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ANALYSIS OF COVID-19 SPREAD IN HIMALAYAN COUNTRIES

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Abstract

In this paper we have discussed the spread of Covid-19 in two Himalayan countries-Bhutan and Nepal, both sharing borders with India. We analyse their cases using RMsDTM through Mathematica. We have also studied relative infected cases of both the countries.

1. Introduction

Covid-19, biggest health crisis of the present century, has affected the world in many ways. It has severely affected the global economy, education sector and tourism industry. No country (as per our knowledge) in the world has been spared from the devastating effects of this disease. It is highly contagious and spreads through SARS-CoV-2 family of virus. Its first case was reported in Wuhan, China in December 2019. It spread very fast to other parts of the world so it was declared a pandemic by WHO on March 11, 2020. Different countries have deployed different measures to mitigate the disease. The purpose of this paper is to study and compare the spread of the disease in two Himalayan countries namely Bhutan and Nepal, having similar physical conditions. Both these countries are bordered by world's most populous countries, China and India which have been severely affected by Covid-19. Bhutan and Nepal are small countries located in Himalayan region in southern part of Asia. Both these countries are landlocked. Nepal is located to the west of Bhutan. Their boundaries do not touch each other, though they are connected through the Indian state of Sikkim.

Bhutan is located almost entirely in the southern slopes of the Great Himalayan mountain range, spreading across 38394 square kilometres [16]. Because of pristine landscapes, lush green mountains, pure and gentle rivers, Kingdom of Bhutan is sometimes referred to as Shangri-la (a beautiful, pure and isolated mountain place where people live a perfect life). It is one of the two carbon-negative countries in the world [7]. Bhutan has diverse climate due to great differences in elevation of different regions. The density of population in Bhutan is sparse-20 persons per square kilometre.

First case of Covid-19 reported in Bhutan on March 05, 2020 was of an American tourist. About two weeks later when another tourist was found to be infected in the country, the Government of Bhutan took prompt action and started monitoring and isolating all those who came in contact with these tourists [4]. All international borders were sealed off on March 23. The import of fruits, vegetables and meat was banned. The quarantine period for Covid-19 was extended from 14 to 21 days [7, 15]. Contact tracing and monitoring of the Covid cases was done very efficiently in Bhutan. People of Bhutan, in general, adhere to the advisory issued by the government and His Majesty. Quick actions taken by the government including nation wise lockdowns (August, 2020 and December, 2020) [15] were instrumental in successfully curtailing further transmission of the disease.

Nepal, another neighbouring country of India, is a developing nation with lower middle income economy which is largely dependent on agriculture and tourism. Nepal's climate is determined by elevation as well as by its location. On one hand, some lower regions are cool and some have warm temperate climate, on the other, there are uninhabited regions as well-covered by snow and ice throughout the year. The population of Nepal is 2.97 crores with density 203 per square kilometer [18]. In Nepal, the first reported case was of a student (January 13, 2020) who had returned from China while the second case was of a girl (reported on March 23, 2020) who had returned from France [5, 2]. Nepal took several measures to stop the spread of the disease. Nationwide lockdown was declared on March 24, 2020. Mountaineering expeditions were cancelled and all international flights were stopped [11]. Nepal had to cancel the "Visit Nepal Year" campaign due to the pandemic.

In April 2021, second wave of Covid-19 started affecting many countries, including Bhutan and Nepal. In the present study, we analyse the Covid-19 cases scenario of both these Himalayan countries for a period chosen from this phase, starting from June 16, 2021 to July 30, 2021. This paper is arranged as follows: The methodology used in the study of this paper is described in Section 2. In Section 3 and Section 4, through the solution of SIR compartmental model using repeated MsDTM, we analyse the Covid cases in Bhutan and Nepal. We attempt a comparison of the spread of the disease in these two regions in Section 5.

2. Methodology

Various real life problems have been formulated into mathematical models to simulate a situation and predict future behaviours or outcomes.

