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DEFUZZIFICATION OF SYMMETRIC OCTAGONAL INTUITIONISTIC FUZZY NUMBER

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Abstract

The aim of this paper is to obtain an optimal solution for the Intuitionistic Fuzzy Assignment Problem using Symmetric Octagonal Intuitionistic Fuzzy Number. The costs of the Intuitionistic Fuzzy Assignment Problem are taken as Symmetric Octagonal Intuitionistic Fuzzy Numbers. The costs are defuzzified into crisp values using the proposed ranking. The Intuitionistic Fuzzy Assignment Problem is solved by Hungarian method. An example illustrates the proposed method.

1. Introduction

Assignment problem is a particular kind of linear programming problem which deals with the allocation of the various resources to the various activities on one to one basis. It does it in such a way that the cost or time involved in the process is minimum and profit or sale is maximum. Suppose

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there are 'n' facilities and 'n' jobs it is clear that in this case, there will be n assignments. Each facility or say machine can perform each job, one at time. But there should be certain procedure by which assignment should be made so that the profit is maximized or the cost or time is minimized.

In real life, there are many situations, where it is impossible to get precise data for the cost parameters, due to complexity and uncertainty of information. Therefore, it is desirable to apply fuzzy sets to eliminate the impreciseness and vagueness.

The concept of fuzzy set was introduced by Zadeh [13] in 1965 and it dealt with imprecision, vagueness in real life situations. In 1970, Bellman and Zadeh [4] proposed the concept of decision making problems involving uncertainty and imprecision. K. T. Attanasov [1], [2], [3] introduced Intuitionistic fuzzy sets and their applications to deal with vagueness. W. L. Gau and D. J. Buehrer [7] discussed the concept of vague sets. Bustine and Burillo [5] insisted that vague sets are intuitionistic fuzzy sets. Dhanalakshmi V. and Felbin C. Kennedy [6] proposed the ranking algorithm for symmetric octagonal Fuzzy Numbers. G. Menaka [8] and Rajarajeswari, G. Menaka [10] proposed a new approach for ranking of octagonal intuitionistic fuzzy numbers. G. Sahaya Sudha, K. R. Vijayalakshmi [11] presented a value and ambiguity based ranking of a symmetric hexagonal intuitionistic fuzzy numbers in decision making. R. Parvathi, C. Malathi [9] listed the Arithmetic Operations on Symmetric Trapezoidal Intuitionistic Fuzzy Numbers using (α, β) cuts. G. Uthra et al. [12] defined the Arithmetic Operations on Symmetric Octagonal Intuitionistic Fuzzy Numbers.

In this paper, an Intuitionistic Fuzzy Assignment Problem is considered. The cost values of the Intuitionistic Fuzzy Assignment Problem are taken as Symmetric Octagonal Intuitionistic Fuzzy Numbers. The Symmetric Octagonal Intuitionistic Fuzzy Numbers are converted into crisp values using proposed ranking procedure. The problem is then solved by the usual Hungarian Method.

The rest of this paper is organized as follows. In Section 2, some basic definitions and arithmetic operations of Symmetric Octagonal Intuitionistic Fuzzy Numbers are given. Section 3 presents introduction of Intuitionistic Fuzzy Assignment Problem. In Section 4, procedure and numerical example for the proposed method are given followed by conclusion in section 5.

2. Definitions

Definition 2.1. Fuzzy Set. Let A be a classical set, $\mu_A(x)$ be a function from A to $[0, 1]$. A fuzzy set A with the membership function $\mu_A(x)$ is defined as $A = \{x, \mu_A(x); x \in A$ and $\mu_A(x) \in [0, 1]\}$.

Definition 2.2. Intuitionistic Fuzzy Set. Let X be a given set. An Intuitionistic Fuzzy Set A in X is given by, where $A = \{(x, \mu_A(x), {}_A\vartheta(x)) | x \in X\}$ $X \to [0, 1]$, where $\mu_A(x)$ is the degree of membership of the element x in A and $\vartheta_A(x)$ is the degree of non membership of x in A and $0 \le \mu_A(x) + \vartheta_A(x) \le 1$.

Definition 2.3. Octagonal Intuitionistic Fuzzy Number (OIFN). An OIFN is specified by $A = [(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8); (a_1', a_2', a_3', a_4', a_5', a_6', a_7', a_8')]$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_1', a_2', a_3', a_4', a_5', a_6', a_7', a_8'$ are real numbers and its membership and non-membership functions are given below.

