edges of Anti Fuzzy Graph

This section investigates the effective edges of anti fuzzy graph. It provides the method to finding the effective degree of a vertex and its minimum, maximum degrees in anti fuzzy graphs. The boundaries of $\delta_E(G_A)$ and $\Delta_E(G_A)$ in a complete AFG is obtained. A necessary and sufficient condition is proved for a vertex in anti fuzzy graph is incident with effective edges are to be an isolated vertex in its complement graph for $(n \geq 1)$. The obtained result is compared with real life application.

Definition 3B1. If $\mu(x, y) = \sigma(x) \vee \sigma(y)$ in $G_A(\sigma, \mu)$ then $\mu(x, y)$ is called an effective edge. It is denoted by $d_E(\mu(x, y))$. Otherwise the edge is called weak.

Definition 3B2. An effective degree of a vertex is defined by, $d_E(x) = \sum_{x \neq y} dE(\mu(x, y))$. $d_E(x) = \sum_{x \neq y} dE(\mu(x, y))$, where $x \in \sigma$. That is an effective degree of a vertex is obtain from the sum of membership value of the effective edges incident with σ.

Definition 3B3. Minimum effective degree of a vertex in AFG is denote and defined by $\delta_E(G_A) = \delta_E(\sigma(G_A)) = \min\{d_E\sigma(x)/x \in \sigma\}$.

Definition 3B4. Maximum effective degree of a vertex in AFG is denote and defined by $\Delta_E(G_A) = \Delta_E(\sigma(G_A)) = \max\{d_E\sigma(x)/x \in \sigma\}$.

From Example (1)

Effective degree of a vertex in AFG	Effective degree of a vertex in complement AFG
$d_E(a) = d(.5) = .8 + 0 = .8$	$d_E(a') = d(.5) = .9 + 0 + 0 = .9$
$d_E(b) = 0; d_E(.c) = .8; d_E(d) = 0$	$d_E(b') = .8; d_E(.c') = .8; d_E(d) = .9$

Result 3B5. If a vertex is not incident with any effective edges iff $\delta_E(G_A) = 0$. It is not necessarily said that the given graphs is disconnected.

Result 3B6. A vertex in AFG is adjacent to each other vertices which are all effective edges then it will be an isolated vertex in complement graph. (Example-3: Vertex ‘a’ is incident with the effective edges. So, the complement graph gives an isolated vertex).

Theorem 3B7. *If AFG is complete, then all the edges are effective. But the converse need not be true.*

Proof. By the definition of complete AFG, $\mu(x, y) = \max(\sigma(x), \sigma(y))$ for all x, y in σ. For $K_n (n > 1)$ every vertex is adjacent with $n - 1$ vertices of $G_A(\sigma, \mu)$. This implies that, every $n - 1$ edges satisfies, $\mu(x, y) = \max(\sigma(x), \sigma(y))$. Clearly, every edge is effective in K_n. Conversely, if all the edges are effective then the underlying graph may be strong.

Theorem 3B8. *For* $K_n (n > 1)$ *Its maximum effective degree cannot exceed* $n - 1$.

Proof. Consider AFG $G_A(\sigma, \mu)$ is complete with $|n|$ vertices, $\mu(x, y) = \max(\sigma(x), \sigma(y))$ for all x, y in σ. Every complete graph having $\frac{n(n-1)}{2}$ vertices.

By the Nature of AFG, the membership value of (σ, μ) is lies between o to 1. Let $|n| = 4$ then the degree of every vertex is less than the sum of membership values of edges incident with it.

Obviously, the effective degree of a vertex cannot exceed 3. (Example-2 shows the result) Therefore, The maximum effective degree $\Delta_E(G_A) \leq n-1$.

Theorem 4B9. *For $K_n(n > 1)$ AFG, its minimum effective degree exist with $\epsilon \leq \delta_E(G_A) \leq n-1$.*

Proof. Consider the given AFG $G_A(\sigma, \mu)$ is complete $K_n(n > 1)$. Every graph satisfies, $\delta_E(G_A) \leq \Delta(G_A)$. If $\mu(x, y) = 0$, then we get a contradiction to the statement. This implies, every $\mu(x, y) > 0$. Moreover by theorem (3B.8), $\Delta(G_A) \leq n-1$. Hence we concluded, $\epsilon \leq \delta_E(G_A) \leq n-1$.

