Advances and Applications in Mathematical Sciences
Volume 18, Issue 11, September 2019, Pages 1295-1307

NUMERICAL SOLUTION OF INTUITIONISTIC FUZZY DIFFERENTIAL EQUATION BY USING RK FEHLBERG METHOD

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Abstract

The aim of this paper is to propose RK_Fehlberg method for solving intuitionistic fuzzy differential equations (IFDEs). Convergence analysis of RK_Fehlberg method has been carried out. The applicability of RK_Fehlberg method is illustrated by solving fuzzy differential equations with triangular intuitionistic fuzzy numbers. Comparison of the numerical solution with exact solution shows good accuracy.

1. Introduction

The concept of fuzzy number and fuzzy arithmetic are introduced by Zadeh [13], Dubois and Parade [3]. The fuzzy sets theory is considered to be one of Intuitionistic fuzzy set (IFS) theory. Atanassov [2] was first proposed Intuitionistic fuzzy set and found to be suitable to deal with unexplored areas. Now-a-days, IFSs are being studied widely and being used in different fields of biology, engineering, physics as well as among other field of science.

Numerical solution of FDE by Runge-Kutta method with intuitionistic treatment treated by Abbasbandy and Allahviranloo [1]. Jayakumar and Kanagarajan, have discussed the numerical solution for hybrid fuzzy differential equations by Ruge-Kutta Fehlberg method. Melliani et al. [7, 8] have discussed differential and partial differential equations under intuitionistic fuzzy environment. Sneh Lata and Amit Kumar [12] have introduced time dependent intuitionistic fuzzy linear differential equation.

2010 Mathematics Subject Classification: 34Axx, 34A07, 65Cxx, 03F55.
Keywords: intuitionistic fuzzy differential equations, FIVP, RK_Fehlberg method, convergence.
Received July 11, 2019; Accepted September 21, 2019

The strong and weak solution of intuitionistic fuzzy ordinary differential equation and introduced second order linear differential equations using the fuzzy boundary value by Sankar Prasad Mondal and Tapan Kumar Roy in [6, 11]. The convergence and stability of fuzzy initial value problems discussed by Kelava [5], Ma et al. [9]. Nirmala and chenthur Pandian [10] proposed for solving IFDE under the differentiability. System of fuzzy differential equation (SFDE) is the one of the most important differential equation for uncertainty modeling.

In this his paper presents Runge-Kutta Fe methods based AM, and CeM and CoM for solving intuitionistic fuzzy IVPs. The efficiency of these methods has been illustrated by numerical example. The RK methods compared with AM, CeM and CoM.

2. Preliminaries

Definition 2.1. A fuzzy set $\widetilde{A}$ is defined by $\widetilde{A} = \{(X, \mu_{\widetilde{A}}(x)) : x \in A, \mu_{\widetilde{A}}(x) \in [0, 1]\}$. In the pair $(x, \mu_{\widetilde{A}}(x))$ the first element $x \in A$ and second element $\mu_{\widetilde{A}}(x) \in [0, 1]$ called membership function.

Definition 2.2. Intuitionistic Fuzzy set (IFS):

Let a set X be fixed. An IFS A in X is an object having the form $A = \{\langle x, \mu_A(x) \wp_A(x) > 0 : x \in X\}$ where the $\mu_A(x) : X \to [0, 1]$ and $\delta_A(x) : X \to [0, 1]$ define the degree of membership and degree of non-membership respectively, of the element $x \in X$ to the set A which is a subset of X, for every element of $x \in X$, $0 < \mu_A(x) + \wp_A(x) \leq 1$.

Definition 2.3. An intuitionistic fuzzy set $A = \{x, \mu_A(x), \wp_A(x) \backslash x \in X\}$ such that $\mu_A(x)$ and $(1 - \wp_A\,(x) = 1 - \wp_A(x), \forall x \in R$ are fuzzy numbers, is called an intuitionistic fuzzy number.