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a_1 \\ k\left(\dfrac{x - a_1}{a_2 - a_1}\right) & \text{if } a_1 \le x \le a_2 \\ k & \text{if } a_2 \le x \le a_3 \\ k + (1 - k)\left(\dfrac{x - a_3}{a_4 - a_3}\right) & \text{if } a_3 \le x \le a_4 \\ 1 & \text{if } a_4 \le x \le a_5 \\ k + (1 - k)\left(\dfrac{a_6 - x}{a_6 - a_5}\right) & \text{if } a_5 \le x \le a_6 \\ k & \text{if } a_6 \le x \le a_7 \\ k\left(\dfrac{a_8 - x}{a_8 - a_7}\right) & \text{if } a_7 \le x \le a_8 \end{cases}$$

$$\vartheta_A(x) = \begin{cases} 1 & \text{if } x < a_1' \\ k + (1-k)\left(\dfrac{a_2' - x}{a_2' - a_1'}\right) & \text{if } a_1' \le x \le a_2' \\ k & \text{if } a_2' \le x \le a_3' \\ k\left(\dfrac{a_4 - x}{a_4 - a_3'}\right) & \text{if } a_3' \le x \le a_4 \\ 0 & \text{if } a_4 \le x \le a_5' \\ k\left(\dfrac{x - a_5}{a_6' - a_5}\right) & \text{if } a_5 \le x \le a_6' \\ k & \text{if } a_6' \le x \le a_7' \\ k + (1-k)\left(\dfrac{x - a_7'}{a_8' - a_7'}\right) & \text{if } a_7' \le x \le a_8' \end{cases}$$

where $k = 1/2$.

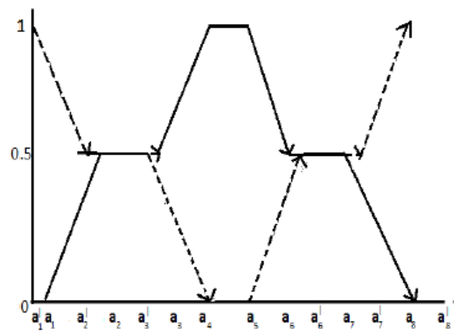


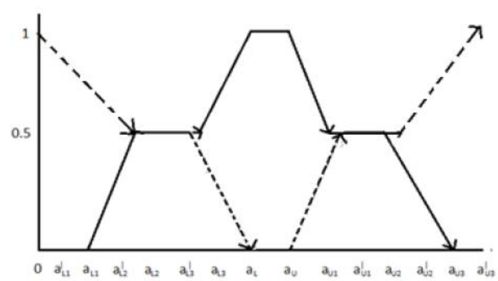
Figure 1. Diagrammatic Representation of OIFN.

Definition 2.4. Symmetric Octagonal Intuitionistic Fuzzy Number (SOIFN).

A SOIFN is given by $A_0 = [(a_L - r - s - t,\ a_L - r - s,\ a_L - r,\ a_L,\ a_U,\ a_U$ $r,\ a_U + r + s,\ a_U + r + s + t);\ (a_L' - r' - s' - t',\ a_L' - r' - s',\ a_L' - r',\ a_L,\ a_U,\ a_U'$ $+ r',\ a_U' + r' + s',\ a_U' + r' + s' + t')]$ where $a_L - r - s - t,\ a_L - r - s,\ a_L - r,$ $a_L,\ a_U,\ a_U + r,\ a_U + r + s,\ a_U + r + s + t,\ a_L' - r' - s' - t',\ a_L' - r' - s',\ a_L' - r',$ $a_L,\ a_U,\ a_U' + r',\ a_U' + r' + s',\ a_U' + r' + s' + t'$ are real numbers such that $(a_L' - r' - s' - t' \le a_L - r - s - t \le a_L' - r' - s' \le a_L - r - s \le a_L' - r' \le a_L - r \le$ $a_L \le a_U \le a_U + r \le a_U' + r' \le a_U + r + s \le a_U' + r' + s' \le a_U + r + s + t \le a_U' +$ $r' + s' + t')$ and its membership and non-membership functions are given below