Mathematical models can be categorized into different types like empirical, stochastic, deterministic etc. [3]. These models have diverse applications in various fields like medicine, physical sciences, ecology and many others. One of the deterministic models is compartmental model where initial conditions and parametric values are used to obtain the desired outcome. There are different types of compartmental models, used according to the requirement and availability of data. To study the behaviour of spread of Covid-19 epidemic also, various compartmental models such as SIS, SIR, SEIR, SEIRD have been used [6, 8, 10]. In our study we use SIR model [8] to estimate the number of susceptibles, infected and recovered of Covid-19 spread in Bhutan and Nepal.

In SIR, (s-Susceptible, i-Infected, r-Recovered) compartmental model, used in this paper, susceptible (s) may get infected with the rate of infection β to move to the next compartment i.e., infected (i). Infected get recovered, with rate of recovery γ, moving to the last compartment, that of recovered (r). There is no exit from the final compartment. N represents the total population, $N = s + i + r$, that remains constant throughout.

The following system of differential equations represents the model used in this paper [3]:

$$\frac{ds}{dt} = -\beta si, \ \frac{di}{dt} = \beta si - \gamma i, \ \frac{dr}{dt} = \gamma_i. \tag{1}$$

The Differential Transform Method, used to solve systems of differential equations with boundary conditions, was introduced by Zhou [14] in 1984.

The kth coefficient of the Taylor's series of an analytical function ϕ about a point c, is called the kth differential transform of ϕ and is denoted by $\Phi(k)$ i.e.,

$$\Phi(k) = \frac{\phi^{(k)}(c)}{k!}.$$

The function ϕ is said to be the inverse differential transform of Φ and is defined as

$$\phi(x) = \sum_{k=0}^{\infty} \Phi(k)(x - c)^k. \tag{2}$$

In DTM, the function ϕ is approximated by a finite degree, say k^{th} degree polynomial obtained from (2)

$$\phi(x) = \sum_{k=0}^{K} \Phi(k) x^k \text{ for some } k \in \mathbb{N}, \tag{3}$$

with remaining terms of the series representing the error in this approximation. Some properties of the transform function, used to solve the system of differential equations in this paper are listed below:

- $\phi(x) = \psi(x) \pm \omega(x);\ \Phi(k) = \Psi(k) \pm \Omega(k),$
- $\phi(x) = \psi(x)\omega(x);\ \Phi(k) = \sum_{m=0}^{k} \Psi(m)\Omega(k - m),$
- $\phi(x) = \alpha\psi(x);\ \Phi(k) = \alpha\Psi(k),$
- $\phi(x) = \frac{d\psi(x)}{dx};\ \Phi(k) = (k + 1)\Psi(k + 1).$

The transformed equations of the system of differential equations for SIR model, given in (1), are given by [1]:

$$\begin{cases} S(k+1) = \frac{1}{k+1}(-\beta\sum_{m=0}^{K}(S(m)I(k-m))), \\ I(k+1) = \frac{1}{k+1}(\beta\sum_{m=0}^{K}(S(m)I(k-m) - \gamma I(k))), \\ R(k+1) = \frac{1}{k+1}\gamma I(k). \end{cases} \tag{4}$$

The susceptibles (s), infected (i) and recovered (r) are obtained using their transformed functions S, I and R respectively, as given in (3).

In Differential Transform Method, the system of equations is solved using some initial conditions. If the method is used to obtain solution of SIR model in an interval $[0, T]$, initial values of S, I and R at $t = 0$ are used. For longer time intervals, the solution thus obtained is not very accurate. This limitation of DTM was considered and the method of Multistage Differential

Transform Method (MsDTM) was used [9]. In MsDTM the interval $[0, T]$ is divided into n sub-intervals $[t_{i-1}, t_i]$, $i = 1, \ldots, n$ of equal length T/n each, where $t_0 = 0$ and $t_n = T$. DTM is applied to the first interval using the given initial values of the function, say $\phi(0)$, to obtain the approximation of the function, given by ϕ_1 in the first interval. $\phi_1(t_1)$ is taken as the initial condition for the function ϕ in the second interval. The process is repeated for all subsequent intervals. The solution using this method [13], is of the form

$$\phi(x) = \begin{cases} \phi_1(x) \\ \phi_2(x) \\ \vdots \\ \phi_n(x) \end{cases} \text{ where } \phi_i(x) = \sum_{k=0}^{K} \Phi(k)(x - t_{i-1})^k, \; x \in [t_{i-1}, t_i]$$

for $i = 1, \ldots, n$.