Therefore IFS $A = \{x, \mu_A(x), \wp_A(x) \backslash x \in X\}$ is a conjecture of two fuzzy numbers, A^+ with a membership function $\mu_{A^+}(x) = \mu_A(x)$ and A^- with a membership function $\mu_{A^-}(x) = 1 - \wp_A(x)$.

Definition 2.4. The α-cut of an IFN $A = \{x, \mu_A(x), \wp_A(x) \backslash x \in X\}$ is defined as follows:

$$A = \{x, \mu_A(x), \wp_A(x) \backslash x \in X, \mu_A(x) \geq \alpha \text{ and } \wp_A(x) \leq 1 - \alpha\} \forall x \in [0, 1].$$

The α-cut representation of IFN A generates the following pair of intervals and is denoted by $[A]_\alpha = \{[A_L^+(\alpha), A_{ij}^+(\alpha)], [A_L^-(\alpha), A_{ij}^-(\alpha)]$

Definition 2.5. A (Triangular Intuitionistic Fuzzy Number A is an intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and non-membership function $\wp_A(x)$ given as follows:

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_2 \\ 0, \text{ otherwise} \end{cases} \quad \wp_A(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1}, & a_1' \leq x \leq a_2 \\ \dfrac{x - a_2}{a_3' - a_2}, & a_2 \leq x \leq a_3', \\ 1, \text{ otherwise} \end{cases}$$

Where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and triangular intuitionistic fuzzy number is denoted by $A = (a_1, a_2, a_3; a_1', a_2, a_3')$.

Definition 2.6. Let mapping $f : I \to W^n$ for some interval I be an intuitionistic fuzzy function. The α, β cut of f is given by $[f(t)]_{\alpha,\beta} = \{[f_1(t; \alpha), f_2(t, \alpha)], [f_3(t; \beta), f_4(t, \beta)]\}$, where

$$f_1(t; \alpha) = \min \{f^+(t; \alpha) | t \in I, 0 \leq \alpha \leq 1\},$$

$$f_2(t; \alpha) = \max \{f^+(t; \alpha) | t \in I, 0 \leq \alpha \leq 1\},$$

$$f_3(t; \alpha) = \min \{f^-(t; \beta) | t \in I, 0 \leq \beta \leq 1\},$$

$$f_4(t; \alpha) = \max \{f^-(t; \beta) | t \in I, 0 \leq \beta \leq 1\}.$$

3. Intuitionistic Fuzzy Cauchy Problem

A first order intuitionistic fuzzy differential equation is a differential equation of the form

$$\begin{cases} y'(t) - f(t, y(t)), t \in [a, b] \\ y(t_0) = y_0 \end{cases}$$

where y is an intuitionistic fuzzy function of the crisp variable t, $f(t, y(y))$ is an intuitionistic fuzzy function of the crisp variable t and the intuitionistic fuzzy variable y and y' is the intuitionistic fuzzy derivative. If an initial value $y(t_0) = y_0$ {intuitionistic fuzzy number}, we get an intuitionistic fuzzy Cauchy problem of first order $y'(t) = f(t, y(t)), y(t_0) = y_0$. As each intuitionistic fuzzy number is a conjecture two fuzzy numbers and above equation can be replaced by an equivalent system as follows:

$$y'(t) = \{[y_1(t; \alpha), y_2(t; \alpha)], [y_3(t; \beta), y_4(t; \beta)]\}, \tag{1}$$

where

$$\begin{aligned} y_1'(t; \alpha) = f_1(t, y^+) &= \min \{f_1(t, u) \mid u \in [y_1, y_2]\} \\ &= F(t, y_1, y_2), y_1(t_0) = y_{1,0} \end{aligned} \tag{2}$$

$$\begin{aligned} y_2'(t; \alpha) = f_2(t, y^+) &= \max \{f_2(t, u) \mid u \in [y_1, y_2]\} \\ &= G(t, y_1, y_2), y_2(t_0) = y_{2,0} \end{aligned} \tag{3}$$