$$\mu_{A_0}(x)=\begin{cases}\dfrac{x-(a_L-r-s-t)}{2t}, & a_L-r-s-t\le x\le a_L-r-s\\ \dfrac{1}{2}, & a_L-r-s\le x\le a_L-r\\ \dfrac{1}{2}+\dfrac{(x-(a_L-r))}{2r} & a_L-r\le x\le a_L\\ 1 & a_L\le x\le a_U\\ \dfrac{1}{2}+\dfrac{((a_U+r)-x)}{2r} & a_U\le x\le a_U+r\\ \dfrac{1}{2}, & a_U+r\le x\le a_U+r+s\\ \dfrac{((a_u+r+s+t)-x)}{2t}, & a_U+r+s\le x\le a_U+r+s+t\end{cases}$$

$$\vartheta_{A_0}(x)=\begin{cases}\dfrac{1}{2}+\dfrac{(a'_L-r'-s')-x}{2t'}, & a'_L-r'-s'-t'\le x\le a'_L-r'-s'\\ \dfrac{1}{2}, & a'_L-r'-s'\le x\le a'_L-r'\\ \dfrac{1}{2}\left(\dfrac{a_L-x}{a_L-(a'_L-r')}\right) & a'_L-r'\le x\le a_L\\ 0 & a_L\le x\le a_U\\ \dfrac{1}{2}\left(\dfrac{x-a_U}{(a'_U-r')-a_U}\right) & a_U\le x\le a'_U+r'\\ \dfrac{1}{2}, & a'_U+r'\le x\le a'_U+r'+s'\\ \dfrac{1}{2}+\left(\dfrac{(x-(a'_U+r'+s'))}{2t'}\right), & a'_U+r'+s'\le x\le a'_U+r'+s'+t'\end{cases}$$



Diagrammatic representation of SOIFN.

where $a_{L1}=a_L-r-s-t$, $a_{L2}=a_L-r-s$, $a_{L3}=a_L-r$, $a_{U1}=a_U+r$, $a_{U2}=a_U+r+s$, $a_{U3}=a_U+r+s+t$, $a'_{L1}=a'_L=a'_L-r'-s'-t'$, $a'_{L2}=a'_L-r'-s'$ $a'_{L3}=a'_L-r'$, $a'_{U1}=a'_U+r'$, $a'_{U2}=a'_U+r'+s'$, $a'_{U3}=a'_U+r'+s'+t'$.

Definition 2.5. Arithmetic Operations on SOIFN.

If $A_0 = [(a_L - r_1 - s_1 - t_1, a_L - r_1 - s_1, a_L - r_1, a_L, a_U, a_U + r_1, a_U + r_1 + s_1, a_U + r_1 + s_1 + t_1); (a'_L - r'_1 - s'_1 - t'_1, a'_L - r'_1 - s'_1, a'_L - r'_1, a_L, a_U, a'_U + r'_1 + a'_U + r'_1 + s'_1 + a'_U + r'_1 + s'_1 + t'_1)]$ $B_o = [(b_L - r_2 - s_2 - t_2, b_L - r_2 - s_2, b_L - r_2, b_L$ $b_U, b_U + r_2, b_U + r_2 + s_2, b_U + r_2 + s_2 + t_2); (b'_L - r'_2 - s'_2 - t'_2, b'_L - r'_2 - s'_2,$ $b'_L - r'_2, b_L, b_U, b'_U + r'_2, b'_U + r'_2 + s'_2, b'_U + r'_2 + s'_2 + t_2{}')]$ are two SOIFNs then.

1. Addition $A_o + B_o = [((a_L + b_L) - (r + s + t), (a_L + b_L) - (r + s),$ $(a_L + b_L) - r, a_L + b_L, a_U + b_U, (a_U + b_U) + r, (a_U + b_U) + (r + s), (a_U + b_u) +$ $(r + s + t)); ((a'_L + b'_L) - (r' + s' + t), (a'_L + b'_L) - (r' + s'), (a'_L + b'_L) - r', a_L + b_L,$ $a_U + b_U, (a'_U + b'_U) + r', (a'_U + b'_U) + (r' + s'), (a'_U + b'_U) + (r' + s' + t'))]$.
Where $r_1 + r_2 = r, s_1 + s_2 = s, t_1 + t_2 = t, t'_1 + t'_2 = t', s'_1 + s'_2 = s', r'_2 + r'_2 = r'$.

