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COMPARISON OF DIFFERENT DEFUZZIFICATION TECHNIQUES FOR THE EVALUATION OF RELIABILITY

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Abstract

In this paper, a new form of Fuzzy Number named as Pendant Fuzzy Number and Intuitionistic Pendant Fuzzy Number are introduced. Arithmetic operations and α-cut of Pendant Fuzzy Number and Intuitionistic Pendant Fuzzy Number are defined with numerical examples. Also a comparative study about Reliability using different Fuzzy Numbers is done and for Defuzzification, different methods such as Signed Distance method, Graded Mean Integration Method and Centroid Method are used.

1. Introduction

In [1], A. Nagoorgani presented a new operation on triangular Fuzzy Number for solving fuzzy linear programming problem. Trident Fuzzy Number and Sub Trident Fuzzy Number and its arithmetic operations were given by Praveen A. Prakash and M. Geetha Lakshmi in [2], [3], [4]. In [5], [6] Fuzzy systems are presented with applications and properties of fuzzy numbers are explained in Fuzzy Systems and Operations Research and Management (Advances in Intelligent Systems and Computing, 367), 2014 and 2016. Pentagonal Fuzzy Numbers are explained in [7] by Avinash J.

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Kamble B. Rama and G. Michael Rosario discussed about the penalty cost and shortage cost using different fuzzy numbers in [8]. Another Fuzzy Number is introduced by B. Rama and G. Michael Rosario [9], named as quadrant Fuzzy Number and its arithmetic operations explained using types of Fuzzy Numbers. In [10] application and detailed theory of Reliability explained using different models and different numbers. In [11], Chin Hsun Hsieh discussed about the heuristic optimization of natural production inventory models with the preference of a decision maker. In [12], Deng-Feng Li has discussed about the Decision and Game theory using intuitionistic fuzzy numbers. G. Geetharamani and P. Jayagouri discussed shortest path using intuitionistic fuzzy numbers in [13]. In [14] G. Menaka explains different Fuzzy Numbers and intuitionistic Fuzzy Numbers and their arithmetic operations and ranking. G. Michael Rosario and A. Dhana Lakshmi [15] discussed the applications of intuitionistic fuzzy equations on reliability evaluation. Fuzzy sets are introduced by Zadeh [16] in 1965 to represent the information possessing non-statistical certainties. In [17] L. S. Srinath explained Reliability using different methods and many examples are solved using series-parallel method, probability method and Boolean Method. In [18], P. Jayagowria and G. Geetharamani are discussed a critical path problem using intuitionistic trapezoidal fuzzy number. In [19], reliability of weaving machine is calculated using different fuzzy numbers by P. Jini Varghese and G. Michael Rosario. In [20] P. Jini Varghese and G. Michael Rosario discussed about the reliability of weaving machine in the textile industry using series and parallel system and it is explained with the help of numerical examples and reliability is evaluated with the help of trapezoidal intuitionistic Fuzzy Numbers and series and parallel system is applied in the reliability analysis. In [21] P. Jini Varghese and G. Michael Rosario explained the arithmetic operations of different Fuzzy Numbers and formulated some properties of trapezoidal intuitionistic Fuzzy Numbers and clearly explained with the help of numerical examples. In [22], Yager explained the clear idea about solving mathematical relationships. Representation of Trapezoidal Fuzzy Numbers and the multiplication operation was introduced by S. Rezvani [23]. S. Rezvani explained Ranking generalized exponential trapezoidal fuzzy numbers based on variance in [24]. In [25], Sankar Kumar Roy and Sudipta Midya are explained Multi objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment, Applied Intelligence, 2019 Publication. In this paper Pendant Fuzzy Numbers are introduced and its arithmetic operations are defined. Different defuzzification techniques are applied on these fuzzy numbers to analyse the reliability of a Weaving Machine.

2. Types of Pendant Fuzzy Number

Definition 1. Triangular Pendant Fuzzy Number (TPFN)

A Triangular Pendant Fuzzy Number is given by $\breve{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p})$ where a_{1p}, a_{2p}, a_{3p} are real numbers and $a_{1p} \le a_{2p} \le a_{3p}$ then its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{bmatrix} 1^{\frac{1}{5}} & \text{for } x < a_{1p} \\ \left[\dfrac{a_{2p} - x}{a_{2p} - a_{1p}}\right]^{\frac{1}{5}} & \text{for } a_{1p} \le x \le a_{2p} \\ 0 & \text{for } x = a_{2p} \\ \left[\dfrac{x - a_{2p}}{a_{3p} - a_{2p}}\right]^{\frac{1}{5}} & \text{for } a_{2p} \le x \le a_{3p} \\ 1^{\frac{1}{5}} & \text{for } x > a_{3p} \end{bmatrix} \tag{1}$$

α-cut of a Triangular Pendant Fuzzy Number is given by

$$A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}), a_{2p} + \alpha^5(a_{3p} - a_{2p})], \alpha \in (0, 1]. \tag{2}$$

Arithmetic Operations of Triangular Pendant Fuzzy Number

1. Addition

If $\breve{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p})$, $\breve{B}_{TPFN} = (b_{1p}, b_{2p}, b_{3p})$ are two Triangular Pendant Fuzzy Numbers then the addition is given by $\breve{A}_{TPFN} + \breve{B}_{TPFN} = (a_{1p} + b_{1p}, a_{2p} + b_{2p}, a_{3p} + b_{3p})$.

2. Subtraction

If $\breve{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p})$, $\breve{B}_{TPFN} = (b_{1p}, b_{2p}, b_{3p})$ are two Triangular Pendant Fuzzy Numbers then the subtraction is given by

$\breve{A}_{TPFN} - \breve{B}_{TPFN} = (a_{1p} - b_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p})$.

Definition 2. Trapezoidal Pendant Fuzzy Number (TrPFN)

A Trapezoidal Pendant Fuzzy Number is given by $\breve{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p})$ where $a_{1p}, a_{2p}, a_{3p}, a_{4p}$ are real numbers and $a_{1p} \leq a_{2p} \leq a_{3p} \leq a_{4p}$ then its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{bmatrix} 1^{\frac{1}{5}} & \text{for } x < a_{1p} \\ \left[\dfrac{a_{2p} - x}{a_{2p} - a_{1p}}\right]^{\frac{1}{5}} & \text{for } a_{1p} \leq x \leq a_{2p} \\ 0 & \text{for } a_{2p} \leq x \leq a_{3p} \\ \left[\dfrac{x - a_{3p}}{a_{4p} - a_{3p}}\right]^{\frac{1}{5}} & \text{for } a_{3p} \leq x \leq a_{4p} \\ 1^{\frac{1}{5}} & \text{for } x > a_{4p} \end{bmatrix} \tag{3}$$

α-cut of a Trapezoidal Pendant Fuzzy Number is given by

$$A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}), a_{3p} + \alpha^5(a_{4p} - a_{3p})], \alpha \in (0, 1]. \tag{4}$$

Arithmetic Operations of Trapezoidal Pendant Fuzzy Number

1. If $\breve{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p})$, $\breve{B}_{TPFN} = (b_{1p}, b_{2p}, b_{3p}, a_{4p})$ are two Trapezoidal Pendant Fuzzy Numbers then the addition is defined by $\breve{A}_{TPFN} + \breve{B}_{TPFN} = (a_{1p} + b_{1p}, a_{2p} + b_{2p}, a_{3p} + b_{3p}, a_{4p} + b_{4p})$.

