Advances and Applications in Mathematical Sciences
Volume 21, Issue 2, December 2021, Pages 937-949

NUMERICAL ANALYSIS OF MAGNETIC FIELD DEPENDENT VISCOSITY AND ANISOTROPY OF POROUS MEDIUM

SURESH GOVINDAN and T. GAYATHRI

Sanskrithi School of Engineering
Department of Mathematics
Puttaparthi, A.P, India
E-mail: suresh.g@sseptp.org

SriManakulaVinayagar Engineering College
Department of Mathematics
Puducherry, India
E-mail: gayathiyagu@gmail.com

Abstract

Ferro convection induced by magnetic field dependent viscosity in an anisotropic porous medium using Darcy model is studied with computational methods. Galerkin method is applied. Linear stability analysis is carried out for both stationary and oscillatory modes. The critical magnetic Rayleigh number is computed for various values of the parameters which characterize the flow. It is found that the increase in magneto viscosity stabilizes the system through stationary mode; Numerical computations are made and illustrated graphically.

I. Introduction

Computational Fluid Dynamics (CFD) is a sophisticated computationally based design and analysis technique. The crucial elements of computational fluid dynamics are discretization, grid generation and coordinate transformation, solution of the coupled algebraic equations, turbulence modeling and visualization. The study of ferroconvection in the fluid saturating a porous medium of very large permeability assumes significance.

2020 Mathematics Subject Classification: 65-XX.
Keywords: Ferro convection; Magneto viscosity; Ferro fluids; Galerkin method; Anisotropic Porous medium; Magneto Rayleigh Number.

Received October 25, 2021; Accepted November 10, 2021

The effect of anisotropy on ferroconvective instability saturating a porous medium of very large permeability has been analysed by Sekar et al. [1]. But the normal ferrofluid will have large concentration of ferromagnetic particle and hence it is advisable to consider the effect of anisotropic character due to anisotropic distribution of particles. Neild [2] investigated the convection in a porous medium with inclined temperature gradient of finite magnitude and confined between perfectly conducting planes using linear stability analysis. The modeling the effects of magnetic field or rotation on flow in a porous medium; momentum equation and anisotropic permeability analogy has been discussed as a technical note by Nield [3].

In the present investigation [4].an attempt is made to find the effect of magnetic field dependent viscosity on the onset of ferroconvetion in an anisotropic densely packed porous medium. The distribution is assumed to be isotropic along the horizontal plane and anisotropic along the vertical plane. The anisotropy comes from the porous medium. Hence the velocity is proportional to the tensor permeability and one can assume that the tensor is diagonal. The finite element method (FEM) [5] is used for finding approximate solution of partial differential equations (PDE) as well as of integral equations such as the heat transport equation. Galerkin method as employed in getting the numerical solutions. Numerical computations are made and are illustrated graphically also.

II. Mathematical Formulation

Consider an infinitely spread layer of Boussinesqferrofluid of thickness ‘d’, in the presence of transverse applied magnetic field, heated from below. The temperature at the bottom surface $z = \frac{-d}{2}$ and at the upper surface $z = \frac{d}{2}$ are $T_0 + \left(\frac{\Delta T}{2}\right)$ and $T_0 - \left(\frac{\Delta T}{2}\right)$ respectively. Further the system is assumed to be an anisotropic densely packed porous medium with anisotropy along the vertical direction which is taken as the z-axis. The fluid is assumed to be incompressible fluid having a variable viscosity given by

$$\mu = \mu_1(1 + \delta \cdot B) \quad (1)$$

The variation coefficient of magnetic field dependent viscosity δ has been taken to be isotropic, $\delta_1 = \delta_2 = \delta_3$. As a first approximation for small field variation, linear variation of magneto viscosity has been used.

The governing mathematical equations used are as follows:

The continuity equation is

$$\nabla \cdot q = 0 \tag{2}$$

The momentum equation for an incompressible ferromagnetic fluid with variable viscosity μ is

$$\rho_0 \frac{Dq}{Dt} = -\nabla p + \rho g + \nabla(H \cdot B) - \frac{\mu}{k} q \tag{3}$$

The temperature equation for an incompressible fluid which obeys the modified Fourier's law as given by Finlayson [8] is

$$\left[\rho_0 C_{V,H} + \mu_0 H \left[\frac{\partial M}{\partial T}\right]_{V,H}\right] \frac{dT}{dt} + \mu_0 T \left[\frac{\partial M}{\partial T}\right]_{V,H} \cdot \frac{dH}{dt} = K_1 \nabla^2 T + \phi \tag{4}$$