$$\begin{aligned} y_3'(t; \alpha) = f_3(t, y^-) &= \min \{f_3(t, u) \mid u \in [y_3, y_4]\} \\ &= H(t, y_3, y_4), y_3(t_0) = y_{3,0} \end{aligned} \tag{4}$$

$$\begin{aligned} y_4'(t; \alpha) = f_4(t, y^-) &= \max \{f_4(t, u) \mid u \in [y_3, y_4]\} \\ &= I(t, y_3, y_4), y_4(t_0) = y_{4,0}. \end{aligned} \tag{5}$$

The system of equations given in (2) and (3) will have unique solution $[y_1, y_2] \in B = \bar{c}[0, 1] \times \bar{c}[0, 1]$ and the system of equations given in (4) and (5) will have unique solution $[y_3, y_4] \in B = \bar{c}[0, 1] \times \bar{c}[0, 1]$.

4. Runge-Kutta-Fehlberg Method for Solving Intutionistic Fuzzy Differential Equation

Consider the fuzzy initial value problem

$$\begin{cases} y'(t) = f(t, y(t)), & t \in [t_0 \in T] \\ y(t_0) = y_0 \end{cases} \tag{4.1}$$

where y is a fuzzy function of $f(t, y)$ is a fuzzy operation of the crisp variable t and the fuzzy variable y, y' is the fuzzy derivative of y and $y(t_0) = y_0$ is a fuzzy number.

The general form of RK for intuitionistic fuzzy initial value problems

$$y_{1n+1} = y_{1n} + h\sum_{i=1}^{s} b_i K_i \, y_{2n+1} = y_{2n} + h\sum_{i=1}^{s} b_i L_i$$

$$y_{3n+1} = y_{3n} + h\sum_{i=1}^{s} b_i M_i \, y_{4n+1} = y_{4n} + h\sum_{i=1}^{s} b_i N_i \tag{4.2a}$$

where

$$K_i = F\left(t_n + c_i h,\ y_{1n} + h\sum_{j=1}^{s} a_{ij} K_j\right)$$

$$L_i = G\left(t_n + c_i h,\ y_{2n} + h\sum_{j=1}^{s} a_{ij} L_j\right),$$

$$M_i = H\left(t_n + c_i h,\ y_{1n} + h\sum_{j=1}^{s} a_{ij} M_j\right),$$

$$N_i = I\left(t_n + c_i h,\ y_{2n} + h\sum_{j=1}^{s} a_{ij} N_j\right),\ i = 1, 2, \ldots, s, \tag{4.2b}$$

where b_i's and c_i's are constants. By setting $s = 6$. Then

$$y_1(t_{n+1}, \alpha) = y_1(t_n, \alpha) + \frac{16}{135} K_1 + \frac{6656}{12825} K_3 + \frac{28561}{56430} K_4 - \frac{9}{50} K_5 + \frac{2}{55} K_6,$$

where

$$K_1 = hF(t_n, y_1(t_n, \alpha) + y_2(t_n, \alpha)),$$

$$K_2 = hF\left(t_n + \frac{1}{4} h, y_1(t_n, \alpha) + \frac{1}{4} K_1, y_2(t_n, \alpha) + \frac{1}{4} L_1\right),$$

$$K_3 = hF\left(t_n + \frac{12}{13} h, y_1(t_n, \alpha) + \frac{3}{32} K_1 + \frac{9}{32} K_2, y_2(t_n, \alpha) + \frac{3}{32} L_1 + \frac{9}{32} L_2\right),$$

$$K_4 = hF\left(t_n + \frac{12}{13} h, y_1(t_n, \alpha) + \frac{1932}{2197} K_1 - \frac{7200}{2197} K_2 + \frac{7296}{2197}\right.$$

$$\left. K_3, y_2(t_n, \alpha) + \frac{1932}{2197} L_1 - \frac{7200}{2197} L_2 + \frac{7296}{2197} L_3\right)$$