2. If $\breve{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p})$, $\breve{B}_{TPFN} = (b_{1p}, b_{2p}, b_{3p}, a_{4p})$ are two Trapezoidal Pendant Fuzzy Numbers then the subtraction is defined by $\breve{A}_{TPFN} - \breve{B}_{TPFN} = (a_{1p} - b_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p}, a_{4p} - b_{4p})$.

Definition 3. Pentagonal Pendant Fuzzy Number (PenPFN).

A Pentagonal Pendant Fuzzy Number is given by $\breve{A}_{PenPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p})$ where $a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p}$ are real numbers and $a_{1p} \leq a_{2p} \leq a_{3p} \leq a_{4p} \leq a_{5p}$, then its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{bmatrix} 1^{\frac{1}{5}} & \text{for } x < a_{1p} \\ \left[\dfrac{a_{2p} - x}{a_{2p} - a_{1p}}\right]^{\frac{1}{5}} & \text{for } a_{1p} \le x \le a_{2p} \\ \left[\dfrac{a_{3p} - x}{a_{3p} - a_{2p}}\right]^{\frac{1}{5}} & \text{for } a_{2p} \le x \le a_{3p} \\ 0 & \text{for } x = a_{3p} \\ \left[\dfrac{x - a_{3p}}{a_{4p} - a_{3p}}\right]^{\frac{1}{5}} & \text{for } a_{3p} \le x \le a_{4p} \\ \left[\dfrac{x - a_{4p}}{a_{5p} - a_{5p}}\right]^{\frac{1}{5}} & \text{for } a_{4p} \le x \le a_{5p} \\ 1^{\frac{1}{5}} & \text{for } x > a_{5p} \end{bmatrix} \quad (5)$$

α-cut of a Pentagonal Pendant Fuzzy Number is given by

$$A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}),\ a_{4p} + \alpha^5(a_{5p} - a_{4p})],\ \alpha \in (0,\ 0.5]. \quad (6)$$

$$A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}),\ a_{3p} + \alpha^5(a_{4p} - a_{3p})],\ \alpha \in (0.5,\ 1]. \quad (7)$$

Arithmetic Operations of Pentagonal Pendant Fuzzy Number

1. If $\breve{A}_{PenPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p})$, $\breve{B}_{PenPFN} = (b_{1p}, b_{2p}, b_{3p}, a_{4p}, a_{5p})$ are two Pentagonal Pendant Fuzzy Numbers then the addition is defined by $\breve{A}_{PenPFN} + B_{PenPFN} = (a_{1p} + b_{1p}, a_{2p} + b_{2p}, a_{3p} + b_{3p}, a_{4p} + b_{4p}, a_{5p} + b_{5p})$.

2. If $\breve{A}_{PenPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p})$, $\breve{B}_{PenPFN} = (b_{1p}, b_{2p}, b_{3p}, a_{4p}, a_{5p})$ are two Pentagonal Pendant Fuzzy Numbers then the subtraction is defined by $\breve{A}_{PenPFN} - \breve{B}_{PenPFN} = (a_{1p} - b_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p}, a_{4p} - b_{4p}, a_{5p} - b_{5p})$.

Definition 4. Triangular Intuitionistic Pendant Fuzzy Number (TIPFN)

A Triangular Intuitionistic Pendant Fuzzy Number is given by $\breve{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p}; a'_{1p}, a_{2p}, a'_{3p})$ where $a_{1p}, a_{2p}, a_{3p}; a'_{1p}, a_{2p}, a'_{3p}$ are real numbers and $a'_{1p} \le a_{1p}, a_{2p} \le a_{2p}, a'_{3p} \le a_{3p}$, then its membership function and non-membership function are defined as

$$\mu_{\tilde{A}}(x) = \begin{bmatrix} 1^{\frac{1}{5}} & \text{for } x < a_{1p} \\ \left[\dfrac{a_{2p} - x}{a_{2p} - a_{1p}}\right]^{\frac{1}{5}} & \text{for } a_{1p} \le x \le a_{2p} \\ 0 & \text{for } x = a_{2p} \\ \left[\dfrac{x - a_{2p}}{a_{3p} - a_{2p}}\right]^{\frac{1}{5}} & \text{for } a_{2p} \le x \le a_{3p} \\ 1^{\frac{1}{5}} & \text{for } x > a_{3p} \end{bmatrix} \tag{8}$$

and

$$\gamma_{\tilde{A}}(x) = \begin{bmatrix} 0 & \text{for } x < a'_{1p} \\ \left[\dfrac{a_{2p} - x}{a_{2p} - a'_{1p}}\right]^{\frac{1}{5}} & \text{for } a'_{1p} \le x \le a_{2p} \\ 1^{\frac{1}{5}} & \text{for } x = a_{2p} \\ \left[\dfrac{x - a_{2p}}{a'_{3p} - a_{2p}}\right]^{\frac{1}{5}} & \text{for } a_{2p} \le x \le a'_{3p} \\ 0 & \text{for } x > a'_{3p} \end{bmatrix} \tag{9}$$

α- cut of a Triangular Intuitionistic Pendant Fuzzy Number is given by

$$A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}), a_{2p} + \alpha^5(a_{3p} - a_{2p})], \alpha \in (0, 1]. \tag{10}$$

β-cut of a Triangular Intuitionistic Pendant Fuzzy Number is given by

$$A_{p\beta} = [a_{2p} - \beta^5(a_{2p} - a'_{1p}), a_{2p} + \beta^5(a'_{3p} - a_{2p})], \beta \in (0, 1]. \tag{11}$$

Arithmetic Operations of Triangular Intuitionistic Pendant Fuzzy Number

1. If $\breve{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p}; a'_{1p}, a_{2p}, a'_{3p})$, $\breve{B}_{TPFN} = (b_{1p}, b_{2p}, b_{3p}; b'_{1p},$

$b_{2p}, b'_{3p})$ are two Triangular Intuitionistic Pendant Fuzzy Numbers then the addition is given by $\breve{A}_{TPFN} + \breve{B}_{TPFN} = (a_{1p} + b_{1p}, a_{2p} + b_{2p}, a_{3p} + b_{3p}; a'_{1p} + b'_{1p}, a_{2p} + b_{2p}, a'_{3p} + b'_{3p})$.

2. If $\breve{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p}; a'_{1p}, a_{2p}, a'_{3p})$, $\breve{B}_{TPFN} = (b_{1p}, b_{2p}, b_{3p}; b'_{1p}, b_{2p}, b'_{3p})$ are two Triangular Intuitionistic Pendant Fuzzy Numbers then the subtraction is given by $\breve{A}_{TPFN} - \breve{B}_{TPFN} = (a_{1p} - b_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p}; a'_{1p} - b'_{1p}, a_{2p} - b_{2p}, a'_{3p} - b'_{3p})$.

