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A METHOD FOR SOLVING BOTTLENECK-COST TRANSPORTATION PROBLEM USING FUZZY OPTIMIZATION TRAPEZOIDAL FUZZY NUMBERS WITH λ-CUT AND RANKING METHOD

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Abstract

In this research, we offer a new algorithm that uses the λ-cut and Ranking Technique to find the best solution to the Bottleneck Transportation Problem using Trapezoidal Fuzzy Number. This approach is used to determine all possible Bottleneck-Cost Transportation Problem (BCTP) solutions. This strategy is illustrated using a numerical example.

1. Introduction

The Transportation Problem is one of the oldest uses of linear programming, and it is also the most successful application of linear programming to solving business problems. Transportation models have a

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wide range of applications in logistics and supply chains for lowering costs. When the price coefficient and deliver and call for portions are acknowledged precisely, an green technique has been created to remedy the transportation problem. The Time-Minimizing Problem (or Bottleneck Transportation Problem) is a first-rate example of transportation in which era is depending on every transportation path. Instead of cutting costs, the goal is to keep the most expensive costs to a minimum by shifting all origins to the destination. Bottleneck transportation reduces the time it takes to move objects from sources to requirements while still meeting specific criteria, such as being accessible at the source and needed at the destination.

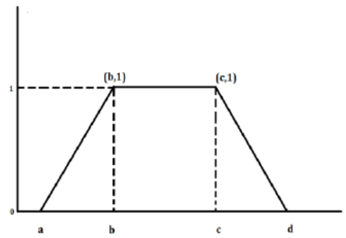
The problem with fuzzy shipping is related to the fact that shipping costs, supply and demand are very vague. Zadeh introduced fuzzy set theory and successfully applied it in a variety of fields to manage decision making. Following Bellman and Zadeh's pioneering work, fuzzy set theory's applications in optimization domains are fast expanding. Several academics have developed a variety of strategies for solving transportation problems in diverse fuzzy settings. Several authors have looked into bottleneck fuzzy transportation. Using the blocking strategy, Akilbasha et al. found the best solution to the fuzzy bottleneck transportation problem.

Minimax technique was introduced by Swati Agarwal and Shambu Sharma to identify an initial basic workable solution to a time-minimizing transportation problem with mixed constraints. Using Pseudo cost, Bhabani Mallia et al. found an early solution to a bottleneck transportation problem. Abirami introduced a signed distance ranking function to determine the best solution to the time-minimizing fuzzy transport problem. Pandian and Natarajan used the fixed-zero method to solve the transportation cost bottleneck and found all efficient solutions. As a result, many authors have investigated the bottleneck fuzzy transfer problem and found solutions. For the most part, they all used the blocking method and the blocking zero approach to arrive at the best and most efficient solution to the problem. We use a time matrix to solve a bottleneck transportation problem utilizing fuzzy trapezoidal numbers and a ranking technique in this study.

2. Preliminaries

$$a_j(n) = (j-1)!\,(-1)^{n-j+1} \sum_{k_{j-1}=0}^{n-j+1} \sum_{k_{j-2}=0}^{n-j+1-k_{j-1}} \cdots$$

Definition 1. A fuzzy number $\overline{A} = (a, b, c, d)$, $a < b < c < d$. Trapezoidal fuzzy number. Its membership function is



$$\mu_{\overline{A}}(x) = \begin{cases} \frac{p-a}{b-a}, & \text{if } a \le p \le b \\ 1, & \text{if } b \le p \le c \\ \frac{d-p}{d-c} & \text{if } c \le p \le d \\ 0, & \text{otherwise} \end{cases}$$

Definition 2. The λ-cut of fuzzy trapezoidal number $\overline{A} = (a, b, c, d)$ is defined by

$$A_\lambda = \{[(b-a)\lambda + a], [d - (d-c)\lambda]\},\ \lambda \in [0, 1]$$

Our main results can be stated as the following theorem.