In many differential equations, parameters other than the variables are also involved. In DTM as well as MsDTM, these parameters are kept constant throughout the method. However, the parameters sometimes represent rates that keep changing with time; like in SIR model, the parameters β and γ stand for rate of transmission and rate of recovery respectively, which are varying with time. Considering this, we use Repeated Multistage Differential Transform Method (RMsDTM). In this method, the time period is divided into equal parts according to the prevalent parametric values in that interval. The parametric values for each interval is accordingly assigned as the value specific to that period. After suitably choosing the parameters, MsDTM is applied to each sub-interval, instead of applying to the original interval.

3. Analysis of Covid-19 cases in Bhutan

We considered the SIR model given in (1) for the study of Covid-19 scenario in Bhutan for a period of 45 days, starting from June 16, 2021. The number of susceptibles (s), infected (i) and recovered (r) for the chosen period were obtained using RMsDTM. Tools of Mathematica [12] were employed for this purpose. The period of 45 days was divided into 5 equal parts of 9 days each. Values of the parameters β and γ, as obtained from the original data [17] and used in the method are given in Table 1. The initial values of susceptibles,

Table 1.

Step	β	γ
I	7.8689767×10^{-8}	0.082244145
II	6.2201453×10^{-8}	0.029917058
III	7.3205908×10^{-8}	0.058522990
IV	7.2028384×10^{-8}	0.061151057
V	3.1442092×10^{-8}	0.097347832

infected and recovered as calculated the from actual data [17] were taken to be $s(0) = 7, 78, 058$, $i(0) = 345$, $r(0) = 29$. The number of infected and recovered obtained using RMsDTM and the actual number of infected and recovered for the chosen period are depicted in the graphs in Figure 1. It is evident from the

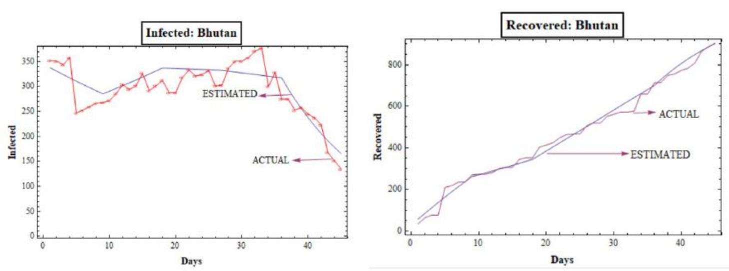

Figure 1. Infected and Recovered: Bhutan.

graphs that the method used to find the estimates, RMsDTM is giving a good approximation to the actual data when applied to the chosen period of 45 days at equal intervals of 9 days. The error in the estimated values of infected and recovered, obtained using RMsDTM, is tabulated in Table 2.

Table 2. Errors in Infected and Recovered: Bhutan.

Day	Infected	Recovered
1	14.18	23.08
2	20.20	21.56

3	20.07	34.44
4	40.81	60.80
5	63.60	45.40
6	51.14	29.16
7	37.82	21.41
8	24.63	2.82
9	17.57	8.48
10	18.89	2.86
11	10.32	5.92
12	2.17	7.86
13	13.46	3.03
14	11.19	0.25
15	6.97	9.70
16	32.98	18.67
17	30.03	17.86
18	25.20	7.87
19	48.67	38.15
20	48.14	28.47
21	17.61	19.81
22	2.08	24.19
23	13.56	19.59
24	10.03	3.03
25	1.50	16.51
26	31.98	3.99
27	29.46	2.48

28	6.24	22.76
29	21.93	10.93
30	23.61	21.00
31	32.29	29.97
32	46.95	49.84
33	54.61	65.60
34	20.75	2.26
35	8.89	21.83
36	42.48	12.71
37	20.18	15.07
38	21.6	8.60
39	2.36	26.87
40	7.31	33.85
41	15.91	45.01
42	17.15	40.37
43	23.40	3.68
44	27.03	0.48
45	30.76	2.09

It can be seen from the table that error in the estimated values of infected and recovered is very low, signifying the efficiency of the method used.