$$K_5 = hF\left(t_n + h, y_1(t_n, \alpha) + \frac{429}{216} K_1 - 8K_2 + \frac{3680}{513} K_3 - \frac{845}{4104} K_4\right.$$

$$\left. y_2(t_n, \alpha) + \frac{439}{216} L_1 - 8L_2 + \frac{3680}{513} L_3 - \frac{845}{4104} L_4\right),$$

$$K_6 = hF\left(t_n + h, y_1(t_n, \alpha) - \frac{8}{27} K_1 + 2K_2 - \frac{3544}{2565} L_3 + \frac{1859}{4104} K_4 - \frac{11}{40} K_5\right.$$

$$\left. y_2(t_n, \alpha) - \frac{8}{27} L_1 + 2L_2 - \frac{3544}{2565} L_3 + \frac{1859}{4104} L_4 - \frac{11}{40} L_5\right),$$

$$y_2(t_{n+1}, \alpha) = y_2(t_n, \alpha) + \frac{16}{135} L_1 + \frac{6656}{12825} L_3 + \frac{28561}{56430} L_4 - \frac{9}{50} L_5 + \frac{2}{55} L_6,$$

where

$$L_1 = hG(t_n, y_1 y_2(t_n, \alpha), y_2(t_n, \alpha)),$$

$$L_2 = hG\left(t_n + \frac{1}{4} h, y_1(t_n + \alpha) + \frac{1}{4} K_1, y_1(t_n + \alpha) + \frac{1}{4} L_1\right),$$

$$L_3 = hG\left(t_n + \frac{3}{8} h, y_1(t_n + \alpha) + \frac{3}{32} K_1 + \frac{9}{32} K_2,\right.$$

$$y_2(t_n+\alpha)+\frac{3}{32}L_1+\frac{9}{32}L_2\Big),$$

$$L_4 = hG\Big(t_n+\frac{12}{13}h,\ y_1(t_n+\alpha)+\frac{1932}{2197}K_1+\frac{7200}{2197}K_2$$

$$+\frac{7296}{2197}K_3,\ y_2(t_n+\alpha)+\frac{1932}{2197}L_1-\frac{7200}{2197}L_2+\frac{7296}{2197}L_3\Big),$$

$$L_5 = hG\Big(t_n+h,\ y_1(t_n+\alpha)+\frac{429}{216}K_1-8K_2+\frac{3680}{513}K_3-\frac{845}{4104}K_5,$$

$$y_2(t_n+\alpha)+\frac{439}{216}L_1-8L_2+\frac{3680}{513}L_3-\frac{845}{4104}L_5\Big),$$

$$L_6 = hG\Big(t_n+h,\ y_1(t_n+\alpha)-\frac{8}{27}K_1+2K_2-\frac{3544}{2565}K_3+\frac{1859}{4104}K_4-\frac{11}{40}K_5,$$

$$y_2(t_n+\alpha)-\frac{8}{27}L_1+2L_2-\frac{3544}{2565}L_3+\frac{1859}{4104}L_4-\frac{11}{40}L_5\Big),$$

$$y_3(t_{n+1},\beta) = y_3(t_n,\beta)+\frac{16}{135}M_1+\frac{6656}{12825}M_3+\frac{28561}{56430}M_4-\frac{9}{50}M_5+\frac{2}{55}M_6,$$

where

$$M_1 = hH(t_n,\ y_3(t_n,\beta),\ y_4(t_n,\beta)),$$

$$M_2 = hH\Big(t_n+\frac{1}{4}h,\ y_3(t_n,\beta)+\frac{1}{4}M_1,\ y_4(t_n,\beta)+\frac{1}{4}N_1\Big),$$

$$M_3 = hH\Big(t_n+\frac{3}{8}h,\ y_3(t_n,\beta)+\frac{3}{32}M_2+\frac{9}{32}M_2,$$