Definition 5. Trapezoidal Intuitionistic Pendant Fuzzy Number (TrIPFN)

A Trapezoidal Intuitionistic Pendant Fuzzy Number is given by $\breve{A}_{TrIPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}; a'_{1p}, a_{2p}, a'_{3p}, a'_{4p})$ where $a_{1p}, a_{2p}, a_{3p}, a_{4p}; a'_{1p}, a_{2p}, a'_{3p}, a'_{4p}$ are real numbers and $a'_{1p} \le a_{1p} \le a_{2p} \le a_{3p}, a'_{3p} \le a_{3p}, a'_{4p} \le a_{4p}$, then its membership function and non-membership function are defined as

$$\mu_{\tilde{A}}(x) = \begin{bmatrix} 1^{\frac{1}{5}} & \text{for } x < a_{1p} \\ \left[\dfrac{a_{2p} - x}{a_{2p} - a_{1p}}\right]^{\frac{1}{5}} & \text{for } a_{1p} \le x \le a_{2p} \\ 0 & \text{for } a_{2p} \le x \le a_{3p} \\ \left[\dfrac{x - a_{3p}}{a_{4p} - a_{3p}}\right]^{\frac{1}{5}} & \text{for } a_{3p} \le x \le a_{4p} \\ 1^{\frac{1}{5}} & \text{for } x > a_{4p} \end{bmatrix} \tag{12}$$

and

$$\gamma_{\tilde{A}}(x) = \begin{bmatrix} 0 & \text{for } x < a'_{1p} \\ \left[\dfrac{a'_{1p} - x}{a_{2p} - a'_{1p}}\right]^{\frac{1}{5}} & \text{for } a'_{1p} \le x \le a_{2p} \\ 1^{\frac{1}{5}} & \text{for } a_{2p} \le x \le a_{3p} \\ \left[\dfrac{x - a'_{4p}}{a'_{4p} - a_{3p}}\right]^{\frac{1}{5}} & \text{for } a_{3p} \le x \le a'_{4p} \\ 0 & \text{for } x > a'_{4p} \end{bmatrix} \tag{13}$$

α-cut of a Trapezoidal Intuitionistic Pendant Fuzzy Number is given by

$$A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}),\ a_{3p} + \alpha^5(a_{4p} - a_{3p})],\ \alpha \in (0, 1]. \tag{14}$$

β-cut of a Trapezoidal Intuitionistic Pendant Fuzzy Number is given by

$$A_{p\beta} = [a_{2p} - \beta^5(a_{2p} - a'_{1p}),\ a_{3p} + \beta^5(a'_{4p} - a_{3p})],\ \beta \in (0, 1]. \tag{15}$$

Arithmetic Operations of Trapezoidal Intuitionistic Pendant Fuzzy Number

1. If $\breve{A}_{TrIPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}; a'_{1p}, a_{2p}, a_{3p}, a'_{4p})$, $\breve{B}_{TrIPFN} = (b_{1p}, b_{2p}, b_{3p}, b_{4p}; b'_{1p}, b_{2p}, b_{3p}, b'_{4p})$ are two Trapezoidal Pendant Fuzzy Numbers then the addition is given by $\breve{A}_{TrIPFN} + \breve{B}_{TrIPFN} = (a_{1p} + b_{1p}, a_{2p} + b_{2p}, a_{3p} + b_{3p}, a_{4p} + a'_{1p}; a'_{1p} + b'_{1p}, a_{2p} + b_{2p}, a_{3p} + a'_{4p}, + b'_{4p})$.

2. If $\breve{A}_{TrIPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}; a'_{1p}, a_{2p}, a_{3p}, a'_{4p})$, $\breve{B}_{TrIPFN} = (b_{1p}, b_{2p}, b_{3p}, b_{4p}; b'_{1p}, b_{2p}, b_{3p}, b'_{4p})$ are two Trapezoidal Pendant Fuzzy Numbers then the subtraction is given by $\breve{A}_{TrIPFN} - \breve{B}_{TrIPFN} = (a_{1p} - b_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p}, a_{4p} - b_{4p}; a'_{1p} - b'_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p}, a'_{4p} - b'_{4p})$.

Definition 6. Pentagonal Intuitionistic Pendant Fuzzy Number (PenIPFN)

A Pentagonal Intuitionistic Pendant Fuzzy Number is given by

$\breve{A}_{PenIPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p}; a'_{1p}, a_{2p}, a_{3p}, a_{4p}, a'_{5p})$, where $a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p}; a'_{1p}, a_{2p}, a_{3p}, a_{4p}, a'_{5p}$ are real numbers and $a'_{1p} \le a_{1p}$

$\leq a_{2p} \leq a_{3p} \leq a_{4p} \leq a_{5p} \leq a'_{5p}$, then its membership function and non-membership function are defined as

$$\mu_{\tilde{A}}(x) = \begin{bmatrix} 1^{\frac{1}{5}} & \text{for } x < a_{1p} \\ \left[\dfrac{x - a_{1p}}{a_{2p} - a_{1p}}\right]^{\frac{1}{5}} & \text{for } a_{1p} \leq x \leq a_{2p} \\ \left[\dfrac{x - a_{2p}}{a_{3p} - a_{2p}}\right]^{\frac{1}{5}} & \text{for } a_{2p} \leq x \leq a_{3p} \\ 0 & \text{for } x = a_{3p} \\ \left[\dfrac{a_{4p} - x}{a_{4p} - a_{3p}}\right]^{\frac{1}{5}} & \text{for } a_{3p} \leq x \leq a_{4p} \\ \left[\dfrac{a_{4p} - x}{a_{5p} - a_{4p}}\right]^{\frac{1}{5}} & \text{for } a_{4p} \leq x \leq a_{5p} \\ 1^{\frac{1}{5}} & \text{for } x > a_{5p} \end{bmatrix} \tag{16}$$

and

$$\gamma_{\tilde{A}}(x) = \begin{bmatrix} 0 & \text{for } x < a'_{1p} \\ \left[\dfrac{x - a'_{1p}}{a_{2p} - a_{1p}}\right]^{\frac{1}{5}} & \text{for } a'_{1p} \leq x \leq a_{2p} \\ \left[\dfrac{x - a_{2p}}{a_{3p} - a_{2p}}\right]^{\frac{1}{5}} & \text{for } a_{2p} \leq x \leq a_{3p} \\ 1^{\frac{1}{5}} & \text{for } x = a_{3p} \\ \left[\dfrac{x - a_{4p}}{a_{4p} - a_{3p}}\right]^{\frac{1}{5}} & \text{for } a_{3p} \leq x \leq a_{4p} \\ \left[\dfrac{x - a_{4p}}{a'_{5p} - a_{4p}}\right]^{\frac{1}{5}} & \text{for } a_{4p} \leq x \leq a'_{5p} \\ 0 & \text{for } x > a'_{5p} \end{bmatrix} \tag{17}$$

α-cut of a Pentagonal Intuitionistic Pendant Fuzzy Number is given by

$$A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}),\ a_{2p} + \alpha^5(a_{5p} - a_{5p})],\ \alpha \in (0,\ 0.5]. \tag{18}$$

$$A_{p\alpha} = [a_{3p} - \alpha^5(a_{3p} - a_{2p}),\ a_{3p} + \alpha^5(a_{4p} - a_{3p})],\ \alpha \in (0.5, 1]. \quad (19)$$

β-cut of a Pentagonal Intuitionistic Pendant Fuzzy Number is given by

$$A_{p\beta} = [a_{2p} - \beta^5(a_{2p} - a'_{1p}),\ a_{4p} + \beta^5(a'_{5p} - a_{4p})],\ \beta \in (0, 0.5]. \quad (20)$$

$$A_{p\beta} = [a_{3p} - \beta^5(a_{3p} - a_{2p}),\ a_{3p} + \beta^5(a_{4p} - a_{3p})],\ \beta \in (0.5, 1]. \quad (21)$$

Arithmetic Operations of Pentagonal Intuitionistic Pendant Fuzzy Number

1. If $\breve{A}_{PenPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p}; a'_{1p}, a_{2p}, a_{3p}, a'p_{4p}, a'_{5p})$, $\breve{B}_{PenPFN} = (b_{1p}, b_{2p}, b_{3p}, b_{4p}, b_{5p}; b'_{1p}, b_{2p}, b_{3p}, b_{4p}, b'_{5p})$ are two Pentagonal Intuitionistic Pendant Fuzzy Numbers then the addition is given by $\breve{A}_{TPFN} + \breve{B}_{TPFN} = (a_{1p} + b_{1p}, a_{2p} + b_{2p}, a_{3p} + b_{3p}; a'_{1p} + b'_{1p}, a_{2p} + b_{2p}, a_{3p} + b_{3p}, a_{4p} + b_{4p}, a'_{5p} + b'_{5p})$.

