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MODELLING SHALLOW WATER WAVES PROPAGATION THROUGH NATURAL-MODIFIED DECOMPOSITION METHOD

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Abstract

Analytical solution to Shallow water wave's equations by Natural - Modified decomposition method [NMDM] is introduced. The proposed analytical approach is an elegant combination of Natural transform and Modified Adomian decomposition method and offered a new standpoint for soliton solution of a model of propagation of Shallow water wave's equations. In this method does not exigency linearization, powerless non-linearity assumptions and constraints or chaos theory and it reduces computational size and avoids round-off errors. Comparative study between numerical and analytical solutions will be considered. The geometric simulations are also offered for the model equations.

Introduction

Nonlinearity exists all over the place of the nature in broad-spectrum. Nonlinear physical phenomena that appear in many areas of physical dynamic systems can be modelled by partial differential equation. Most of physical phenomena are describe by appropriate set of nonlinear partial differential equations. A modal which describes the propagation of shallow water waves with diverse scattering relations have been studied in [1-4] is called coupled Whitham-Broer-Kaup (CWBK) systems as follows

$$E_t - \beta E_{xx} + \alpha \eta_{xxx} + (\eta E)_x = 0 \tag{1}$$

$$\eta_t + \beta \eta_{xx} + \eta \eta_x + E_x = 0, \tag{2}$$

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where $E = E(x, t)$ is the height from symmetry location of the liquid, $\eta = \eta(x, t)$ is the horizontal velocity, α, β are parameter which are described in diverse limit. If $\alpha = 0$ and $\beta \neq 0$ then it's as model of shallow water wave with diffusion [3]. If $\alpha = 1$ and $\beta = 0$ then it's became as a model for water wave which was derived by Sachs [5] in 1988 know as coupled Variant Boussinesq Equation. Z. H. Khan and W. A. Khan [6] introduced an integral transform called N-transform and it was recently renamed as the Natural Transform by Belgacem and Silambarasan [7-8] this is analogous to Laplace transform and Sumudu transform when $u = 1$ and when $s = 1$ respectively [9-11]. Belgacem and Silambarasan [8, 12] have proposed a detailed theory and applications of the Natural Transform. A trustworthy adjustment of Adomian decomposition method has been prepared by Wazwaz [13] known as Modified Decomposition Method. Laplace-Modified Decomposition Method is functional for Solitary Wave Solutions of nonlinear partial differential equation [14-17]. In this task, we shared Natural Transform and Modified decomposition method (in short NMDM) to achieve the Solitons for shallow water waves propagation.

Implementation of Method

In this method first we are taking Natural Transform of both sides of equations (1) and (2), applying with derivative character and after that interpolate inverse transformation of it on those equations. We obtain

$$E(x, t) = E(x, 0) + N^{-1}\left[\frac{u}{s} N^{+}\{(\beta E_{xx} - \alpha\eta_{xx} - M)\}\right]. \tag{3}$$

$$\eta(x, t) = \eta(x, 0) - N^{-1}\left[\frac{u}{s} N^{+}\{(\beta\eta_{xx} + H + E_{x})\}\right]. \tag{4}$$

Where $M(E, \eta) = (\eta E)_x$, $H(E, \eta) = \eta\eta_x$ are indicate nonlinear terms, Infinite series solutions of unknown function $E(x, t)$, $\eta(x, t)$ will be given by modified-Adomian decomposition method as

$$E(x, t) = \sum_{n=0}^{\infty} E_n,\ \eta(x, t) = \sum_{n=0}^{\infty} \eta_n. \tag{5}$$

The nonlinear terms $M(E, \eta) = (\eta E)_x$, $H(E, \eta) = \eta\eta_x$ will be discovered by Adomian polynomials as such

$$M(E, \eta) = \sum_{n=0}^{\infty} A_n(E_0, E_1, \ldots, E_n, \eta_0, \eta_1, \ldots, \eta_n) \tag{6}$$

$$H(E, \eta) = \sum_{n=0}^{\infty} B_n(E_0, E_1, \ldots, E_n, \eta_0, \eta_1, \ldots, \eta_n). \tag{7}$$

They are determined by the following relations:

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} M\left(\sum_{n=0}^{\infty} \lambda^i E_i, \sum_{n=0}^{\infty} \lambda^i \eta_i\right)\right], n \geq 0 \tag{8}$$

$$B_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} H\left(\sum_{n=0}^{\infty} \lambda^i E_i, \sum_{n=0}^{\infty} \lambda^i \eta_i\right)\right], n \geq 0 \tag{9}$$

only some Modified-Adomian decomposition polynomials for $M(E, \eta) = (\eta E)_x$, $H(E, \eta) = \eta\eta_x$ of the nonlinearity as

$$A_0 = (\eta_0 E_0)_x \qquad B_0 = \eta_0 \eta_{0x}$$

$$A_1 = (\eta_0 E_1 + \eta_1 E_0 + \eta_1 E_1)_x \quad B_1 = \eta_1 \eta_{0x} + \eta_0 \eta_{1x} + \eta_1 \eta_{1x}$$

$$A_2 = (\eta_0 E_2 + \eta_2 E_0 + \eta_1 E_2 + \eta_2 E_1 + \eta_2 E_2)_x$$

$$B_2 = \eta_2 \eta_{0x} + \eta_0 \eta_{2x} + \eta_1 \eta_{2x} + \eta_2 \eta_{2x}. \tag{10}$$