$$y_4(t_n,\beta)+\frac{3}{32}N_2+\frac{9}{32}N_2\Big),$$

$$M_4 = hH\Big(t_n+\frac{12}{13}h,\ y_3(t_n,\beta)+\frac{1932}{2197}M_1-\frac{7200}{2197}M_2$$

$$+\frac{7296}{2197}M_3,\ y_4(t_n,\beta)+\frac{1932}{2197}N_1-\frac{7200}{2197}N_2+\frac{7296}{2197}N_3\Big),$$

$$M_5 = hH\Big(t_n+h,\ y_3(t_n,\beta)+\frac{439}{216}M_1-8M_2+\frac{3680}{513}M_3-\frac{845}{4104}M_5,$$

$$y_4(t_n, \beta) + \frac{429}{216} N_1 - 8N_2 + \frac{3680}{513} N_3 - \frac{845}{4104} N_5 \Big),$$

$$M_6 = hH\Big(t_n + h,\ y_3(t_n, \beta) - \frac{8}{27} M_1 + 2M_2 - \frac{3544}{2565} M_3 + \frac{1859}{4104} M_4 - \frac{11}{40} M_5,$$

$$y_4(t_n, \beta) - \frac{8}{27} N_1 - 2N_2 - \frac{3544}{2565} N_3 + \frac{1859}{4104} N_4 - \frac{11}{40} N_5 \Big),$$

$$y_4(t_{n+1}, \beta) = y_4(t_n, \beta) + \frac{16}{135} N_1 + \frac{6656}{12825} N_3 + \frac{28561}{56430} N_4 - \frac{9}{50} N_5 + \frac{2}{55} N_6,$$

where

$$N_1 = hI(t_n,\ y_3(t_n, \beta),\ y_4(t_n, \beta)),$$

$$N_2 = hI\Big(t_n + \frac{1}{4} h,\ y_3(t_n, \beta) + \frac{1}{4} M_1,\ y_4(t_n, \beta) + \frac{1}{4} N_1 \Big),$$

$$N_3 = hI\Big(t_n + \frac{3}{8} h,\ y_3(t_n, \beta) + \frac{3}{32} M_1 + \frac{9}{32} M_2,$$

$$y_4(t_n, \beta) + \frac{3}{32} N_1 + \frac{9}{32} N_2 \Big),$$

$$N_4 = hI\Big(t_n + \frac{12}{13} h,\ y_3(t_n, \beta) + \frac{1932}{2197} M_1 - \frac{7200}{2197} M_2$$

$$+ \frac{7296}{2197} M_3,\ y_4(t_n, \beta) + \frac{1932}{2197} N_1 - \frac{7200}{2197} N_2 + \frac{7296}{2197} N_3 \Big),$$

$$N_5 = hI\Big(t_n + h,\ y_3(t_n, \beta) + \frac{439}{216} M_1 - 8M_2 + \frac{3680}{513} M_3 - \frac{845}{4104} M_5,$$

$$y_4(t_n, \beta) + \frac{429}{216} N_1 - 8N_2 + \frac{3680}{513} N_3 - \frac{845}{4104} N_4 \Big),$$

$$N_6 = hI\Big(t_n + h,\ y_3(t_n, \beta) - \frac{8}{27} M_1 + 2M_2 - \frac{3544}{2565} M_3 + \frac{1859}{4104} M_4 - \frac{11}{40} M_5,$$

$$y_4(t_n, \beta) - \frac{8}{27} N_1 + 2N_2 - \frac{3544}{2565} N_3 + \frac{1859}{4104} N_4 - \frac{11}{40} N_5 \Big).$$

5. Convergence of Fuzzy Runge-Kutta-Fehlberg Method

The solution is obtained by grid points at

$$a = t_0 \leq t_1 \leq \ldots \leq t_N = b \text{ and } h = \frac{b-a}{N} = t_{n+1} - t_n. \tag{5.1}$$