The density equation of state for Boussinesq magnetic fluid is

$$\rho = \rho_0[1 - \alpha(T - T_\alpha)] \tag{5}$$

The magnetization depends on the magnitude the magnetic field and temperature which can be written as

$$M = \frac{H}{H} M(H, T) \tag{6}$$

In order to evaluate the partial derivatives of magnetization M, the linearised magnetic equation of state, as followed by Finlayson [8] is

$$M = M_0 + \chi(H - H_0) - K_2(T - T_a) \tag{7}$$

The Maxwell's equation for non-conducting fluids is

$$\nabla \times H = 0 \tag{8}$$

$$\nabla \cdot B = 0 \tag{9}$$

The magnetic field and magnetic induction are related by

$$B = \mu_0(M + H) \tag{10}$$

Basic state is assumed to be quiescent. A small perturbation has been imparted on all the dynamical variables and linear theory is used. Modified Navier Stokes equations on linearization:

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu_0(M_0 + H_0)\frac{\partial H_1^1}{\partial z} - \frac{\mu_1}{k_1} u \tag{11}$$

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial y} + \mu_0(M_0 + H_0)\frac{\partial H_2^1}{\partial z} - \frac{\mu_1}{k_1} v \tag{12}$$

$$\rho_0 \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \rho_0 g\alpha T^1 - \mu_0 K_2 \beta H_3^1 + \frac{\mu_0 k_2^2 \beta T^1}{(1+\chi)} + \mu_0(M_0 + H_0)\frac{\partial H_3^1}{\partial z}$$

$$- \mu_0(M_0 + H_0)\frac{K_2\beta}{(1+\chi)} - \frac{\mu_1}{k_2} w - \frac{\mu_1}{k_2}\delta\mu_0(M_0 + H_0)w \tag{13}$$

Further analysis and techniques has been carried out as followed by Vaidyanathan et al. [6]. Which leads to the following vertical component of momentum equation:

$$\rho_0\left(\frac{\partial \nabla^2 w}{\partial t}\right) = \frac{\mu_1}{k_1}\frac{\partial^2 w}{\partial z^2} + \rho_0 g\alpha \nabla_1^2 T^1 - \mu_0 K_2 \beta \frac{\partial}{\partial z}(\nabla_1^2 \phi^1) + \frac{\mu_0 K_2^2 \beta}{(1+\chi)}\nabla_1^2 T^1$$

$$- \frac{\mu_1}{k_2}\nabla_1^2 w - \frac{\mu_1}{k_2}\delta\mu_0(M_0 + H_0)\nabla_1^2 w \tag{14}$$

III Normal mode Analysis

The normal mode solutions of all dynamical variables can be written as

$$f(x, y, z, t) = f(z, t)\exp\{i(k_x x + k_y y)\} \tag{15}$$

$$\rho_0 \frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial z^2} - k^2\right)w = -\frac{\mu_1}{k_1}\frac{\partial^2 w}{\partial z^2} - \rho_0 g\alpha k^2\theta + \frac{\mu_0 K_2\beta}{(1+\chi)}\left[(1+\chi)\frac{\partial \phi}{\partial z} - K_2\theta\right]k^2$$

$$+ \frac{\mu_1}{k_2}k^2 w + \frac{\mu_1}{k_2}\delta\mu_0(M_0 + H_0)k^2 w \tag{16}$$

Equation (4) is linearised and the resulting equation gives upon using $H^1 = \nabla\phi^1$

$$\rho c \frac{\partial \theta}{\partial t} - \mu_0 T_0 K_2 \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z}\right) = K_1^2\left[\frac{\partial^2}{\partial z^2}\right]0 + \left(\rho c \beta - \frac{\mu_0 T_0 K_2^2 \beta}{(1+\chi)}\right)w \tag{17}$$

where,

$$\rho c = \rho C_{V \cdot H} + \mu_0 K_2 H_0. \tag{18}$$

On simplification,

$$(1+\chi)\frac{\partial^2 \theta}{\partial z^2}\left(1 + \frac{M_0}{H_0}\right)k^2 \theta - K_2 \frac{\partial \theta}{\partial z} = 0. \tag{19}$$

