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GEOMETRIC MEAN CORDIAL LABELING OF SOME GRAPHS

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Abstract

Let $G = (V, E)$ be a graph and f be a mapping from $V(G) \to \{0, 1, 2\}$. For each edge uv assign the label $\lceil \sqrt{f(u)\, f(v)} \rceil$, f is called a geometric mean cordial labeling if $| v_f(i) - v_f(j) | \leq 1$ and $| e_f(i) - e_f(j) | \leq 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x, $x \in \{0, 1, 2\}$. A graph with a geometric mean cordial labeling is called geometric mean cordial graph [3]. In this paper we investigate the geometric mean cordial labeling of Jewel graph J_n, Friendship graph F_n and Tadpole graph $T(n, n)$.

1. Introduction

In this paper we consider only finite, simple, connected and undirected graphs. A graph labeling is an assignment of integers to the vertices or edges, or both subject to certain conditions. The concept of mean labeling was first introduced by S. Somasundaram and R. Ponraj and it was developed in their further research papers [8]. S. K. Vaidya has discussed the mean labeling in the context of some graph operations [9]. A. Durai Baskar introduced geometric mean labeling of graph [4]. The concept of geometric mean cordial

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labeling was developed by K. Chitra Lakshmi and K. Nagarajan [3]. In our previous research article, V. Annamma and Jawahar Nisha M. I. [2], we have proved that the graphs such as comb graph, crown graph and quadrilateral snake graph admits geometric mean cordial labeling. In this paper we investigate the geometric mean cordial labeling of Jewel graph J_n, Friendship graph F_n and Tadpole graph $T(n, n)$.

2. Preliminaries

Definition 2.1[1]**.** The Jewel Graph J_n is the graph with vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and edge set $E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.

Definition 2.2[6]**.** The Friendship graph F_n is a set of n triangles having a common central vertex.

Definition 2.3[7]**.** Tadpole graph $T(n, n)$ is obtained by joining a cycle graph C_n to a path graph P_n with a bridge.

3. Main Results

Theorem 3.1. *The jewel graph* J_n *is a geometric mean cordial graph if* $n \equiv 0 (mod\ 3)$ *and* $n \geq 4$.

Proof. Let G be a Jewel graph J_n with vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and edge set $E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.

Let l denote the number of vertices and k denote the number of edges.

Then the Jewel graph J_n is as in figure 3.1(a)

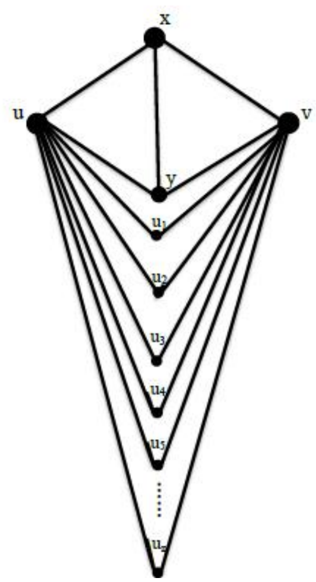

Figure 3.1(a). Jewel graph J_n.

Define $f : V(G) \to \{0, 1, 2\}$ as follows:

Here $n \equiv 0(\bmod 3)$, $n \geq 4$. Let $n = 3t$, $t \geq 2$.

Case (i) When $n = 6$ **i.e. When** $t = 2$

$f(u) = f(v) = f(x) = f(y) = 1$

$f(u_{2i-1}) = 0$

$f(u_{2i}) = 2, \forall 1 \leq i \leq 3.$

Here $l = 10$ and $k = 17$.

So $v_f(0) = v_f(2) = 3$, $v_f(1) = 4$, $e_f(0) = e_f(2) = 6$, $e_f(1) = 5$.

Case (ii) When $n > 6$ **i.e. When** $t > 2$

$f(u) = f(v) = f(x) = f(y) = 1$

$f(u_{2i-1}) = 0$

$f(u_{2i}) = 2, \quad \forall 1 \leq i \leq 3.$

$$f(u_i) = \begin{cases} 0, & \text{i} \equiv 1(\text{mod } 3) \\ 1, & \text{i} \equiv 2(\text{mod } 3) \\ 2, & \text{i} \equiv 0(\text{mod } 3) \end{cases} \quad \forall i \geq 7.$$

Here $l \equiv 1(\text{mod } 3)$ i.e. $l = 3x + 1,$ so $v_f(0) = v_f(2) = x,\ v_f(1) = x + 1.$

Also $k \equiv 2(\text{mod } 3)$ i.e. $k = 3y + 2,$ so $e_f(0) = e_f(2) = y + 1,\ e_f(1) = y.$

From the above two cases, we see that $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$ and hence f is a geometric mean cordial labeling.

Illustration 3.1.

Geometric Mean Cordial Labeling of Jewel graph J_6 is shown in Figure 3.1(b)

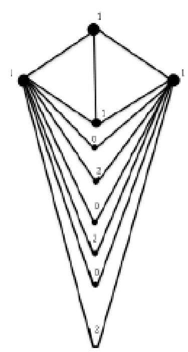

Figure 3.1(b). Jewel graph J_6.

Theorem 3.2. *Friendship graph F_n is a geometric mean cordial graph if* $n \equiv 0(mod\ 3)$.

Proof. Let G be a Friendship graph F_n with vertex set $V(F_n) = \{v, v_i : 1 \leq i \leq n\}$ and edge set $E(F_n) = \{v_{2i-1}v_{2i} : 1 \leq i \leq n\}$ $\bigcup \{vv_i : 1 \leq i \leq n\}$.