Then obtain the following mechanism by means of the standard MADM, after identify E_0, η_0

$$E_{n+1}(x, t) = N^{-1}\left[\frac{u}{s} N^+\left\{\left(\beta \sum_{n=0}^{\infty} E_{nxx} - \alpha \sum_{n=0}^{\infty} \eta_{nxx} - \sum_{n=0}^{\infty} A_n\right)\right\}\right] \tag{11}$$

$$\eta_{n+1}(x, t) = N^{-1}\left[\frac{u}{s} N^+\left\{\left(\beta \sum_{n=0}^{\infty} \eta_{nxx} + \sum_{n=0}^{\infty} B_n + \sum_{n=0}^{\infty} E_{nx}\right)\right\}\right]. \tag{12}$$

The sensible solutions for χ_n, μ_n

$$\chi_n = \sum_{n=0}^{\infty} E_n,\ n \geq 0 \text{ with } \lim_{n\to\infty} \chi_n = E(x, t) \tag{13}$$

$$\mu_n = \sum_{n=0}^{\infty} \eta_n,\ n \geq 0 \text{ with } \lim_{n\to\infty} \mu_n = \eta(x, t). \tag{14}$$

We consider initial condition E_0 and η_0 for the system and we can compute the other components of solution by equation (11) and equation (12)

$$E_0 = E(x, 0) = -2(\alpha + \beta^2 + \beta\sqrt{\alpha + \beta^2})k^2 \csc h^2(kx)$$

$$\eta_0 = \eta(x, 0) = \omega - 2(\sqrt{\alpha + \beta^2})k \coth(kx)$$

$$E_1(x, t) = N^{-1}\left[\frac{u}{s} N^{+}\{(\beta E_{0xx} - \alpha\eta_{0xxx} - A_0)\}\right]$$

$$E_1 = -4t\sqrt{\alpha + \beta^2}k^4\{(\sqrt{\alpha + \beta^2} + \beta)^2 + \alpha\}[\csc h^4(kx) + 2\csc h^2(kx)\coth^2(kx)]$$

$$-4t\omega k^4\sqrt{\alpha + \beta^2}(\sqrt{\alpha + \beta^2} + \beta)\csc h^2(kx)\coth kx$$

$$\eta_1(x, t) = N^{-1}\left[\frac{u}{s} N^{+}\{(\beta\eta_{0xx} + B_0 + E_{0x})\}\right]$$

$$\eta_1(x, t) = 2t\omega k^2\sqrt{\alpha + \beta n^2}\ \csc h^2(kx)$$

$$E_2(x, t) = N^{-1}\left[\frac{u}{s} N^{+}\{(\beta E_{1xx} - \alpha\eta_{1xxx} - A_1)\}\right]$$

$$E_2(x, t) = 4pt^2k^6\{(p + \beta)^2 + \alpha\}[2(5p - 9\beta)\csc h^4(kx)\coth^2(kx)$$

$$+ (p - 3\beta)\csc h^6(kx) + (4p - 3\beta)\csc h^2(kx)\coth^4(kx)]$$

$$-4pt^2k^5\omega[\{3((p + \beta)^2 + \alpha) + 2(\alpha + \beta)\}\csc h^4(kx)\coth^2(kx)$$

$$+ \{(2k - 3)\alpha - 3(p + \beta)^2\}\csc h^2(kx)\coth^3(kx)]$$

$$-2pt^2k^4\omega\{k\omega(p + \beta) + \alpha\}(\csc h^4(kx) + 2\csc h^2(kx)\coth^2(kx))$$

$$-\frac{16}{3}t^3p^2k^7\omega((p+\beta)^2+\alpha)[5\csc h^6(kx)\coth(kx)+4\csc h^2(kx)\coth^3(kx)]$$

$$-\frac{8}{3}t^3p^2k^7\omega^2(p+\beta)(\csc h^6(kx)+4\csc h^4(kx)\coth^2(kx))$$

where $p^2 = \alpha + \beta^2$

$$\eta_2(x,t)=N^{-1}\left[\frac{u}{s}N^+\{(\beta\eta_{1xx}+B_1+E_{1x})\}\right]$$

$$\eta_2(x, t) = -2\beta t^2\omega k^2\sqrt{\alpha+\beta^2}[\coth^4(kx)+2\csc h^2(kx)\coth^2(kx)]$$

