Advances and Applications in Mathematical Sciences
Volume 21, Issue 2, December 2021, Pages 763-771

SOLVING INTUITIONISTIC FUZZY TRANSPORTATION PROBLEM USING INTUITIONISTIC HEXAGONAL FUZZY NUMBERS

THANGARAJ BEAULA[1], ANGELINE SARGUNA GIFTA[2] and ERAIANBU[3]

[1]Associate Professor and Head
[2]Assistant Professor
[3]M. Phil. Research Scholar

T. B. M. L College (Affiliated to
Bharathidasan University) Porayar, India

Email: edwinbeaula@yahoo.co.in
anbuerai520@gmail.com

Abstract

In this paper, intuitionistic fuzzy transportation problem is solved using intuitionistic hexagonal fuzzy number. The problem is illustrated by a numerical example.

1. Introduction

The transportation problem is a unique sort of linear programming problems where the goal is to limit the expense of conveying a product starting with one source then onto the another objective. Many authors have investigated to find an appropriate solution for the fuzzy transportation problem using many techniques. Sometimes the concept of fuzzy set theory is not adequate to handle the transportation problems. So intuitionistic fuzzy set theory is developed to manipulate the transportation problems. It has prominent role in logistics and supply chain management problem with fuzzy co-efficient. Atanassov in 1986 [1-3] explored the notion of intuitionistic fuzzy sets to deal with vagueness or uncertainty. The main advantage of IFS is that include both the degree of membership and non-membership of each element in the set. Chanas and Kutcha [4] have proposed a unique method to obtain

2020 Mathematics Subject Classification: 40G15.
Keywords: I-Convergent, I-Cauchy, I-Statistical convergent, I-Statistical Cauchy.
Received May 17, 2021; Accepted June 7, 2021

the feasible solution to the transportation problem with fuzzy co-efficient. Gupta [5] has expanded an efficient method for solving intuitionistic fuzzy transportation problem of type 2.

This paper organized as follows, in section 2, basic definitions related to this work are garnered. In Section 3, proposed algorithms are explained. In section 4, a numerical example is illustrated to validate the efficiency of the proposed methods. Section 5 ends this paper with conclusions and references.

2. Preliminaries

In this section, some basic definitions which are related to this work are given.

Fuzzy set 2.1. A fuzzy set is characterized by a membership function that maps the element of universe of discourse X to $[0,1]$. A fuzzy set A in a universal set X is defined as $A = \{(x, \mu_A(x)); x \in X\}$ where $\mu_A : x \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x in X for the fuzzy set A.

Fuzzy number 2.2. A fuzzy set $\widetilde{A}$ defined on the set of real number R is said to be a fuzzy number if its satisfies the following conditions

(i) $\widetilde{A}$ is normal

(ii) $\widetilde{A}$ is convex

(iii) The support of $\widetilde{A}$ is closed and bounded.

Hexagonal Fuzzy Number 2.3. An hexagonal fuzzy number is given by 6-tuples is denoted as $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ where a_1, a_2, a_3, a_4, a_5 and a_6 are real numbers and $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6$ its membership function is given by $\mu_{\widetilde{A}}(x) = \begin{cases} \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right) & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right) & \text{for } a_2 \le x \le a_3 \\ 1 & \text{for } a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x - a_4}{a_5 - a_4}\right) & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6 - x}{a_6 - a_5}\right) & \text{for } a_5 \le x \le a_6 \\ 0 & \text{otherwise.} \end{cases}$

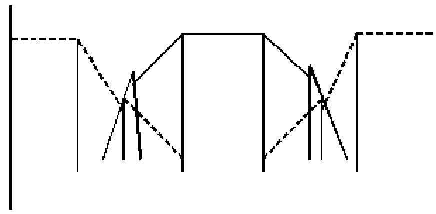
Hexagonal Intuitionistic Fuzzy Numbers 2.4. A hexagonal intuitionistic fuzzy number is defined as $\widetilde{R}_H^I = ((a_1, a_2, a_3, a_4, a_5, a_6)$ $(b_1, b_2, b_3, b_4, b_5, b_6))$ where $a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5, b_6$ are real numbers and $b_1 \le a_1 \le b_2 \le a_2 \le b_3 \le a_3 \le b_4 \le a_4 \le b_5 \le a_5 \le b_6$ $\le a_6$ its membership and non-membership functions are given by

$$\widetilde{A}(x) = \begin{cases} \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right) & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right) & \text{for } a_2 \le x \le a_3 \\ 1 & \text{for } a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x - a_4}{a_5 - a_4}\right) & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6 - x}{a_6 - a_5}\right) & \text{for } a_5 \le x \le a_6 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\widetilde{A}^I(x) = \begin{cases} 1 - \frac{1}{2}\left(\frac{x - b_1}{b_2 - b_1}\right) & \text{for } b_1 \le x \le b_2 \\ \frac{1}{2}\left(\frac{b_3 - x}{b_3 - b_2}\right) & \text{for } b_2 \le x \le b_3 \\ 0 & \text{for } b_3 \le x \le b_4 \\ \frac{1}{2}\left(\frac{x - b_4}{b_5 - b_4}\right) & \text{for } b_4 \le b_5 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - b_5}{b_6 - b_5}\right) & \text{for } b_5 \le x \le b_6 \\ 1 & \text{otherwise.} \end{cases}$$