Theorem 3B10. *A vertex is adjacent to $n-1$ vertices incident with effective edges in $G_A(\sigma, \mu)$ is corresponding to an isolated vertex of $\overline{G}_A(\overline{\sigma}, \overline{\mu})$. The converse is also true.*

Proof. Consider the AFG $G_A(\sigma, \mu)$ with a vertex is adjacent to $n-1$ vertices and having the edges are all effective.

$\leftrightarrow$ Then $\mu(x, y) = \max(\sigma(x), \sigma(y))$ for all x, y.

$\leftrightarrow$ By the complement, $\overline{\mu}(x, y) = \mu(x, y) - \max(\sigma(x), \sigma(y)) = 0$ for all x, y.

$\leftrightarrow$ Hence it is an isolated vertex in $\overline{G}_A(\overline{\sigma}, \overline{\mu})$.

Theorem 3B11. *Complement of $K_n(n > 1)$ is a null graph.*

Proof. If the AFG $G_A(\sigma, \mu)$ is complete $K_n(n > 1)$ then all the edges are effective. $\mu(x, y) = \max(\sigma(x), \sigma(y))$ for all x, y. Therefore, $\overline{\mu}(x, y) = 0$ for all

x, y. By the theorem (4.10), every vertex is an isolated. Hence, we get a null x, y.

By the theorem (3B.10), every vertex is an isolated. Hence, we get a null graph for complement of K_n.

Remark. In this case we can say that, K_n cannot be the self complementary.

4. Application of Effective Edges in AFG Model of Coin Splitting System

This section displays the relevance of effective edges in anti fuzzy graph and its complement graph. This scenario shows by an example of coin splitting system. This model is the best example of theorem 3B.10 and 3B.11.

Our Indian money contains 4 types of coins in rupees such as 1, 2, 5 and 10 which are used by peoples in every day. This model is converted to anti fuzzy graph by considering the rupees as vertices and their relationship as edges. The relationship is considered by splitting the coins by one rupee.

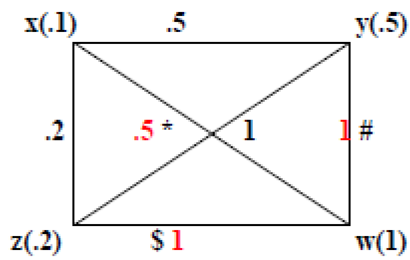

Figure 6. Model of coin splitting system: K_n Anti fuzzy Graph.

From the above graph x, y, z and w represents the vertices which considered by the coins of 1, 5, 2 and 10 respectively with fuzzy value. Take every vertices is adjacent with each other, because every coins can be written by the sum of 1 rupee coin. To make a complete anti fuzzy graph, we should assign each edge membership value is exactly the maximum of vertex value.

A 2, 5 and 10 rupee coin contains the same number of one rupee coin itself. On this basis, we could assign the edge value between 1 to other coins 2, 5and 10 is .2, .5 and 1 respectively. By the same way for *,$ and # we can assign .5, 1 and 1 respectively.

From these values of coin splitting system, we get complete anti fuzzy graph (K_4). If we are finding a CAFG for this model, we get a null graph. Because all the coins are exactly divide to one rupee coin. Hence, Complement of K_n is a null graph.

6. Conclusion

The complement of anti fuzzy graph is obtained with its underlying graph. Maximum, minimum degree of a vertex, order and size of anti fuzzy graph are computed and compared with its complement graph. By the observation, Every Anti Fuzzy Graph satisfies Order (G_A) = order $(\overline{G}_A)$ and $\text{Size}(G_A) + \text{Size}(\bar{G}_A) \geq 2\sum_{u \neq v} \sigma(x) \vee \sigma(y)$. In a complete AFG $(K_n, n > 1)$ its maximum effective degree $\Delta(G_A) \leq n-1$. The minimum effective degree satisfies $\epsilon \leq \delta_E(G_A) \leq n-1$. In a complete AFG, the edges are effective then we got an isolated vertex in its complement. That is, K_n does not preserves the self complementary. This outcome is proved by taking the model of one rupee coin splitting system in AFG. We hope that this idea will lead to find more application around us.

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