Using appropriate non-dimensional terms, we get

$$\left(\frac{\partial}{\partial t^*}\right)(D^2 - a^2)w^* = aR^{\frac{1}{2}}[M_1 D\phi^* - (1+M_1)T^*]$$

$$+ \left(\frac{-D^2}{k_1^*} + \frac{a^2}{k_2^*} + \frac{\delta^* M_3 a^2}{k_2^*}\right)w^* \tag{20}$$

$$P_r \frac{\partial T^*}{\partial t^*} - P_r M_2 \frac{\partial}{\partial t^*}(D\phi^*) = (D^2 - a^2)T^* + (1 - M_2)aR^{\frac{1}{2}}w^* \tag{21}$$

$$D^2\phi^* - a^2 M_3 \phi^* - DT^* = 0, \tag{22}$$

where the following non-dimensional parameters are used

$$k_1^* = \frac{k_1}{d^2},\ k_2^* = \frac{k_2}{d^2},\ M_1 = \left(\frac{\mu_0 K_2^2 \beta}{(1+\chi)\alpha \rho_0 g}\right) M_2 = \left(\frac{\mu_0 T_0 K_2^2}{\rho_0 c(1+\chi)}\right)$$

$$M_3 = \left(\frac{1 + \dfrac{M_0}{H_0}}{(1+\chi)}\right),\ P_r = \frac{\mu c}{K_1}, \tag{23}$$

IV. Numerical Solution Using Galerkin Method

The boundary conditions for stress free boundaries are

$$w^* = D^2 w^* = T^* = D\phi^* = 0$$

$$\text{at } z = \frac{-1}{2} \text{ and } z = \frac{1}{2} \tag{24}$$

Using Galerkin technique the power series expansion for the variables are taken as

$$w^*(z, t) = Aw_1(z)e^{i\sigma t}$$

$$T^*(z, t) = BT_1(z)e^{i\sigma t}$$

$$\phi^*(z, t) = C\phi_1(z)e^{i\sigma t}$$

Using these, we get,

$$\left[\left(\sigma + \frac{1}{k_1}\right)D^2 w_1(z) - \sigma a^2(z) - \left(\frac{a^2 + \delta M_3 a^2}{k_2}\right)w_1(z)\right]A$$

$$+ aR^{\frac{1}{2}}(1 + M_1)T_1(z)B - aR^{\frac{1}{2}}M_1 D\phi_1(z)C = 0 \tag{25}$$

$$\left[-(1 - M_1)aR^{\frac{1}{2}}w_1(z)\right]^{A+[(P_r\sigma+a^2)T_1(z)-D^2T_1(z)]B-[P_rM_2\sigma D\phi_1(z)]C=0} \tag{26}$$

$$D^2\phi_1(z)C - a^2 M_3\phi_1(z)C - DT_1(z)B = 0 \tag{27}$$

Taking

$$w_1(z) = \frac{z^4}{2} - \frac{3z^2}{4} + \frac{5}{32}$$

$$T_1(z) = z^4 + z^2 - \frac{5}{16}$$

$$\phi_1(z) = \frac{z^3}{3} - \frac{z}{4}$$

so as to satisfy the boundary conditions

We get the equations

$$\left(\sigma + \frac{1}{k_1}\right)(-0.1214285) - (0.01230158)\left[\sigma a^2 + \left(\frac{a^2 + \delta M_3 a^2}{k_2}\right)\right]A$$

$$+ [aR^{\frac{1}{2}}(1 + M_1)(-0.0261367)]B - [aR^{\frac{1}{2}} M_1(-0.02023809)]C = 0 \tag{28}$$

$$[-(1 - M_1)aR^{\frac{1}{2}}(-0.025992)]A + \lfloor(P_r\sigma + a^2)(0.05515873) - (-0.569047619)\rfloor$$

$$B - [P_r.M_2\sigma(0.04285714)]C = 0 \tag{29}$$

$$(0.042857)B + \lfloor(-0.03333) + a^2 M_3(0.003373)\rfloor C = 0 \tag{30}$$

For the existence of non-trivial solutions for the above equations the determinant of the coefficients of A, B, and C in "(28)", "(29)", "(30)" is equated to 0.

Stationary stability is analysed using the expression

$$R = \frac{x_6 x_7 [x_1 + x_2]}{x_3 x_5 x_7 + (0.042857) x_4 x_5}.$$

Where

$$x_1 = \frac{(0.121428)}{k_1};\ x_2 = \frac{(0.01230158)a^2(1 + \delta M_3)}{\varepsilon k_1}$$