We define

$$F[t_n,\ y(t_n;\ a)] = \sum_{i=1}^{s} b_i K_i(t_n,\ y(t_n;\ a))\, G[t_n,\ y(t_n;\ a)] = \sum_{i=1}^{s} b_i L_i(t_n,\ y(t_n;\ a))$$

$$H[t_n,\ y(t_n;\ \beta)] = \sum_{i=1}^{s} b_i M_i(t_n,\ y(t_n;\ \beta))\, I[t_n,\ y(t_n;\ \beta)]$$

$$= \sum_{i=1}^{s} b_i N_i(t_n,\ y(t_n;\ \beta)). \tag{5.2}$$

The exact and approximate solutions at $t_n,\ 0 \leq n \leq N$ are denoted respectively by

$$[Y(t_n)]_{\alpha,\beta} = [\underline{Y^+}(t_n;\ \alpha),\ \overline{Y^+}(t_n;\ \alpha),\ \underline{Y^-}(t_n;\ \beta),\ \overline{Y^-}(t_n;\ \beta)]$$

and

$$[Y(t_n)]_{\alpha,\beta} = [\underline{Y^+}(t_n;\ \alpha),\ \overline{Y^+}(t_n;\ \alpha),\ \underline{Y^-}(t_n;\ \beta),\ \overline{Y^-}(t_n;\ \beta)].$$

We have

$$\underline{Y^+}(t_n;\ \alpha) \approx \underline{Y^+}(t_n;\ \alpha) + hF[t_n,\ \underline{Y^+}(t_n;\ \alpha),\ \overline{Y^+}(t_n;\ \alpha)],$$

$$\overline{Y^+}(t_n;\ \alpha) \approx \overline{Y^+}(t_n;\ \alpha) + hG[t_n,\ \underline{Y^+}(t_n;\ \alpha),\ \overline{Y^+}(t_n;\ \alpha)],$$

$$\underline{Y^-}(t_n;\ \beta) \approx \underline{Y^-}(t_n;\ \beta) + hH[t_n,\ \underline{Y^-}(t_n;\ \beta),\ \overline{Y^-}(t_n;\ \beta)],$$

$$\overline{Y^-}(t_n;\ \beta) \approx \overline{Y^-}(t_n;\ \beta) + hI[t_n,\ \underline{Y^-}(t_n;\ \beta),\ \overline{Y^-}(t_n;\ \beta)],$$

and

$$\underline{y}^{+}(t_{n+1};\ \alpha) = \underline{y}^{+}(t_n;\ \alpha) + hF[t_n,\ \underline{y}^{+}(t_n;\ \alpha),\ \overline{y^{+}}(t_n;\ \alpha)],$$

$$\overline{Y^{+}}(t_{n+1};\ \alpha) = \overline{y^{+}}(t_n;\ \alpha) + hG[t_n,\ \underline{y}^{+}(t_n;\ \alpha),\ \overline{y^{+}}(t_n;\ \alpha)],$$

$$\underline{Y}^{-}(t_n;\ \beta) = \underline{y}^{-}(t_n;\ \beta) + hH[t_n,\ \underline{y}^{-}(t_n;\ \beta),\ \overline{Y^{-}}(t_n;\ \beta)],$$

$$\overline{Y^{-}}(t_n;\ \beta) = \overline{Y^{-}}(t_n;\ \beta) + hI[t_n,\ \underline{y}^{-}(t_n;\ \beta),\ \overline{y^{-}}(t_n;\ \beta)],$$

We need the following lemmas to show the convergence of these approximates, that is, $\underline{y}^{+}(t_n;\ \alpha),\ \overline{y^{+}}(t_n;\ \alpha),\ \underline{y}^{-}(t_n;\ \alpha)$, and $\overline{y^{-}}(t_n;\ \alpha)$ converges to $\underline{Y}^{+}(t_n;\ \alpha),\ \overline{Y^{+}}(t_n;\ \alpha),\ \underline{Y}^{-}(t_n;\ \alpha)$, and $\overline{Y^{-}}(t_n;\ \alpha)$ respectively whenever $h \to 0$.