2. If $\breve{A}_{TrPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p}; a'_{1p}, a_{2p}, a_{3p}, a'p_{4p}, a'_{5p})$, $\breve{B}_{TrPFN} = (b_{1p}, b_{2p}, b_{3p}, b_{4p}, b_{5p}; b'_{1p}, b_{2p}, b_{3p}, b_{4p}, b'_{5p})$ are two Pentagonal Intuitionistic Pendant Fuzzy Numbers then the subtraction is given by $\breve{A}_{TrPFN} - \breve{B}_{TrPFN} = (a_{1p} - b_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p}; a'_{1p} - b'_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p}, a_{4p} - b_{4p}, a'_{5p} - b'_{5p})$.

3. Bow Tie Diagram

In a simple qualitative cause-and-effect diagram, a bow tie is a graphical representation of pathways from the causes of an occurrence or danger to its effects. A ‘bowtie’ is a visual representation of the risk that is dealing with in a single, easy-to-understand picture. The diagram is in the form of a bow tie.

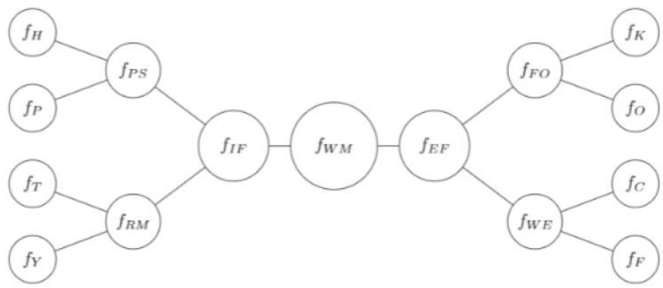

Figure 1. Represents the Bow Tie diagram of the failure rate of the weaving machine with the corresponding factors.

Here,

$\breve{f}_{WM}$ Failure rate of weaving machine

$\breve{f}_{H}$ Failure rate of controlled humidity

$\breve{f}_{IF}$ Failure rate of internal factors

$\breve{f}_{P}$ Failure rate of balanced pressure

$\breve{f}_{EF}$ Failure rate of external factors

$\breve{f}_{T}$ Failure rate of quality of the thread

$\breve{f}_{PS}$ Failure rate of physical stabilities

$\breve{f}_{Y}$ Failure rate of continuous filament yarn

$\breve{f}_{RM}$ Failure rate of row materials

$\breve{f}_{F}$ Failure rate of availability of the fuel

$\breve{f}_{WE}$ Failure rate of working energy

$\breve{f}_{C}$ Failure rate of the balanced current flow

$\breve{f}_{FO}$ Failure rate of flow of lubricants oil

$\breve{f}_{O}$ Failure rate of shortage of lubricants oil

$\breve{f}_N$ Failure rate due to nozzles blocked

The Failure Rate of weaving machine can be calculated when the failures of the occurrence of basic fault events are known. Failure of weaving machine $\breve{f}_{WM}$ and reliability of the system can be evaluated by using the following steps:

$$\breve{f}_{PS} = 1 \ominus (1 \ominus \breve{f}_H)(1 \ominus \breve{f}_P) \tag{22}$$

$$\breve{f}_{RM} = 1 \ominus (1 \ominus \breve{f}_T)(1 \ominus \breve{f}_Y) \tag{23}$$

$$\breve{f}_{WE} = 1 \ominus (1 \ominus \breve{f}_F)(1 \ominus \breve{f}_C) \tag{24}$$

$$\breve{f}_{FO} = 1 \ominus (1 \ominus \breve{f}_O)(1 \ominus \breve{f}_N) \tag{25}$$

$$\breve{f}_{IF} = 1 \ominus (1 \ominus \breve{f}_{PS})(1 \ominus \breve{f}_{RM}) \tag{26}$$

$$\breve{f}_{EF} = 1 \ominus (1 \ominus \breve{f}_{WE})(1 \ominus \breve{f}_{FO}) \tag{27}$$

$$\breve{f}_{WM} = 1 \ominus (1 \ominus \breve{f}_{IF})(1 \ominus \breve{f}_{EF}) \tag{28}$$

Since $\text{Reliability} = 1 - \text{Failure Rate}$

Using equations (22), (23), (24) and (25)

$$\breve{f}_{PS} = 1 \ominus R_H R_P \tag{29}$$

$$\breve{f}_{RM} = 1 \ominus R_T R_Y \tag{30}$$

$$\breve{f}_{WE} = 1 \ominus R_F R_C \tag{31}$$

$$\breve{f}_{FO} = 1 \ominus R_O R_C \tag{32}$$

Substituting in equations (26) and (27)

$$\breve{f}_{IF} = 1 \ominus R_{PS} R_{RM} = 1 \ominus (R_H R_P)(R_T R_Y) \tag{33}$$

$$\breve{f}_{EF} = 1 \ominus R_{WE} R_{FO} = 1 \ominus (R_F R_C)(R_O R_N) \tag{34}$$

(33) and (34) in (28) gives the following result

$$\breve{f}_{WM} = 1 \ominus (R_{IF} R_{EF})$$

$$f_{WM} = 1 \ominus (R_H R_P)(R_T R_Y)(R_F R_C)(R_O R_N) \tag{35}$$

Apply the formula, $R_{WM} = 1 - \widetilde{f_{WM}}$ in (35) we get the Reliability of the Weaving Machine Therefore,

$$R_{WM} = (R_H R_P)(R_T R_Y)(R_F R_C)(R_O R_N) \tag{36}$$

4. Numerical Evaluation

Numerical Example 1: Using Triangular Intuitionistic Fuzzy Number and Triangular Intuitionistic Pendant Fuzzy Number

Let us consider the values as follows:

$R_H = (0.8, 0.85, 0.9; 0.7, 0.85, 0.95)$ $R_P = (0.6, 0.7, 0.85, 0.5, 0.7, 0.9)$

$R_T = (0.7, 0.75, 0.8; 0.6, 0.75, 0.85)$ $R_Y = (0.7, 0.8, 0.85; 0.6, 0.8, 0.9)$

$R_F = (0.8, 0.85, 0.9; 0.7, 0.85, 0.8)$ $R_C = (0.5, 0.6, 0.7; 0.4, 0.6, 0.8)$

$R_O = (0.6, 0.7, 0.8; 0.5, 0.7, 0.85)$ $R_N = (0.5, 0.7, 0.8, 0.4, 0.7, 0.9)$

$$R_O R_N = (0.3, 0.49, 0.64 : 0.20, 0.49, 0.765) \tag{37}$$

$$R_C R_F = (0.4, 0.51, 0.63; 0.28, 0.51, 0.64) \tag{38}$$

$$R_T R_Y = (0.49, 0.6, 0.68; 0.36, 0.6, 0.765) \tag{39}$$

$$R_H R_P = (0.48, 0.595, 0.765; 0.35, 0.595, 0.855) \tag{40}$$

Substitute equations (37), (38), (39) and (40) are in (36) gives

$$R_{WM} = (0.028224, 0.0892143, 0.20974464; 0.007056, 0.0892143, 0.32023512). \tag{41}$$

The representation of Triangular Intuitionistic Fuzzy Number and Triangular Intuitionistic Pendant Fuzzy Number are same, but it's (α, β)-cut are different. So the values of all the failure factors are same and the Reliability is also same for both Triangular Intuitionistic Fuzzy Number and Triangular Intuitionistic Pendant Fuzzy Number.