$$x_3 = a(1 + M_1)(0.0261367),$$

$$x_4 = aM_1(0.02023809)$$

$$x_5 = (1 - M_2)a(0.025992)$$

$$x_6 = a^2(0.0551587) + 0.569047$$

$$x_7 = -\lfloor 0.03333 + a^2 M_3(0.003373)\rfloor.$$

V. Results and Discussions

The effect of magnetic field dependent viscosity on ferroconvection in an anisotropic densely packed porous medium has been analysed using Darcy model. The permeability value of the porous medium has been taken using the values proposed by Walker and Homsy [7]. The magnetization parameter M_1 has been taken to be 1000 [9]. For these fluids M_2 is assumed to have negligible value and hence taken to be zero. The magnetization parameter M_3 is varied from 1 to 10. The effect of anisotropy is studied by taking the anisotropic parameter ε, which is the ratio of vertical to horizontal permeability and is varied form 1 to 70 [8]. The permeability of the porous medium is varied from 0.0001 to 0.001, the coefficient of magnetic field dependent viscosity δ has been analysed form 0.01 to 0.05 [6].

The Critical magnetic thermal Rayleigh number N_c is obtained for different values of permeability, anisotropic parameter ε, coefficient of magnetic field dependent viscosity parameter δ, and the magnetization parameter M_3. From the Figure 1 and 2, one can find that oscillatory instability is not possible for a densely packed anisotropic porous medium having a variable viscosity. One can also observe from the figures as the coefficient of magnetic field dependent viscosity δ is increased, the critical magnetic thermal Rayleigh number N_c also increases, this would imply that the magnetic field dependent viscosity stabilizes through viscosity variations with respect to magnetic field.

It is clear from the Figures 1 and 2 that, as the anisotropy parameter ε increases, the critical magnetic thermal Rayleigh number N_c is found to decrease. This indicates that, the system destabilizes. Similar results were also found for different values of the permeability parameter.

From the above discussion and analysis one can conclude that the variable viscosity tends to stabilizes the system, when compared with the constant viscosity system. The increase in magnetization tends to destabilize the system. The presence of anisotropic densely packed porous medium destabilizes the system. On comparison with theoretical results obtained by the author in an earlier paper [4], the present computational results are found to be fully in agreement to the possible extent of accuracy.

Table 1. Marginal stability of magnetic field dependent viscosity of Ferro fluid saturating a densely packed porous medium destabilize by stationary mode having $M_1 = 1000, M_2 = 0$.

k_1	ε	δ	M_3	(a_c)	$N_c(RM_1)_c$
0.0001	10	0.01	1	5.17	237643
			3	5.17	195968
			5	5.17	188211
			7	5.17	185308
		0.03	1	5.17	238653
			3	5.17	198458
			5	5.17	192179
			7	5.17	190755
		0.05	1	5.17	239664
			3	5.17	200947
			5	5.17	196148
			7	5.17	196202
	30	0.01	1	5.17	203619
			3	5.17	167477
			5	5.17	160434
			7	5.17	157555
		0.03	1	5.17	203956
			1	5.17	203956
			3	5.17	168307
			5	5.17	161757
		0.05	7	5.17	159371
			1	5.17	204293
			3	5.17	169136

			5	5.17	163079

Table 2. Marginal stability of magnetic field dependent viscosity of Ferro fluid saturating a densely packed porous medium destabilize by stationary mode having $M_1 = 1000$, $M_2 = 0$.

k_1	ε	δ	M_3	(a_c)	$N_c(RM_1)_c$
0.0001	10	0.01	1	5.17	237643
			3	5.17	195968
			5	5.17	188211
			7	5.17	185308
		0.03	1	5.17	238653
			3	5.17	198458
			5	5.17	192179
			7	5.17	190755
		0.05	1	5.17	239664
			3	5.17	200947
			5	5.17	196148
			7	5.17	196202
	30	0.01	1	5.17	203619
			3	5.17	167477
			5	5.17	160434
			7	5.17	157555
		0.03	1	5.17	203956
			1	5.17	203956
			3	5.17	168307
			5	5.17	161757
			7	5.17	159371

			1	5.17	204293
			3	5.17	169136
			5	5.17	163079

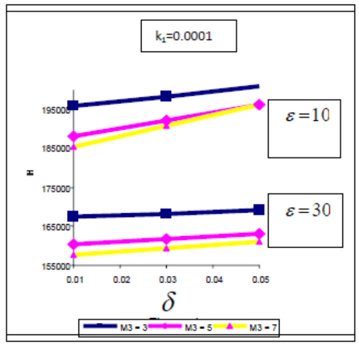

Figure 1.