Lemma 5.1. *Let the sequence of numbers* $\{W_n^{+}\}_{n=0}^{N},\ \{W_n^{-}\}_{n=0}^{N}$ *satisfy*

$$|\ W_{n+1}\ | \le A|\ W_n\ | + B\ \ 0 \le n \le N-1$$

for some given positive constants A and B. Then

$$|\ W_n\ | \le A^{n}|\ W_0\ | + B\,\frac{A^{n}-1}{A-1}.$$

Lemma 5.2. *Let the sequence of numbers* $\{W_n\}_{n=0}^{N},\ \{V_n\}_{n=0}^{N}$, *satisfy*

$$|\ W_{n+1}\ | \le |\ W_n\ | + A.\max\{|\ W_n\ ||\ V_n\ |\} + B,$$

$$|\ V_{n+1}\ | \le |\ V_n\ | + A.\max\{|\ W_n\ ||\ V_n\ |\} + B,$$

for some given positive constants A and B and denote

$$U_n = |\ W_n\ | + +|\ V_n\ |,\ 0 \le n \le N.$$

Then

$$U_n \le \overline{A^{n}}U_0 + \overline{B}\,\frac{\overline{A^{n}}-1}{\overline{A}-1},\ 0 \le n \le N\ \ \textit{where}\ \overline{A} = 1+2A\ \textit{and}\ \overline{B} = 2B.$$

Let $F(t,\ u,\ v)$ and $G(t,\ u,\ v)$ are obtained by substituting $[y(t)]_{r^{\cdot}} = [u,\ v]$ in (5.2).

The domain where F and G are defined is therefore

$$K = \{(t, u, v)/0 \le t \le T, -\infty < v < \infty, -\infty < u \le v\}$$

Theorem 5.1. *Let* $F(t, u, v), G(t, u, v), H(t, u', v')$ *and* $I(t, u', v')$ *belonging to* $c^P(K)$ *and let the partial derivatives of* F, G, H *and* I *be bounded over K. Then for arbitrary fixed* $\alpha, \beta, 0 \le \alpha, \beta \le 1,$ *the approximate solution of (5.1),* $[\underline{y}^+(t_n; \alpha), \overline{y}^+(t_n; \alpha), \underline{y}^-(t_n; \beta), \overline{y}^-(t_n; \beta)]$ *converge to the exact solution* $[Y(t_n; \alpha), \overline{Y}^+(t_n; \alpha), \underline{Y}^-(t_n; \beta), \overline{Y}^-(t_n; \beta)]$.

6. Numerical Examples

Example 6.1. Consider the fuzzy ordinary differential equations $y' = y$ with $y(t_0) = (3, 5, 7; 1.5, 5, 8)$.

Solution. The solution is given by

$$y_1(t; \alpha) = (3 + 2\alpha)e^t \qquad y_2(t; \alpha) = (7 - 2\alpha)e^t$$

$$y_3(t; \alpha) = (5 - 3.5)e^t \qquad y_4(t; \alpha) = (5 + 3\beta)e^t.$$

The Error results at $t = 1$ are shown in the Table 1. The solution graph is given in Figure 1.

Table 1

α, β	Absolute Error for IFRK6 at $t = 1$			
	$y_1(t; \alpha)$	$y_2(t; \alpha)$	$y_3(t; \beta)$	$y_4(t; \beta)$
0.00	1.105e-08	2.579e-08	5.526e-09	2.947e-08
0.20	1.253e-08	2.431e-08	8.105e-09	2.726e-08
0.40	1.400e-08	2.284e-08	1.068e-08	2.505e-08
0.60	1.547e-08	2.137e-08	1.326e-08	2.284e-08
0.80	1.695e-08	1.989e-08	1.584e-08	2.063e-08
1.00	1.842e-08	1.842e-08	1.842e-08	1.842e-08