Numerical Example 2. Using Trapezoidal Intuitionistic Fuzzy Number and Trapezoidal Intuitionistic Pendant Fuzzy Number

Let us consider the values as follows:

$$R_H = (0.8, 0.85, 0.9, 0.95; 0.7, 0.85, 0.9, 0.98)$$

$$R_P = (0.6, 0.7, 0.85, 0.9; 0.5, 0.7, 0.85, 0.98)$$

$$R_T = (0.7, 0.75, 0.8, 0.85; 0.65, 0.75, 0.8, 0.9)$$

$$R_Y = (0.7, 0.8, 0.85, 0.88; 0.65, 0.8, 0.85, 0.9)$$

$$R_F = (0.8, 0.85, 0.9, 0.95; 0.75, 0.85, 0.9, 0.99)$$

$$R_C = (0.5, 0.6, 0.7, 0.8; 0.45, 0.6, 0.7, 0.9)$$

$$R_O = (0.6, 0.7, 0.8, 0.85; 0.55, 0.7, 0.8, 0.88)$$

$$R_N = (0.5, 0.7, 0.8, 0.85; 0.45, 0.7, 0.8, 0.9)$$

$$R_O R_N = (0.3, 0.49, 0.64, 0.7225; 0.2475, 0.49, 0.64, 0.792) \tag{42}$$

$$R_C R_F = (0.4, 0.51, 0.63, 0.76; 0.3375, 0.51, 0.63, 0.891) \tag{43}$$

$$R_T R_Y = (0.49, 0.6, 0.68, 0.748; 0.4225, 0.6, 0.68, 0.81) \tag{44}$$

$$R_H R_P = (0.48, 0.595, 0.765, 0.855; 0.35, 0.595, 0.765, 0.96041) \tag{45}$$

Substitute equations (42), (43), (44) and (45) are in (36) gives

$$R_{WM} = (0.028224, 0.0892143, 0.20974464; 0.351171414; 0.0123521836, 0.0892143, 0.20974464, 0.5489649009) \tag{46}$$

The representation of Trapezoidal Intuitionistic Fuzzy Number and Trapezoidal Intuitionistic Pendant Fuzzy Number are same, but it's (α, β)-cut are different. So the values of all the failure factors are same and the Reliability is also same for both Trapezoidal Intuitionistic Fuzzy Number and Trapezoidal Intuitionistic Pendant Fuzzy Number.

Numerical Example 3. Using Pentagonal Intuitionistic Fuzzy Number and Pentagonal Intuitionistic Pendant Fuzzy Number

Let us consider the values as follows:

$$R_H = (0.8,\ 0.85,\ 0.9,\ 0.95,\ 0.97;\ 0.7,\ 0.85,\ 0.9,\ 95,\ 0.98)$$

$$R_P = (0.6,\ 0.7,\ 0.85,\ 0.88,\ 0.9;\ 0.5,\ 0.7,\ 0.85,\ 0.88,\ 0.98)$$

$$R_T = (0.7,\ 0.75,\ 0.8,\ 0.83,\ 0.85;\ 0.65,\ 0.75,\ 0.8,\ 0.83,\ 0.9)$$

$$R_Y = (0.7,\ 0.8,\ 0.85,\ 0.86,\ 0.88;\ 0.65,\ 0.8,\ 0.85,\ 0.86,\ 0.9)$$

$$R_F = (0.8,\ 0.85,\ 0.9,\ 0.95,\ 0.98;\ 0.75,\ 0.85,\ 0.9,\ 0.95,\ 0.99)$$

$$R_C = (0.5,\ 0.6,\ 0.7,\ 0.75,\ 0.8;\ 0.45,\ 0.6,\ 0.7,\ 0.75,\ 0.9)$$

$$R_O = (0.6,\ 0.7,\ 0.8,\ 0.82,\ 0.85;\ 0.55,\ 0.7,\ 0.8,\ 0.82,\ 0.88)$$

$$R_N = (0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.85,\ 0.45,\ 0.6,\ 0.7,\ 0.8,\ 0.9)$$

$$R_O R_N = (0.3,\ 0.42,\ 0.56,\ 0.656,\ 0.7225;\ 0.247,\ 0.42,\ 0.56,\ 0.656,\ 0.792) \tag{47}$$

$$R_C R_F = (0.4,\ 0.51,\ 0.63,\ 0.7125,\ 0.0784;\ 0.3375,\ 0.51,\ 0.63,\ 0.7125,\ 0.891) \tag{48}$$

$$R_T R_Y = (0.49,\ 0.6,\ 0.68,\ 0.7138;\ 0.748;\ 0.4225,\ 0.6,\ 0.68,\ 0.7138,\ 0.81) \tag{49}$$

$$R_H R_P = (0.48,\ 0.595,\ 0.765,\ 0.836,\ 0.873;\ 0.35,\ 0.595,\ 0.765,\ 0.836,\ 0.9604) \tag{50}$$

Substitute equations (47), (48), (49) and (50) are in (36) gives

$$R_{WM} = (0.028224,\ 0.0764694,\ 0.18352656;\ 0.2789147803,\ 0.36988758;\ 0.0123521,\ 0.0764694,\ 0.18352656,\ 0.2789147803,\ 0.548959) \tag{51}$$

The representation of Pentagonal Intuitionistic Fuzzy Number and Pentagonal Intuitionistic Pendant Fuzzy Number are same, but its (α, β)-cut are different. So the values of all the failure factors are same and the Reliability is also same for both Pentagonal Intuitionistic Fuzzy Number and Pentagonal Intuitionistic Pendant Fuzzy Number.

5. Defuzzification Methods

Let R_M and R_{NM} denotes the crisp values for the membership and non-membership function respectively and R denotes the Reliability of Weaving Machine.

5.1 Signed Distance Method

The fuzzy numbers such as Triangular Pendant Fuzzy Number, Triangular Intuitionistic Pendant Fuzzy Number, Trapezoidal Pendant Fuzzy Number, Trapezoidal Intuitionistic Pendant Fuzzy Number, Pentagonal Pendant Fuzzy Number and Pentagonal Intuitionistic Pendant Fuzzy Number can be defuzzified by Signed Distance Method and the defuzzified values for R_M, R_{NM} and R are given as follows:

$$R_M = \frac{1}{2}\left[\int_0^1 (a_1(\alpha) + a_2(\alpha))\, d\alpha\right];\ R_{NM} = \frac{1}{2}\left[\int_0^1 (a_1(\beta) + a_2(\beta))\, d\beta\right];$$

$$R = \frac{1}{2}(R_M + R_{NM}) \tag{52}$$

5.1.1 Signed Distance Method for Triangular Intuitionistic Fuzzy Number

Considering Triangular Intuitionistic Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Signed Distance Method are given by

$$R_M = \frac{1}{2}\left[\int_0^1 [a_2 - \alpha(a_2 - a_1) + a_2 + \alpha(a_3 - a_2)]\right] d\alpha = \frac{a_1 + 2a_2 + a_3}{4} \tag{53}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_2 - \beta(a_2 - a_1') + a_2 + \beta(a_3' - a_2)]\right] d\beta = \frac{a_1' + 2a_2 + a_3'}{4} \tag{54}$$

$$R = \frac{a_1 + 4a_2 + a_3 + a_1' + a_3'}{8} \tag{55}$$

5.1.2 Signed Distance Method for Triangular Intuitionistic Pendant Fuzzy Number

Considering Triangular Intuitionistic Pendant Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Signed Distance Method are given by