Graphical Representation of Hexagonal Intuitionistic Fuzzy Number 2.5.



b_1 $a_1 b_2 a_2$ a_3 a_4 $a_5 a_5 a_6 a_6$ b_3 b_4

- Membership Function

– Non-Membership Function

Ranking Method 2.6. The ranking of hexagonal intuitionistic fuzzy number $R(A) = (a_1, a_2, a_3, a_4, a_5, a_6)$ $(b_1, b_2, b_3, b_4, b_5, b_6)$ which maps the set of all fuzzy numbers to set of all real numbers is given by,

$$R(A) = \frac{S(\mu_A) + S(\gamma_A)}{2}$$

where $S(\mu_A) = \dfrac{6a_1 + 8a_2 + 13a_3 + 13a_4 + 8a_5 + 6a_6}{54}$

and $S(\gamma_A) = \dfrac{6b_1 + 8b_2 + 13b_3 + 13b_4 + 8b_5 + 6b_6}{54}$.

Fuzzy Transportation Problem 2.7. A fuzzy transportation problem is a transportation problem in which the transportation expenditures, supply and demand quantities are fuzzy quantities.

3. Vogel Approximation Method (VAM)

The Vogel approximation (unit penalty) method is an iterative procedure for computing a basic feasible solution of a fuzzy transportation problem.

Algorithm

Step 1. Find the cells having smallest and next to smallest cost in each row and write the difference (called penalty) along the side of the table in row penalty.

Step 2. Find the cells having smallest and next to smallest cost in each column and write the difference (called penalty) along the side of the table.

Step 3. Select the row or column with the maximum penalty and final cell that has least cost in selected row or column. Allocate as much as possible in the cell. If there is a tie in the value of penalties then select the cell where maximum allocation can be possible.

Step 4. Adjust the supply and demand and cross out (strike out) the satisfied row or column.

Step 5. Repeat this steps until all supply and demand values are 0.

Modified Distribution Method (MODIM) 3.1.

To find the best solution for FTP using a modified method of distribution.

Algorithm of MODIM is illustrated as follows:

Step 1. Find initial basic feasible solution by proposed Vogels approximation method.

Step 2. Compute dual variables R_i and K_j and all row and column, respectively, satisfying $R_i + K_j = C_{ij}$ set $R_1 = 0$.

Step 3. Calculate the improvement index value for unoccupied cells by the equation $E_{ij} = C_{ij} - R_i - E_j$.

Step 4. Consider valued of E_{ij},

Case (i). IBFS is fuzzy optimal solution, if $E_{ij} \geq 0$ for every unoccupied cells.

Case (ii). IBFS is not fuzzy optimal solution, for at least one $E_{ij} < 0$. Go to step 5.

Step 5. Choose the unoccupied cell for the most negative value of E_{ij}.

Step 6. To construct the closed loop below.

At first start the closed loop with choose the empty cell and move vertically and horizontally with corner cells occupied and comeback to choose the empty cell to complete the loop. Use sign "+" and "–" at the corners of the closed loop, by assigning the "+" sign to the selected empty cell first.

Step 7. Look for the minimum allocation value from the cells which have "–" sign. After that, allocate this value to choose empty cell and subtract it to the other occupied cell having "–" sign and it to the other occupied cells having "+" sign.

Step 8. Allocation in step 7 will result an improved basic feasible solution (IBFS).

Step 9. Test the optimally condition for improved BFS. The process is complete when $E_{ij} \geq 0$ for all the empty cell.

4. Numerical Example

Consider a 3×3 hexagonal intuitionistic fuzzy fractional transportation problem given in the following table.