2. Trapezoidal Fuzzy Number Ranking Approach

The classification technique responds to the compensatory linear and additive properties and produces results that are consistent with human intuition. If â is a fuzzy number, then the rank is determined by

$$R(\hat{a}) = \int_0^1 (0.5)(a_\lambda^L,\ a_\lambda^R)\,d\lambda$$

Where, $(a_\lambda^L,\ a_\lambda^R)$ is the λ level cut of the fuzzy number

$$(a_\lambda^L, a_\lambda^R) = \{[(b-a)\lambda + a], [d-(d-c)\lambda]\}.$$

Mathematical Formulation

Let the following Bottleneck-Cost Transportation Problem as

$$Min\, \mathcal{C} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} p_{ij}$$

$$Min\, \mathcal{T} = \left\{ \begin{matrix} \max imize \\ (i,\ j) \end{matrix} t_{ij} \mid p_{ij} > 0 \right\}$$

Subject to

$$\sum_{j=1}^{n} p_{ij} = a_i,\ i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{m} p_{ij} = b_i,\ i = 1, 2, \ldots, n$$

$$\text{and } p_{ij} \geq 0, \text{ for all } i \text{ and } j$$

where,

m = number of supply points

n = number of demand points

p_{ij} = number of units sent from supply point i to demand point j

t_{ij} = time of shipping goods from supply point i to demand point j

a_i = supply at supply point i

b_j = demand at demand point j

In fuzzy bottleneck-cost transportation problem mode of the transportation is faster (or) slower results in higher (or) lower time of transportation but with a low (or) high transportation cost. Therefore, it states that the commission of a target is affected by the bottleneck. For a sequence of different period in a specified time interval, one can have a sequence of low-cost transportation schedules. This kind of analysis provides the opportunity to the decision makers to select an appropriate transportation plan and the extent of Bottleneck that they can afford.

Definition 3. A point $(\mathcal{P}, \mathcal{T})$ where $\mathcal{P} = \{p_{ij}, i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n\}$ and $\mathcal{T}$ is a time is said to be a feasible solution of problem if $\mathcal{P}$ satisfies the constraints.

Definition 4. Executable points $(\mathcal{P}_0, \mathcal{T}_0)$ are said to be efficient for the problem if there are no other executable points $(\mathcal{P}, \mathcal{T})$ for such a problem.

$\mathcal{Z}_1(\mathcal{P}) \leq \mathcal{Z}_1(\mathcal{P}_0)$ and $\mathcal{Z}_2(\mathcal{T}) < \mathcal{Z}_2(\mathcal{T}_0)$

(or)

$\mathcal{Z}_1(\mathcal{P}) \leq \mathcal{Z}_1(\mathcal{P}_0)$ and $\mathcal{Z}_2(\mathcal{T}) \leq \mathcal{Z}_2(\mathcal{T}_0)$

Definition 5. If the minimum transit time associated with the cost shipping issue is $\mathcal{M}$, then the shipping cost issue of the shipping bottleneck is said to be in effect at any time $\mathcal{M}$.

Step 1. Represent all fuzzy costs and fuzzy time in terms of Trapezoidal fuzzy numbers.

Step 2. Construct the time transportation problem using Ranking technique and solve by any one of the existing methods such that $\bar{\mathcal{T}} = \mathcal{T}' = Max\{t_{ij} \mid x_{ij} > 0\}$.

Step 3. The time matrix,

$$\bar{t} = (t_{ij}) = \begin{cases} \mathcal{M}, \ if \ t_{ij} > \mathcal{T}' \\ 1, \ if \ t_{ij} = \mathcal{T}' \\ 0, \ if \ t_{ij} < \mathcal{T}' \end{cases}$$

Step 4. Using the preceding matrix, solve the new transportation problem with time coefficient t_{ij} by using any of the current approaches.

Step 5. Determine the time $\bar{\mathcal{T}} = \sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} p_{ij}$. If $\mathcal{T} = 0$ repeat the step-2 to 5. If $\mathcal{T} \neq 0$ then current transportation plan is optimal and it is $\mathcal{T}_0$.

Step 6. Formulate the cost transportation table from the given fuzzy bottleneck cost transportation problem using ranking technique.

Step 7. Solve it by any one of the existing methods and determine corresponding time transportation. Let it be $\mathcal{T}_N$.

Step 8. Construct the active transportation problem for each time T in $[\mathcal{T}_0, \mathcal{T}_N]$. For each instantaneous T, an optimal solution to the transport cost problem $\mathcal{P}$ is determined from the previous steps. Then the pair $(\mathcal{P}, N)$ is an effective solution to the bottleneck fuzzy cost problem.

Step 9. For each time T in $[\mathcal{T}_0, \mathcal{T}_N]$ compute $\lambda = \dfrac{\mathcal{T}_N - T}{\mathcal{T}_N - \mathcal{T}_0}$ which is the level of time satisfaction for the time $\mathcal{P}$.