$$R_M = \frac{1}{2}\left[\int_0^1 [a_{2p} - \alpha^5(a_{2p} - a_{1p}) + a_{2p} + \alpha^5(a_{3p} - a_{2p})]\right] d\alpha$$

$$= \frac{a_{1p} + 10a_{2p} + a_{3p}}{12} \tag{56}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_{2p} - \beta^5(a_{2p} - a'_{1p}) + a_{2p} + \beta^5(a'_{3p} - a_{2p})]\right] d\beta$$

$$= \frac{a'_{1p} + 10a_{2p} + a'_{3p}}{12} \tag{57}$$

$$R = \frac{a_{1p} + 20a_{2p} + a_{3p} + a'_{1p} + a'_{3p}}{24} \tag{58}$$

5.1.3 Signed Distance Method for Trapezoidal Intuitionistic Fuzzy Number

Considering Trapezoidal Intuitionistic Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Signed Distance Method are given by

$$R_M = \frac{1}{2}\left[\int_0^1 [a_2 - \alpha(a_2 - a_1) + a_3 + \alpha(a_4 - a_3)]\right] d\alpha = \frac{a_1 + a_2 + a_3 + a_4}{4} \tag{59}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_2 - \beta(a_2 - a'_1) + a_3 + \beta(a'_4 - a_3)]\right] d\beta = \frac{a'_1 + a_2 + a_3 + a'_4}{4} \tag{60}$$

$$R = \frac{a_1 + 2a_2 + 2a_3 + a_4 + a'_1 + a'_4}{8} \rightarrow \tag{61}$$

5.1.4 Signed Distance Method for Trapezoidal Intuitionistic Pendant Fuzzy Number

Considering Trapezoidal Intuitionistic Pendant Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Signed Distance Method are given by

$$R_M = \frac{1}{2}\left[\int_0^1 [a_{2p} - \alpha^5(a_{2p} - a_{1p}) + a_{3p} + \alpha^5(a_{4p} - a_{3p})]\right] d\alpha$$

$$= \frac{a_{1p} + 5a_{2p} + 5a_{3p} + a_{4p}}{12} \tag{62}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_{2p} - \beta^5(a_{2p} - a'_{1p}) + a_{3p} + \beta^5(a'_{4p} - a_{3p})]\right] d\beta$$

$$= \frac{a'_{1p} + 5a_{2p} + 5a_{3p} + a'_{4p}}{12} \tag{63}$$

$$R = \frac{a_{1p} + 10a_{2p} + 10a_{3p} + a_{4p} + a'_{1p} + a'_{4p}}{24} \tag{64}$$

5.1.5 Signed Distance Method for Pentagonal Intuitionistic Fuzzy Number

Considering Pentagonal Intuitionistic Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Signed Distance Method are given by

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_2 - \alpha(a_2 - a_1) + a_4 + \alpha(a_5 - a_4) + a_3 - \alpha(a_3 - a_2) + a_3 + \alpha(a_4 - a_3)]\right]$$

$$d\alpha = \frac{a_1 + 2a_2 + 2a_3 + 2a_4 + a_5}{4} \tag{65}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_2 - \beta(a_2 - a'_1) + a_4 + \beta(a'_5 - a_4) + a_3 - \beta(a_3 - a_2) + a_3 + \alpha(a_4 - a_3)]\right]$$

$$d\beta = \frac{a'_1 + 2a_2 + 2a_3 + 2a_4 + a'_5}{4} \tag{66}$$

$$R = \frac{a_1 + 4a_2 + 4a_3 + 4a_4 + a_5 + a'_1 + a'_5}{8} \tag{67}$$

5.1.6 Signed Distance Method for Pentagonal Intuitionistic Pendant Fuzzy Number

Considering Pentagonal Intuitionistic Pendant Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Signed Distance Method are given by

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_{2p} - \alpha^5(a_{2p} - a_{1p}) + a_{4p} + \alpha^5(a_{5p} - a_{4p}) + a_{3p} - \alpha^5(a_{3p} - a_{2p}) + a_{3p} + \alpha^5(a_{4p} - a_{3p})]\right]$$

$$d\alpha = \frac{a_{1p} + 6a_{2p} + 10a_{3p} + 6a_{4p} + a_{5p}}{12} \tag{68}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_{2p} - \beta^5(a_{2p} - a'_{1p}) + a_{4p} + \beta^5(a'_{5p} - a_{4p}) + a_{3p} - \beta^5(a_{3p} - a_{2p}) + a_{3p} + \beta^5(a_{4p} - a_{3p})]\right]$$

$$d\alpha = \frac{a'_{1p} + 6a_{2p} + 10a_{3p} + 6a_{4p} + a'_{5p}}{12} \tag{69}$$

$$R = \frac{a_{1p} + 12a_{2p} + 20a_{3p} + 12a_{4p} + a_{5p} + a'_{1p} + a'_{5p}}{24} \tag{70}$$

5.2 Graded Mean Integration Method

Again for different fuzzy numbers using Graded Mean Integration Method, the defuzzified values for R_M, R_{NM} and R are given as follows:

$$R_M = \frac{1}{2}\left[\int_0^1 (a_1(\alpha)) + a_2(\alpha)\, d\alpha\right]\alpha d\alpha;\ R_M = \frac{1}{2}\left[\int_0^1 (a_1(\beta)) + a_2(\beta)\, d\beta\right]\beta d\beta;$$

$$R = \frac{1}{2}(R_M + R_{NM}) \tag{71}$$

5.2.1 Graded Mean Integration Method for Triangular Intuitionistic Fuzzy Number

Considering Triangular Intuitionistic Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Graded Mean Integration Method are given by

$$R_M = \frac{1}{2}\left[\int_0^1 [a_2 - \alpha(a_2 - a_1) + a_2 + \alpha(a_3 - a_2)]\right]\alpha d\alpha = \frac{a_1 + a_2 + a_3}{6} \tag{72}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_2 - \beta(a_2 - a'_1) + a_2 + \beta(a'_3 - a_2)]\right]\beta d\beta = \frac{a'_1 + a_2 + a'_3}{6} \tag{73}$$

$$R = \frac{a_1 + 2a_2 + a_3 + a'_1 + a'_3}{12} \tag{74}$$

5.2.2 Graded Mean Integration Method for Triangular Intuitionistic Pendant Fuzzy Number

Considering Triangular Intuitionistic Pendant Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Graded Mean Integration Method are given by

$$R_M = \frac{1}{2}\left[\int_0^1 [a_{2p} - \alpha^5(a_{2p} - a_{1p}) + a_{2p} + \alpha^5(a_{3p} - a_{2p})]\right]\alpha d\alpha$$

$$= \frac{a_{1p} + 5a_{2p} + a_{3p}}{14} \tag{75}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_{2p} - \beta^5(a_{2p} - a'_{1p}) + a_{2p} + \beta^5(a'_{3p} - a_{2p})]\right]\beta d\beta$$

$$= \frac{a'_{1p} + 5a_{2p} + a'_{3p}}{14} \tag{76}$$

$$R = \frac{a_{1p} + 10a_{2p} + a_{3p} + a'_{1p} + a'_{3p}}{28} \tag{77}$$

5.2.3 Graded Mean Integration Method for Trapezoidal Intuitionistic Fuzzy Number

Considering Trapezoidal Intuitionistic Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Graded Mean Integration Method are given by

$$R_M = \frac{1}{2}\left[\int_0^1 [a_2 - \alpha(a_2 - a_1) + a_3 + \alpha(a_4 - a_3)]\right]\alpha d\alpha$$