2. Subtraction

$A_o - B_o = (((a_L - b_U) - (r + s + t), (a_L - b_U) - (r + s), (a_L - b_U) - r, a_L$ $-b_L, a_U - b_U, (a_U - b_L) + r, (a_U - b_L) + (r + s), (a_U - b_L) + (r + s + t));$ $((a'_L - b'_U) - (r' + s' + t), (a'_L - b'_U) - (r' + s'), (a'_L - b'_U) - r', a_L - b_L, a_U -$ $b_U, (a'_U - b_L{}') + r', (a'_U - b'_L) + (r' + s'), (a'_U - b'_L) + (r' + s' + t')))$. Where $r_1 + r_2 = r, s_1 + s_2 = s, t_1 + t_2 = t, t'_1 + t'_2 = t', s'_1 + s'_2 = s', r'_1 + r'_2 = r'$

3. Scalar Multiplication

$$C_o = \begin{cases} [k(a_L - r - s - t), k(a_L - r - s), k(a_L - r), ka_L, ka_U, \\ k(a_U + r), k(a_U + r + s), k(a_U + r + s + t); k(a'_L - r' - s' - t')k(a'_L - r' - s'), \\ k(a'_L - r'), ka_L, ka_U, k(a'_U + r'), k(a'_U + r' + s'), k(a'_U + r' + s' + t')]; k > 0 \\ [-k(a_U + r + s + t), -k(a_U + r + s), -k(a_U + r), -ka_U, -ka_L, -k(a_U - r), \\ -k(a_U - r - s), -k(a_U - r - s - t); -k(a'_U + r' + s' + t'), -k(a'_U + \dot{r}' + s'), \\ -k(a'_L + \dot{r}'), -ka_U, -ka_L, -k(a'_U - \dot{r}'), -k(a'_U - \dot{r}' - s'), \\ -k(a'_U - \dot{r}' - s' - t')]; k < 0. \end{cases}$$

4. Additive Image

$A'_o = (-(a_U + r + s + t), -(a_U + r + s), -(a_U + r), -a_U, -a_L - (a_L - r), -(a_L - r - s),$

$-(a_L - r - s - t) - (a'_U + r' + s' + t'), -(a'_U + r' + s'), (-(a_U + r'), -a_U, -a_L, -(a'_L - r'),$
$-(a'_L - r' - s'), -(a'_L - r' - s' - t').$

Definition 2.6. Ranking of Symmetric Octagonal Intuitionistic Fuzzy Number (SOIFN)

The ranking function of Symmetric Octagonal Intuitionistic Fuzzy Number $A_{so} = (a_1, a_2, a_3, a_L, a_U, a_6, a_7, a_8; a'_1, a'_2, a'_3, a_L, a_U, a'_6, a'_7, a'_8)$ maps the set of all Fuzzy Numbers to a set of real numbers defined as $R[A_{so}] = Max[Mag_{\mu}(A_{so}), Mag_{\vartheta}(A_{so})]$ and similarly $R[B_{so}] = Max$ $[Mag_{\mu}(B_{so}), Mag_{\vartheta}(B_{so})]$ where $Mag_{\mu}(A_{so}) = a_1 + a_2 + a_3 + a_L + a_U + a_6$ $+ a_7 + a_8 /8 Mag_{\vartheta}(A_{so}) = \dfrac{a'_1 + a'_2 + a'_3 + a_L + a_U + a'_6 + a'_7 + a'_8}{8}.$

Remark. If A_{so} and B_{so} are two Symmetric Octagonal Intuitionistic Fuzzy Numbers. Then

(i) $A_{so} < B_{so}$ *if* $Mag_{\mu}(A_{so}) < Mag_{\mu}(B_{so})$ and $Mag_{\vartheta}(A_{so}) < Mag_{\vartheta}(B_{so})$

(ii) $A_{so} > B_{so}$ *if* $Mag_{\mu}(A_{so}) > Mag_{\mu}(B_{so})$ and $Mag_{\vartheta}(A_{so}) > Mag_{\vartheta}(B_{so})$

(iii) $A_{so} = B_{so}$ *if* $Mag_{\mu}(A_{so}) = Mag_{\mu}(B_{so})$ and $Mag_{\vartheta}(A_{so}) = Mag_{\vartheta}(B_{so})$.

3. Intuitionistic Fuzzy Assignment Problem

Consider the situation of assigning n machines to n jobs and each machine is capable of doing each job at different costs. Let C^*_{ij} be an Intuitionistic fuzzy cost of assigning the i^{th} machine to the j^{th} job. Let x_{ij} be the decision variable denoting the assignment of the i^{th} machine to the j^{th} job. The objective is to minimize the total cost.