1. Numerical Example

The fuzzy bottleneck transport cost table given in Table 1 has the upper coefficient in each cell representing the fuzzy transit time along the respective path and the lower coefficient representing the unit fuzzy transport cost per cell on the road.

Table-1

Sources	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	(8,10,12,18) (4,5,6,7)	(45,55,69,107) (4,6,8,14)	(51,68,75,106) (9,10,11,14)	(35,49,53,75) (9,11,13,19)	(2,6,10,22)
S_2	(36,42,67,123) (4,6,8,14)	(78,86,97,127) (6,7,8,11)	(18,25,31,50) (10,12,14,20)	(6,15,23,48) (13,14,15,18)	(10,16,21,37)
S_3	(87,95,99,115) (13,14,15,18)	(46,57,64,89) (9,11,13,19)	(14,18,21,31) (8,9,10,13)	(12,18,24,42) (5,7,9,15)	(11,17,19,29)
Deman d	(8,10,12,18)	(1,3,4,8)	(8,14,16,26)	(4,14,18,36)	

Now, let built the time transportation problem of BCTP.

Table-2

Sources	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	(8,10,12,18)	(45,55,69,107)	(51,68,75,106)	(35,49,53,75)	(2,6,10,22)

S_2	(36,42,67,123)	(78,86,97,127)	(18,25,31,50)	(6,15,23,48)	(10,16,21,37)
S_3	(87,95,99,115)	(46,57,64,89)	(14,18,21,31)	(12,18,24,42)	(11,17,19,29)
Demand	(8,10,12,18)	(1,3,4,8)	(8,14,16,26)	(4,14,18,36)	

Using ranking method, we have to convert fuzzy trapezoidal numbers into crisp value and verify the transportation table is balanced.

Table-3

Sources	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	10 12	69	75	53	10
S_2	2 67	4 97	15 31	23	21
S_3	99	64	1 21	18 24	19
Demand	12	4	16	18	

The required time $\mathcal{T}'$ is

$$\mathcal{T}_1 = Max\{12, 67, 97, 31, 21, 24\}$$

$$\mathcal{T}_1 = 97$$

Time matrix,

$$t_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \mathcal{M} & 0 & 0 & 0 \end{bmatrix}$$

Table-4

Sources	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	10 12	69	75	53	10
S_2	2 67	97	16 31	3 23	21
S_3	99	4 64	21	15 24	19
Demand	12	4	16	18	

required time $\mathcal{T}'$ is

$$\mathcal{T}_2 = Max\{12, 67, 31, 23, 64, 24\}$$

$$\mathcal{T}_2 = 67$$

Corresponding time matrix,

$$t_2 = \begin{bmatrix} 1 & \mathcal{M} & \mathcal{M} & 0 \\ 1 & \mathcal{M} & 0 & 0 \\ \mathcal{M} & 0 & 0 & 0 \end{bmatrix}$$

Table-5

Sources	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	10 12	69	75	53	10
S_2	2 67	97	16 31	3 23	21
S_3	99	4 64	21	15 24	19
Demand	12	4	16	18	

The required time $\mathcal{T}'$ is

$$\mathcal{T}_3 = Max\{12, 67, 31, 23, 64, 24\}$$

$$\mathcal{T}_3 = 67$$

Hence $\mathcal{T}_0 = 67$ by step-5 is special solution of the time transportation problem.

Let construct fuzzy cost transportation table of BCTP.

Table-6

Sources	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	(4,5,6,7)	(4,6,8,14)	(9,10,11,14)	(9,11,13,19)	(2,6,10,22)
S_2	(4,6,8,14)	(6,7,8,11)	(10,12,14,20)	(13,14,15,18)	(10,16,21,37)

S_3	(13,14,15,18)	(9,11,13,19)	(8,9,10,13)	(5,7,9,15)	(11,17,19,29)
Demand	(8,10,12,18)	(1,3,4,8)	(8,14,16,26)	(4,14,18,36)	

Using Ranking Method, we have to convert fuzzy trapezoidal number into crisp value and verify the transportation table is balanced.

Table-7

Sources	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	6	8	11	13	10
S_2	8	8	14	15	21
S_3	15	13	10	9	19
Demand	12	4	16	18	

The special solution is

$$p_{13} = 10,\ p_{21} = 12,\ p_{22} = 4,\ p_{23} = 5,\ p_{33} = 1,\ p_{34} = 18$$

Transportation Cost= Rs.480 and Transportation Time = 97

Now we have, $\mathfrak{T}_0 = 67$ and $\mathfrak{T}_N = 97$

$$N = \{67, 69, 75, 97\}$$

Now, take $N = 67$ using algorithm we can solve Cost Transportation Problem.