$$= \frac{2a_1 + a_2 + a_3 + 2a_4}{12} \tag{78}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_2 - \beta(a_2 - a'_1) + a_3 + \beta(a'_4 - a_3)]\right]\beta d\beta$$

$$= \frac{2a'_1 + a_2 + a_3 + 2a'_4}{12} \tag{79}$$

$$R = \frac{2a_1 + 2a_2 + 2a_3 + 2a_4 + 2a'_1 + 2a'_4}{24} \tag{80}$$

5.2.4 Graded Mean Integration Method for Trapezoidal Intuitionistic Pendant Fuzzy Number

Considering Trapezoidal Intuitionistic Pendant Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Graded Mean Integration Method are given by

$$R_M = \frac{1}{2}\left[\int_0^1 [a_{2p} - \alpha^5(a_{2p} - a_{1p}) + a_{3p} + \alpha^5(a_{4p} - a_{3p})]\right]\alpha d\alpha$$

$$= \frac{2a_{1p} + 5a_{2p} + 5a_{3p} + 2a_{4p}}{28} \tag{81}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_{2p} - \beta^5(a_{2p} - a'_{1p}) + a_{3p} + \beta^5(a'_{4p} - a_{3p})]\right]\beta d\beta$$

$$= \frac{2a'_{1p} + 5a_{2p} + 5a_{3p} + 2a'_{4p}}{28} \tag{82}$$

$$R = \frac{2a_{1p} + 10a_{2p} + 10a_{3p} + 2a_{4p} + 2a'_{1p} + 2a'_{4p}}{56} \tag{83}$$

5.2.5 Graded Mean Integration Method for Pentagonal Intuitionistic Fuzzy Number

Considering Pentagonal Intuitionistic Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Graded Mean Integration Method are given by

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_2 - \alpha(a_2 - a_1) + a_4 + \alpha(a_5 - a_4) + a_3 - \alpha(a_3 - a_2) + a_3 + \alpha(a_4 - a_3)]\right]$$

$$\alpha d\alpha = \frac{2a_1 + 3a_2 + 2a_3 + 3a_4 + 2a_5}{12} \tag{84}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_2 - \beta(a_2 - a'_1) + a_4 + \beta(a'_5 - a_4) + a_3 - \beta(a_3 - a_2) + a_3 + \alpha(a_4 - a_3)]\right]$$

$$\beta d\beta = \frac{2a'_1 + 3a_2 + 2a_3 + 3a_4 + 2a'_5}{12} \tag{85}$$

$$R = \frac{2a_1 + 6a_2 + 4a_3 + 6a_4 + 2a_5 + 2a'_1 + 2a'_5}{24} \tag{86}$$

5.2.6 Graded Mean Integration Method for Pentagonal Intuitionistic Pendant Fuzzy Number

Considering Pentagonal Intuitionistic Pendant Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Graded Mean Integration Method are given by

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_{2p} - \alpha^5(a_{2p} - a_{1p}) + a_{4p} + \alpha^5(a_{5p} - a_{4p}) + a_{3p} - \alpha^5(a_{3p} - a_{2p}) + a_{3p} + \alpha^5(a_{4p} - a_{3p})]\right]$$

$$\alpha d\alpha = \frac{2a_{1p} + 7a_{2p} + 10a_{3p} + 7a_{4p} + 2a_{5p}}{28} \tag{87}$$

$$R_{NM} = \frac{1}{2}\left[\int_0^1 [a_{2p} - \beta^5(a_{2p} - a'_{1p}) + a_{4p} + \beta^5(a'_{5p} - a_{4p}) + a_{3p} - \beta^5(a_{3p} - a_{2p}) + a_{3p} + \beta^5(a_{4p} - a_{3p})]\right]$$

$$\beta d\beta = \frac{2a'_{1p} + 7a_{2p} + 10a_{3p} + 7a_{4p} + 2a'_{5p}}{28} \tag{88}$$

$$R = \frac{2a_{1p} + 14a_{2p} + 20a_{3p} + 14a_{4p} + 2a_{5p} + 2a'_{1p} + 2a'_{5p}}{56} \tag{89}$$

5.3 Centroid Method

Again for different fuzzy numbers using Centroid Method, the defuzzified values for R_M, R_{NM} and R are given as follows:

5.3.1 Centroid Method Triangular Intuitionistic Fuzzy Number

Considering Triangular Intuitionistic Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Centroid Method are given by

$$R_M = \frac{a_1 + a_2 + a_3}{3} \tag{90}$$

$$R_M = \frac{a'_1 + a_2 + a'_3}{3} \tag{91}$$

$$R = \frac{a_1 + 2a_2 + a_3 + a'_1 + a'_3}{6} \tag{92}$$

5.3.2 Centroid Method Triangular Intuitionistic Pendant Fuzzy Number

Considering Triangular Intuitionistic Pendant Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Centroid Method are given by

$$R_M = \frac{a_{1p} + a_{2p} + a_{3p}}{3} \tag{93}$$

$$R_M = \frac{a'_{1p} + a_{2p} + a'_{3p}}{3} \tag{94}$$

$$R = \frac{a_{1p} + 2a_{2p} + a_{3p} + a'_{1p} + a'_{3p}}{6} \tag{95}$$

5.3.3 Centroid Method Trapezoidal Intuitionistic Fuzzy Number

Considering Trapezoidal Intuitionistic Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Centroid Method are given by

$$R_M = \frac{a_1 + a_2 + a_3 + a_4}{4} \tag{96}$$

$$R_M = \frac{a'_1 + a_2 + a_3 + a'_4}{4} \tag{97}$$

$$R = \frac{a_1 + 2a_2 + 2a_3 + a_4 + a'_1 + a'_4}{8} \tag{98}$$

5.3.4 Centroid Method Trapezoidal Intuitionistic Pendant Fuzzy Number

Considering Trapezoidal Intuitionistic Pendant Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Centroid Method are given by

$$R_M = \frac{a_{1p} + a_{2p} + a_{3p} + a_{4p}}{4} \tag{99}$$

$$R_M = \frac{a'_{1p} + a_{2p} + a_{3p}}{4} \tag{100}$$

$$R = \frac{a_{1p} + 2a_{2p} + a_{3p} + a'_{1p} + a'_{3p}}{8} \tag{101}$$

5.3.5 Centroid Method Pentagonal Intuitionistic Fuzzy Number

Considering Pentagonal Intuitionistic Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Centroid Method are given by

$$R_M = \frac{a_1 + a_2 + a_3 + a_4 + a_5}{5} \tag{102}$$

$$R_M = \frac{a'_1 + a_2 + a_3 + a_4 + a'_5}{5} \tag{103}$$

$$R = \frac{a_1 + 2a_2 + 2a_3 + 2a_4 + a_5 + a_1' + a_5'}{10} \tag{104}$$

5.3.6 Centroid Method Pentagonal Intuitionistic Pendant Fuzzy Number

Considering Pentagonal Intuitionistic Pendant Fuzzy Number the defuzzified values of R_M, R_{NM} and R using Centroid Method are given by

$$R_M = \frac{a_{1p} + a_{2p} + a_{3p} + a_{4p} + a_{5p}}{5} \tag{105}$$

$$R_M = \frac{a_{1p}' + a_{2p} + a_{3p} + a_{4p} + a_{5p}'}{5} \tag{106}$$

$$R = \frac{a_{1p} + 2a_{2p} + 2a_{3p} + 2a_{4p} + a_{5p} + a_{1p}' + a_{5p}'}{10} \tag{107}$$

6. Analysis

6.1 Defuzzified Values

The following table gives the Reliability of Weaving machine using Triangular Intuitionistic Fuzzy Number, Triangular Intuitionistic Pendant Fuzzy Number, Trapezoidal Intuitionistic Fuzzy Number, Trapezoidal Intuitionistic Pendant Fuzzy Number, Pentagonal Intuitionistic Fuzzy Number and Pentagonal Intuitionistic Pendant Fuzzy Number using the Defuzzification Techniques: Signed Distance Method, Graded Mean Integration Method and Centroid Method

Table 1. Reliability of Weaving Machine.