	D_1	D_2	D_3	Supply
S_1	[(4,6,7,8,12,13) (3,4,7,8,14,15)]	[(8,9,11,12,15,18) (7,8,11,12,18,20)]	[(9,11,13,15,18,22) (7,9,13,15,21,24)]	30
S_2	[(4,6,8,10,13,16) (3,5,8,10,14,18)	[(9,11,15,17,21,26) (8,10,15,17,24,29)]	[(7,9,12,15,20,26) (5,7,12,15,22,28)]	45
S_3	[(7,9,12,15,20,26) (5,8,12,15,22,28)]	[(3,4,5,6,8,10) (2,4,5,6,10,12)]	[(7,8,10,12,14,17) (6,7,10,12,17,19)]	15
Demand	25	35	30	90

Fuzzy transportation problem is balanced.

$$\sum_{i=1}^{m} R_i = \sum_{j=1}^{n} K_j = 90.$$

For table it is clear that it is a balanced transportation problem. By using the proposed algorithm the solution of the problem is given as follows.

Now applying ranking method to each row and column, we have

$$R(A) = \frac{S(\mu_A) + S(\gamma_A)}{2}$$

where

$$S(\mu_A) = \frac{6a_1 + 8a_2 + 13a_3 + 13a_4 + 8a_5 + 6a_6}{54}$$

and $S(\gamma_A) = \dfrac{6b_1 + 8b_2 + 13b_3 + 13b_4 + 8b_5 + 6b_6}{54}$

$$C_{11} = [(4, 6, 7, 8, 12, 13)\ (3, 4, 7, 8, 14, 15)]$$

$$S(\mu_A) = \frac{4(6) + 6(8) + 7(13) + 8(13) + 14(8) + 13(6)}{54}$$

$$= \frac{435}{54}$$

$$= 8.0555$$

$$S(\gamma_A) = \frac{3(6) + 4(8) + 7(13) + 8(13) + 14(8) + 15(6)}{54}$$

$$= \frac{447}{54}$$

$$= 8.2777$$

$$R(A) = \frac{S(\mu_A) + S(\gamma_A)}{2} = \frac{8.0555 + 8.2777}{2}$$

$$= 8.1666.$$

Similarly, $C_{11} = 8$, $C_{12} = 12$, $C_{13} = 15$, $C_{21} = 9$, $C_{22} = 17$, $C_{23} = 14$, $C_{31} = 14$, $C_{32} = 6$, $C_{33} = 11$.

The reduced crisp transportation table is given below.

Reduced Table.

	D_1	D_2	D_3	**Supply**
S_1	8	12	15	25
S_2	9	17	14	30
S_3	14	6	11	35
Demand	30	45	15	90

Using VAM procedure,

we get,

10 8	20 12	15	30
15 9	17	30 14	45
14	15 6	11	15
25	35	30	

$$\text{Transportation Cost } = (10 \times 8) + (20 \times 12) + (15 \times 9) + (30 \times 14) + (15 \times 6)$$
$$= 965.$$

Using MODI Method.

First we check number of allocation equal to number of column and number of row minus one.

(i.e.) $m + n - 1 =$ Number of allocation

where m is number of rows and n is number of columns.

10 8	20 12	15	R_1
15 9	17	30 14	R_2
14	15 6	11	R_3
K_1	K_2	K_3	

Take $K_1 = 0$, we get another value

$C_{ij} = R_i + K_j$.

We get $R_1 = 8$, $R_2 = 9$, $R_3 = 2$, $K_2 = 4$, $K_3 = 5$

$E_{ij} = C_{ij} - R_i - K_j$, where E_{ij} is unoccupied cells, $E_{13} = 2$, $E_{22} = 4$, $E_{31} = 12$, $E_{33} = 7$

$$E_{ij} > 0$$

(i.e.) every unoccupied cells are positive, we get IBFS.

$$\text{Transportation Cost } = (10 \times 8) + (20 \times 12) + (15 \times 9) + (30 \times 14) + (15 \times 6)$$

$$= 965.$$

5. Conclusion

The area of intuitionistic transportation problem is immense and has drastic importance, especially in our day to day life and the solution to such have same importance as well. Hexagonal intuitionistic fuzzy number plays an efficient role to solve intuitionistic fuzzy transportation problems. In the present paper hexagonal intuitionistic fuzzy number used to solve IFTP. Numerical examples are given to justify the proposed algorithms. Eventually, conclusion and references are appended.

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986), 87-96.

[2] K. T. Atanassov, Intuitionistic Fuzzy Sets, Theory and Applications, Physica-Verlag, Heidelberg, New York 20 (1999).

[3] K. T. Atanassov, Ideas for intuitionistic fuzzy equations, inequalities and optimization, Notes on Intuitionistic Fuzzy Sets 1 (1995), 17-24.

[4] S. Chanas and D. Kuchta, A concept of the optimal solution of the transportation problem with fuzzy cost co-efficients, Fuzzy Sets and Systems 82 (1996), 299-305.

[5] G. Gupta and A. Kumari, An efficient method for solving intuitionistic fuzzy transportation problem of type-2, Int. J. Appl. Comput. Math. 3 (2017), 3795-3804