Table-8

Sources	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	6	-	-	13	10
S_2	8	-	14	15	21
S_3	-	13	10	9	19

Demand	12	4	16	18

The special solution is

$$p_{11} = 10,\ p_{21} = 2,\ p_{23} = 14,\ p_{24} = 3,\ p_{32} = 4,\ p_{34} = 15$$

Transportation Cost = Rs.532

Taking $N = 69,$ then using algorithm

Table-9

Sources	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	6	8	-	13	10
S_2	8	-	14	15	21
S_3	-	13	10	9	19
Demand	12	4	16	18	

The special solution is

$$p_{11} = 6,\ p_{21} = 4,\ p_{21} = 6,\ p_{23} = 15,\ p_{33} = 1,\ p_{34} = 18i$$

Transportation Cost = Rs.498

Now take $N = 75$ using algorithm we get

Table-10

Sources	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	6	8	11	13	10
S_2	8	-	14	15	21
S_3	-	13	10	9	19
Demand	12	4	16	18	

The special solution is

$$p_{12} = 4,\ p_{13} = 6,\ p_{21} = 12,\ p_{23} = 9,\ p_{33} = 1,\ p_{34} = 18$$

Transportation Cost = Rs.492

Therefore, the special solution of the fuzzy Bottleneck-Cost Transportation Problem.

Table-11

S. No	Fuzzy Bottleneck-Cost Transportation Problem Special Solutions	Objective value of BCTP	Satisfaction λ level
1	$p_{11} = 10,\ p_{21} = 2,\ p_{23} = 14,\ p_{24} = 3,\ p_{32} = 4,\ p_{34} = 15$ at time 67	(532,67)	1
2	$p_{11} = 6,\ p_{12} = 4,\ p_{21} = 6,\ p_{23} = 15,\ p_{33} = 1,\ p_{34} = 18$ at time 69	(498,69)	$\frac{27}{29} \approx 0.93$
3	$p_{12} = 4,\ p_{13} = 6,\ p_{21} = 12,\ p_{23} = 9,\ p_{33} = 1,\ p_{34} = 18$ at time 75	(492,75)	$\frac{22}{29} \approx 0.76$
4	$p_{13} = 10,\ p_{21} = 12,\ p_{22} = 4,\ p_{23} = 5,\ p_{33} = 1,\ p_{34} = 18$ at time 97	(480,97)	0

Conclusion

In this paper we have considered a fuzzy bottleneck transportation problem containing Trapezoidal fuzzy numbers and solve it using ranking technique by converting into crisp equivalent problem then by Bottleneck-Cost Transportation Problem algorithm we get minimum transportation cost and time.

References

[1] Bhabani Mallia, Manjula Das and Chakradhar Das, A new algorithm for Bottleneck Transportation Problem, International Journal of Innovative Technology and Exploring Engineering, June 2019.

[2] V. E. Sobana and D. Anuradha, On Solving Bottleneck Bi-criteria Fuzzy Transportation Problems, International Journal of Engineering and Technology, 2018

[3] Dr. Kirtiwant, P. Ghadle and Dhanashri A. Munot, A Freezing method for solving bottleneck-cost transportation problem, Turkish Journal of Computer and Mathematics education, 2021.

[4] P. Pandian and G. Natarajan, A new method for solving Bottleneck-Cost Transportation Problems, International Mathematical forum, 2011.

[5] V. Vidhya and K. Ganesan, A simple method for the solution of bottleneck-cost transportation problem under fuzzy environment, AIP Conference Proceedings, 2020.

[6] Shugani Poonam, S. H. Abbas and V.K. Gupta, Fuzzy Transportation Problem of Trapezoidal Numbers with λ-cut and ranking technique, International Journal of fuzzy Mathematics and Systems 2(3) (2012), 263-267.

[7] S. Kayalvizhi, Gunasekar and Thenmozhi, Evaluation on Aggregation risk rate for defuzzification in fuzzy sets, IJRDO-Journal of Computer Science and Engineering, 2016.

[8] De Dipankar, A method for solving fuzzy transportation problem of trapezoidal number, 2010.

[9] R. Srinivasan, N. Karthikeyan and A. Jayaraja, A proposed technique to resolve transportation problem by trapezoidal fuzzy numbers, Indian Journal of Science and Technology, 2021.