Let l denote the number of vertices and k denote the number of edges. Then $l = 2n + 1,\ k = 3n$.

Then the Friendship graph F_n is as in figure 3.2(a)

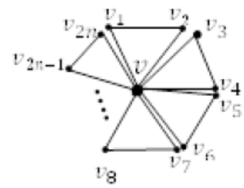

Figure 3.2(a). Jewel graph F_n.

Define $f : V(G) \to \{0, 1, 2\}$ as follows:

Here $n \equiv 0 (\text{mod } 3)$. Let $n = 3t, t \geq 1$.

Then $l \equiv 1 (\text{mod } 6)$. Let $l = 6s + 1$, s ≥ 1. Also $k \equiv 1 (\text{mod } 9)$. Let $k = 9m$, $m \geq 1$.

$f(v) = 1$

$$f(v_i) = \begin{cases} 0, & 1 \leq i \leq 2s \\ 1, & 2s + 1 \leq i \leq 4s \\ 2, & 4s + 1 \leq i \leq 6s. \end{cases}$$

So $v_f(0) = v_f(2) = 2s$, $v_f(1) = 2s + 1$ and $e_f(0) = e_f(2) = e_f(1) = 3m$.

From the above case, we see that $| v_f(i) - v_f(j) | \leq 1$ and $| e_f(i) - e_f(j) | \leq 1$ for all $i, j \in \{0, 1, 2\}$ and hence f is a geometric mean cordial labeling.

Illustration 3.2.

Geometric Mean Cordial Labeling of Friendship graph F_6 is shown in Figure 3.2(b)

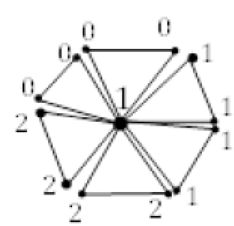

Figure 3.2(b). Jewel graph F_6.

Theorem 3.3. *Tadpole graph $T_{(n,\, n)}$ is a geometric mean cordial graph except when $n \equiv 0(mod\ 3)$.*

Proof. Let G be a Tadpole graph $T_{(n,\, n)}$ with vertex set $V(T_{(n,\, n)}) = \{v_i,\, u_i : 1 \le i \le n\}$ and edge set $E(J_n) = \{v_iv_{i+1},\, u_iu_{i+1} : 1 \le i \le n-1\} \cup \{v_nv_1\} \cup \{v_1u_1\}$.

Let l denote the number of vertices and the number of edges. Then $l = 2n$.

Then the Tadpole graph $T_{(n,\, n)}$ is as in figure 3.3(a)

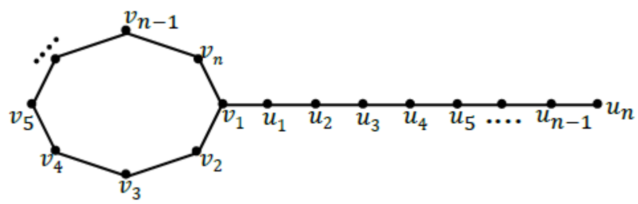

Figure 3.3 (a). Tadpole Graph $T_{n,\, n}$.

Define $f : V(G) \to \{0, 1, 2\}$ as follows:

Case (i) $n \equiv 1(\text{mod}\ 3)$. Let $n = 3t + 1$.

$$f(v_i) = \begin{cases} 2, & 1 \le i \le t \\ 1, & t+1 \le i \le 3t+1 \end{cases} \text{ and } f(u_i) = \begin{cases} 2, & 1 \le i \le t+1 \\ 0, & t+1 \le i \le 3t+1. \end{cases}$$

Then $l \equiv 2(\text{mod}\ 3)$ i.e. $1 = 3s + 1$ so $v_f(2) = s,\, v_f(1) = v_f(0) = s+1$, $e_f(0) = e_f(2) = s+1,\, e_f(1) = s$.

Case (ii) $n \equiv 2(\text{mod}\ 3)$. Let $n = 3t + 2$.

$$f(v_i) = \begin{cases} 2, & 1 \le i \le t \\ 1, & t+1 \le i \le 3t+2 \end{cases} \text{ and } f(u_i) = \begin{cases} 2, & 1 \le i \le t+1 \\ 0, & t+2 \le i \le 3t+2. \end{cases}$$

Then $l \equiv 1(\text{mod}\ 3)$ i.e. $1 = 3s + 1$ so $v_f(0) = v_f(2) = s,\, v_f(1) = s+1$, $e_f(2) = s+1,\, e_f(0) = e_f(1) = s$.

From the above two cases, we see that $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$ and hence f is a geometric mean cordial labeling.

Illustration 3.3.

Geometric Mean Cordial Labeling of Tadpole graph $T_{(8,\, 8)}$ is shown in Figure 3.3(b)

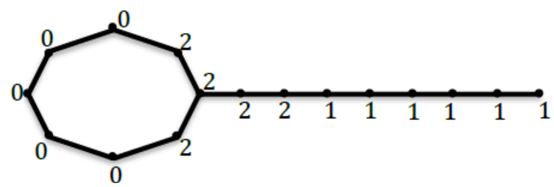

Figure 3.3 (b). Tadpole Graph $T_{8,\, 8}$.

4. Conclusion

In this paper we have examined the graphs such as Jewel graph J_n, Friendship graph F_n and Tadpole graph $T_{(n,\, n)}$ admits geometric mean cordial labeling. Similar results on various graphs are an open area of research. Geometric mean cordial labeling has various applications, one of its prominent applications being the communications network addressing. The vertices of the graph can be considered as the nodes of the communication network, which helps in transmission of messages over the links of communication.

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