Numbers	Signed Distance Method			Graded Mean Integration Method			Centroid Method		
	R_M	R_{NM}	R	R_M	R_{NM}	R	R_M	R_{NM}	R
TIFN	0.10409931	0.12642993	0.1526462	0.05453049	0.06941757	0.06197403	0.10906098	0.13883514	0.123948
TIPFN	0.09417597	0.10161951	0.09789	0.04886001	0.055240187	0.0520501	0.10906098	0.13883514	0.123948
TrIFN	0.16958859	0.21506901	0.192328797	0.088145814	0.118466092	0.10330595	0.16958859	0.215069006	0.1923288
TrIPFN	0.15618251	0.17134265	0.16376257	0.080485197	0.093479603	0.0869824	0.16958859	0.215069006	0.1923288
PenIFN	0.36898327	0.40978315	0.389383	0.185785735	0.212985655	0.1993857	0.18740446	0.220044368	0.2037244
PenIPFN	0.3638018	0.27740682	0.370603408	0.182827787	0.194484895	0.18865634	0.18740446	0.220044368	0.2037244

6.2 Comparative study through graph

6.2.1 Comparison of Triangular Intuitionistic Fuzzy Number and Triangular Intuitionistic Pendant Fuzzy Number using Bar Diagram

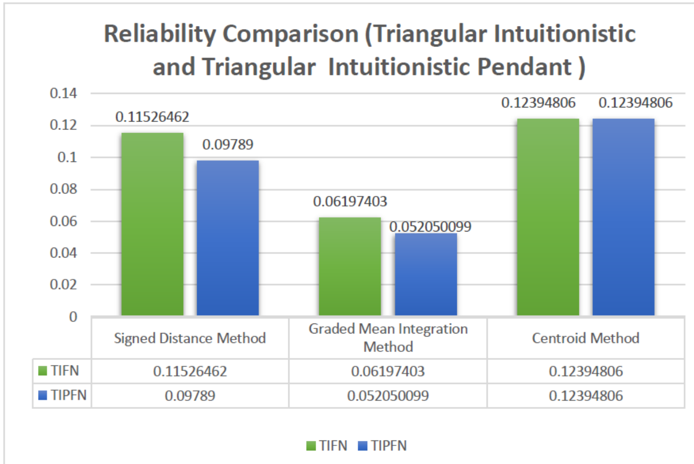

Figure 1. Reliability Graph 1.

6.2.2 Comparison of Trapezoidal Intuitionistic Fuzzy Number and Trapezoidal Intuitionistic Pendant Fuzzy Number using Bar Diagram

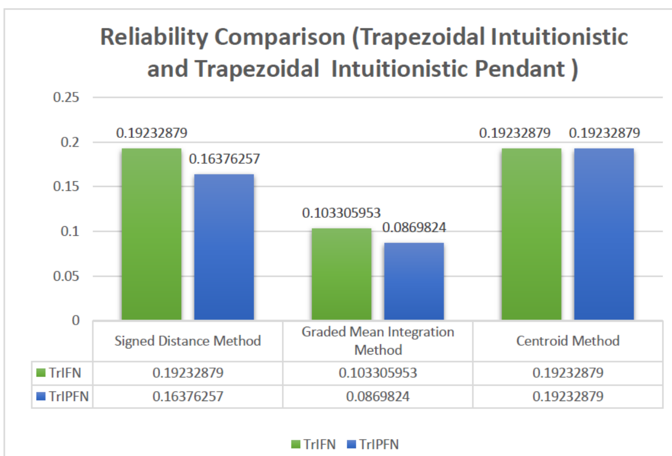

Figure 2. Reliability Graph 2.

6.2.3 Comparison of Pentagonal Intuitionistic Fuzzy Number and pentagonal Intuitionistic Pendant Fuzzy Number using Bar Diagram

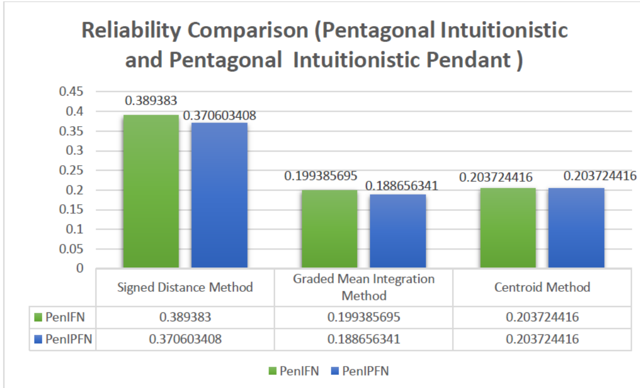

Figure 3. Reliability Graph 3.

6.3 Representation of Reliability through graph

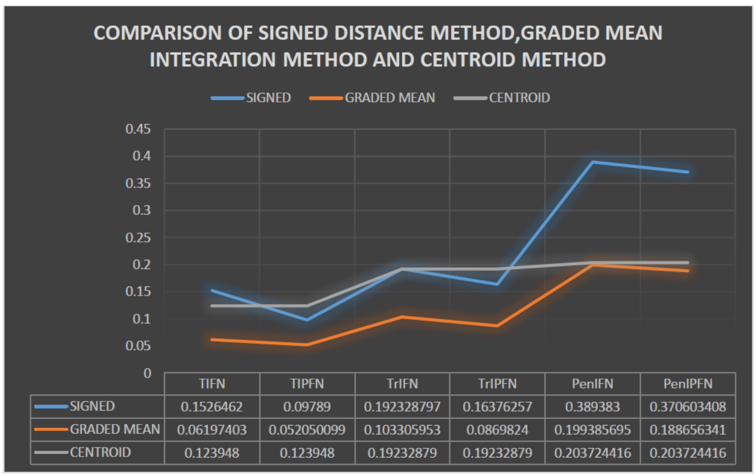

Figure 4. Reliability Graph 4.

7. Conclusion

From the above analysis it is found that when Signed Distance Method is used for defuzzification, the value obtained using Triangular Intuitionistic Fuzzy Number is more reliable than the value obtained using Triangular Intuitionistic Pendant Fuzzy Number and it is same for Trapezoidal Intuitionistic and Pentagonal Intuitionistic fuzzy numbers. Reliability value become less when Graded mean Integration Method is used for

Defuzzification for all the Fuzzy Numbers in this study. Using Centroid Method , the reliability value obtained from Triangular Intuitionistic Fuzzy Number and Triangular Intuitionistic Pendant Fuzzy number are same and similarly for Trapezoidal Intuitionistic and Pentagonal Intuitionistic fuzzy numbers. Also Centroid Method gives better reliability value than the values obtained from Signed Distance Method and Graded Mean Integration Method for all the Fuzzy Numbers in this study. When Pentagonal Intuitionistic Fuzzy Number and Pentagonal Intuitionistic Pendant Fuzzy Number are used the reliability is high irrespective of defuzzification techniques. Comparative study shows that the reliability values derived by considering Pentagonal Intuitionistic Fuzzy Number and Pentagonal Intuitionistic Pendant Fuzzy Number and using Signed Distance Method for defuzzification are more reliable than other fuzzy numbers and other defuzzification techniques in this study.

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