The mathematical model of an Intuitionistic Fuzzy Assignment Problem is given by

$$\text{Minimize } z^* = \sum_{i=1}^{n} \sum_{j=1}^{n} C^*_{ij} x_{ij}.$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \ for\ j = 1, 2 \cdots n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \ for\ i = 1, 2 \cdots n$$

$$x_{ij} \in [0, 1] \text{ where } x_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{machine is assigned to } j^{th} \text{job} \\ 0, \text{ if the } i^{th} \text{machine is assigned to } j^{th} \text{job} \end{cases}$$

$$C_{ij}^{*} = (C_{ij}^{1}, C_{ij}^{2}, C_{ij}^{3}, C_{ij}^{L}, C_{ij}^{U}, C_{ij}^{6}, C_{ij}^{7}, C_{ij}^{8})$$

$$(C_{ij}^{1*}, C_{ij}^{2*}, C_{ij}^{3*}, C_{ij}^{L}, C_{ij}^{U}, C_{ij}^{6*}, C_{ij}^{7*}, C_{ij}^{8*}).$$

4. Procedure

1. Consider the Intuitionistic Fuzzy Assignment Problem whose cost values are Symmetric Octagonal Intuitionistic Fuzzy Numbers.

2. Symmetric Octagonal Intuitionistic Fuzzy Numbers are defuzzified into crisp values using proposed ranking.

3. Check whether the Intuitionistic Fuzzy Assignment Problem is balanced.

(i) If balanced go to step 5 (no. of rows = no. of columns)

(ii) If not balanced go to step 4 (no. of rows ≠ no. of columns)

4. If the given Intuitionistic Fuzzy assignment problem is not balanced then add dummy row (or) dummy column to change the problem into balanced. (the cost value is zero for dummy row/dummy column).

5. Solve the assignment problem by the usual Hungarian method to get the optimal solution.

Example. An Intuitionistic Fuzzy Assignment problem with rows representing 3 machines M_1, M_2, M_3 and columns representing the 3 jobs J_1, J_2, J_3 is considered. The cost matrix C_{ij}^{*} whose elements are Symmetric

Octagonal Intuitionistic Fuzzy Numbers is given below.

$$\begin{pmatrix} (4, 6, 8, 10, 12, 14, 16, 18; 1, 4, 7, 10, 12, 15, 18, 21) & (6, 8, 10, 12, 14, 16, 18, 20; 3, 6, 9, 12, 14, 17, 20, 23) & (9, 11, 13, 15, 17, 19, 21, 23; 6, 9, 12, 15, 17, 20, 23, 26) \\ (10, 12, 14, 16, 18, 20, 22, 24; 9, 10, 13, 16, 18, 21, 24, 27) & (3, 5, 7, 9, 11, 13, 15, 17; 0, 3, 6, 9, 11, 14, 17, 20) & (4, 6, 8, 10, 12, 14, 16, 18; 1, 4, 7, 10, 12, 15, 18, 21) \\ (3, 5, 7, 9, 11, 13, 15, 17; 0, 3, 6, 9, 11, 14, 17, 20) & (8, 10, 12, 14, 18, 20, 22; 5, 8, 11, 14, 16, 19, 22, 25) & (7, 9, 11, 16, 15, 17, 19, 21; 4, 7, 10, 13, 15, 18, 21, 24) \end{pmatrix}.$$

Using proposed ranking, the defuzzification of the above problem is

$$\begin{pmatrix} 11 & 13 & 16 \\ 17 & 10 & 11 \\ 10 & 15 & 14 \end{pmatrix}.$$

Here Number of Rows = Number of Columns.

Therefore the Intuitionistic Fuzzy Assignment Problem is balanced.

Now, the Intuitionistic Fuzzy Assignment Problem is solved by Hungarian Method.

The assignment Schedule of the Intuitionistic Fuzzy Assignment Problem is as follows.

$$\begin{pmatrix} 0 & (0) & 2 \\ 9 & 0 & (0) \\ (0) & 5 & 3 \end{pmatrix}.$$

The Fuzzy Assignment Schedule is $M_1 \to J_2,\ M_2 \to J_3,\ M_3 \to J_1$.

The Optimum Assignment $Cost = 13 + 11 + 10 = 33$.

5. Conclusion

In this paper, we obtain the optimal solution of Intuitionistic Fuzzy Assignment problem whose cost values are taken as Symmetric Octagonal Fuzzy Numbers. The Symmetric Octagonal Intuitionistic Fuzzy Numbers are converted into crisp values using proposed ranking. The optimum assignment schedule of the Intuitionistic Fuzzy Assignment problem is obtained by Hungarian Method. In Intuitionistic Fuzzy Assignment Problems involving imprecise data, we hope that this approach will be effective.

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