$$-2t^2\omega^2k^3\sqrt{\alpha+\beta^2}\csc h^2(kx)\coth(kx)$$

$$+4t^2\omega k^4(\alpha+\beta^2)\csc h^2(kx)\coth^2(kx)$$

$$+2t^2\omega k^5(\alpha+\beta^2)\csc h^4(kx)$$

$$+2t^2k^4\sqrt{\alpha+\beta^2}\{(\sqrt{\alpha+\beta^2}+\beta)^2+\alpha\}[6\csc h^4(kx)\coth(kx)$$

$$+4\csc h^2(kx)\coth^3(kx)]$$

$$+2t^2\omega k^5\sqrt{\alpha+\beta^2}(\sqrt{\alpha+\beta^2}+\beta)[\csc h^4(kx)$$

$$+2\csc h^2(kx)\coth^2(kx)]$$

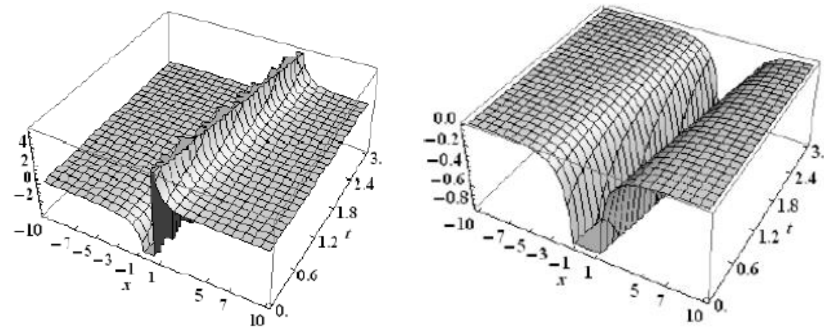
$$-\frac{4}{3}t^3\omega^2k^5(\alpha+\beta^2)\csc h^4(kx)\coth(kx)$$

and so on. In a similar way, then the series solutions expression by NMDM can be written in the close form:

$$E(x, t) = -2(\alpha+\beta^2+\beta\sqrt{\alpha+\beta^2})k^2\csc h^2\{k(x-\omega t)\}$$

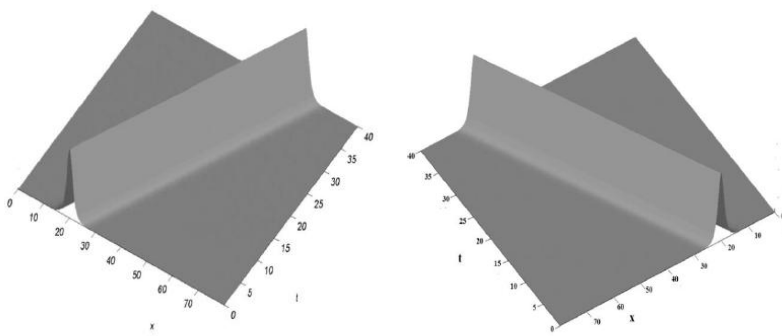
$$\eta(x, t) = \omega - 2(\sqrt{\alpha+\beta^2})k\coth\{k(x-\omega t)\}.$$

For soliton if $\alpha = 1 = \beta$, $\omega = 1$, $k = 0.7$ with $-10 \le x \ge 10$, $0 \le t \ge 3$.

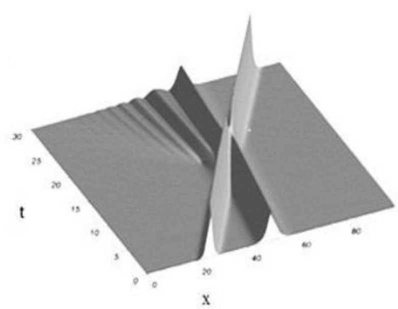


$|E(x,\, t)|$ $|\eta(x, t)|$.

If we take the time footpace $\Delta(t) = 0.1,\ 0 \leq t \leq 40$ and space gait $\Delta x = 0.1,\ 0 \leq x \leq 70$. We can see after the Synovial of the two soliton waves, they move propulsion in the same direction and the same velocity as before of colloid for $|E_2(x,\, t)|$ and $|\eta_2(x,\, t)|$.



$|E_2(x,\, t)|$ $|\eta_2(x,\, t)|$



$\{|E|_2 + |\eta|_2\}$.

Simulation results of the interaction of the two waves with $\alpha = 1$, $\beta = 0$, $\omega = 1$, $k = 0.7$.

Conclusions

The Natural-Modified Adomian decomposition method [NMDM] is a dominant method which has provided a well-organized prospective for nonlinear Whitham-Broer-Kaup (WBK) Equation with initial condition. This scheme is embellish to locate solitary wave solution of Shallow Water Wave systems. The analytical solution of Shallow Water Wave model has been designed without any require to a transformation techniques and linearization by this method [NMDM]. As well as, any discretization method to obtain geometric solutions by this method does not require. In the mechanism of this method may be applied on nonlinear partial differential equations without any required to difficult calculations apart from for straightforward and basic operators.

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