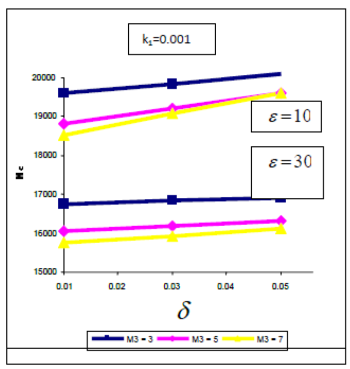

Figure 2.

where

k_1 - Brinkman Number

ε- Anisotropy parameter

δ- Coefficient of Magnetic field dependent viscosity

M_3 - Magnetization

a_c - Critical value Number

N_c - Thermal Rayleigh Number

References

[1] R. Sekar, G. Vaidyanathan and A. Ramanathan, Ferro convection in an anisotropic porous medium, Int. J. Engng. Sci. 34(4) (1996), 399 -405.

[2] D. A. Nield, Convection in a porous medium with inclined temperature gradient, Int J. Heat Transfer, (1991).

[3] D. A. Neild, Convection in porous media, Int. J. Heat Transfer 42 (1999), 37-15.

[4] A. Ramanathan and G. Suresh, Effect of magnetic field dependent viscosity and anisotropy of porous medium on ferroconvection, Int. J. of Engng. Sci. 42 (2004), 411-425.

[5] J. Donea and A. Huerta, Finite Element Methods for Flow Problems, Wiley, (2005).

[6] G. Vaidyanathan, R. Sekar and A. Ramanathan, Ferro thermohaline convection in a porous medium, J. Magn. Magn. Mater. 149 (1995), 137 -142.

[7] K. Walker and G. M. Hosmy, A note on convective instabilities in Boussinesq fluids and porous media, ASME J. Heat Transfer 99 (1977), 338.

[8] A. K. Goel and S. C. Agrawal, A numerical study of hydromagnetic thermal convection in a visco-elastic dusty fluid in a porous medium, Indian J. Pure Appl. Math. 29(9) (1998), 929-940.

[9] B. A. Finlayson, Convection instability of ferromagnetic fluids, J. Fluid Mech. 40(4) (1970), 753.

[10] R. E. Rosensweig and Ferrodynamics, Cambridge University Press, Cambridge, (1985).

[11] R. E. Rosensweig, G. Miskolczy and F. D. Ezekiel, Magnetic fluid seals, machine design 40 (1968), 145.

[12] R. Moskowitz, Dynamic sealing with magnetic fluids ASLE Trans. 18(2) (1975), 135.

[13] R. E. Rosensweig, Nature (London) 210 (1966), 613-614

[14] E. L, Jr. Resler and R. E. Rosensweig, Magnetocalonic power, AIAA J., 2(8) (1964), 14-18.

[15] M.I. Shliomis, Effective viscosity of magnetic suspensions, Magn. Fluids. Soviet Phys, Uspekhi (Engl. Transl) 17(2) (1974), 153.

[16] D. P. Lalas and S. Carmi, Thermoconvective stability of ferro fluids, Phys. Fluids 14(2) (1971), 436.

[17] G. Vaidyanathan, R. Sekar and R. Balasubramanian, Ferroconvective instability of fluids saturating a porous medium, Int. J. Eng. Sci. 29(10) (1991), 1259.

[18] R. Sekar, G. Vaidyanathan and A. Ramanathan, The ferroconvection in fluids saturating a rotating densely packed porous medium, Int. J. Eng. Sci. 31(2) (1993), 241.

[19] M. I. Shliomis, Effective viscosity of magnetic suspensions, Sov. Phys. JETP 34(6) (1972), 1291.

[20] M. I. Shliomis, Yu. L. Raiker, IEEE. Trans. Magn. MAG. 16(2) (1980), 237.

[21] R. C. Sharma, Sunil and S. Chand, Thermosolutal instability of Rivin-Erickson Rotating fluid in porous medium, Indian J. Pure Appl. Math., 29(4) (1998), 433-440.

[22] M. S. Malashety and V. Padmavathi, Effect of gravity modulation on the onset of convection in a fluid and porous layer, Int. J. Engng. Sci. 35(9) (1997), 829-840.

[23] J. Donea, T. Belytschko and P. Smolinski, A generalized Galerkin method for steady state convection-diffusion problems with application to quadratic shape function, Comp. Meth Appl. Mech. Eng. 48 (1985), 25-43.

[24] T. J. R. Hughes, L. P. Franca, G. M. Hulbert, Z. Johan and F. Sakhib, The Galerkin least square method for advective diffusion equations, in Recent Developments in Computational Fluid Mechanics (edsTezduyar T. E. and Hughes T.J.R), AMD95, ASME, (1998).

[25] R. Sekar and G. Vaidyanathan, Int. J. Engng. Sci. 31 (1991), 1139.