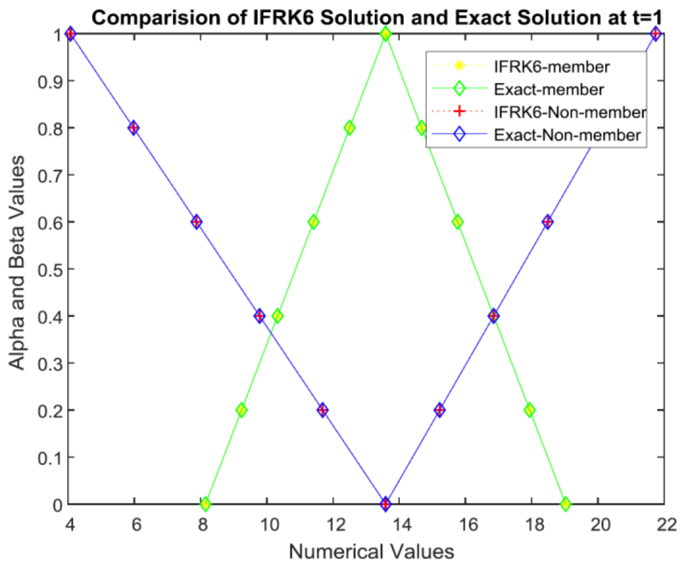

Figure 1

7. Conclusion

In this paper, the Runge Kutta Felhberg method has been presented to solve the intuitionistic fuzzy IVPs. The efficiency of the method has been illustrated through the numerical examples of Triangular type. The numerical results obtained agree well with the corresponding exact solutions. This shows that RK Fehlberg method much applicable to solve intuitionistic fuzzy IVPs.

References

[1] S. Abbasbandy and T. Allahviranloo, Numerical solution of fuzzy differential equation by Runge-Kutta method and intuitionistic treatment, Notes on IFS, 8(3) (2002), 45-53.

[2] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and Systems, 20 (1986), 87-96.

[3] D. Dubois and H. Parade, Operation on Fuzzy Number, International Journal of Fuzzy system, 9 (1978), 613-626.

[4] T. Jayakumar and K. Kanagarajan, Numerical solution for hybrid fuzzy differential equations by Ruge-Kutta Fehlberg method, International Journal of Mathematical Analysis, vol. 6.

[5] O. Kaleva, Fuzzy differerential equations, Fuzzy sets and systems 24 (1987), 301-317.

[6] G. S. Mahapatra and T. K. Roy, Reliability Evaluation using Triangular Intuitionistic Fuzzy Numbers Arithmetic Operations, World Academy of Science, Engineering and Technology 50 (2009), 574-581.

[7] S. Melliani and L. S. Chadli, Introduction to intuitionistic fuzzy differential equations, Notes on IFS 6(2) (2000), 31-41.

[8] S. Melliani and L. S. Chadli, Introduction to intuitionistic fuzzy partial differential equations, Notes on IFS 7(3) (2001), 39-42.

[9] Ming Ma, Menahem Friedman and Abraham Kandel, Numerical solutions of fuzzy differential equations, Fuzzy Sets and Systems, 105 (1999), 133-138.

[10] V. Nirmala and S. Chenthur Pandian, Numerical Approach for Solving Intuitionistic Fuzzy diffrential Equation under Generalised Differentiability Concept, Applied Mathematical Sciences 9(67) (2015), 3337-3346.

[11] Sankar Prasad Mondal and Tapan Kumar Roy, First order homogeneous ordinary differential equation with initial value as triangular intuitionistic fuzzy number, Journal of Uncertainty in Mathematics Science, (2014).

[12] Sneh Lata and Amit Kumar, A new method to solve time-dependent intuitionistic fuzzy differential equations and its application to analyze the intuitionistic fuzzy reliability of industrial systems, Concurrent Engineering: Research and applications (2012), 1-8.

[13] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.