The pattern of active infected (I) and recovered (R) for Bhutan in different time intervals can be observed from Figure 2. These are depending on the rate of transmission and rate of recovery for respective period as given in Table 1.

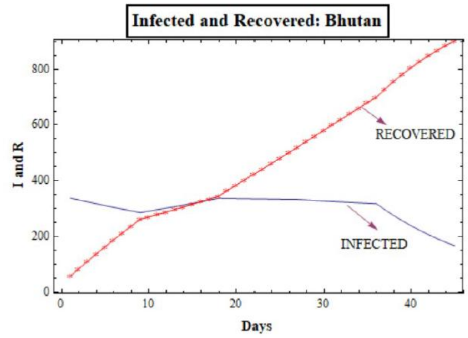

Figure 2. Infected-Recovered: Bhutan.

4. Analysis of Covid-19 Cases in Nepal

In this section, we estimate the Covid-19 cases in Nepal. We considered the period of June 16, 2021 to July 30, 2021. This period was divided into 5 intervals of 9 days length each. The values of parameters in each in interval, estimated from the original data, is given in Table 3. The initial conditions for $s, i,$ and r, also taken from the original data are $s(0) = 2, 96, 74, 920$, $i(0) = 63, 257$ and $r(0) = 4, 228$.

Using RMsDTM for SIR Model, the values of $s, i,$ and r were obtained for the chosen period of 45 days. The estimated value of infected and recovered along-with the actual numbers are plotted in Figure 3.

We see from Figure 3 that the values estimated using RMsDTM are very close to the actual data. The error in estimation is listed in Table 4.

Table 3.

Step	β	γ
I	1.1555055×10^{-9}	0.060280078
II	1.4572281×10^{-9}	0.104755842
III	$1.97339037 \times 10^{-9}$	0.061103027
IV	2.2914386×10^{-9}	0.065817385
V	2.7507269×10^{-9}	0.063411509

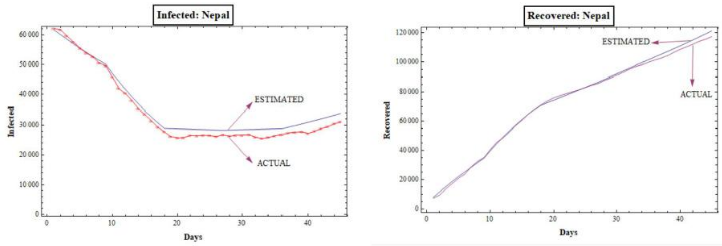

Figure 3. Infected and Recovered: Nepal.

Table 4. Errors in Infected and Recovered: Nepal.

Day	Infected	Recovered
1	502.80	629.58
2	1671.02	2114.06
3	1107.68	1615.44
4	602.85	1402.97
5	21.37	1285.82
6	190.98	1407.99
7	94.31	366.39
8	870.09	869.59
9	519.32	360.59
10	1294.40	758.36
11	2267.85	940.85
12	1348.66	456.95
13	1249.02	794.34
14	1561.05	390.09
15	1139.73	447.19
16	1263.55	79.52
17	1419.63	264.87

18	1170.97	273.85
19	2634.03	1090.03
20	3049.18	1565.73
21	2837.42	1391.95
22	2048.04	1003.65
23	2071.75	573.83
24	1855.56	169.51
25	1883.74	112.38
26	2127.02	290.81
27	1515.39	866.79
28	2027.02	636.64
29	1701.66	876.48
30	1770.31	954.31
31	1643.33	1008.14
32	2579.35	698.99
33	3107.39	888.86
34	2816.80	1482.75
35	2296.22	1750.69
36	1944.64	2137.67
37	1928.89	2667.01
38	2179.44	2845.49
39	2629.30	2540.11
40	3715.03	2416.45
41	3601.58	2684.78
42	3249.94	2901.10

43	3229.81	2928.19
44	2882.03	3629.23
45	2866.60	3913.22

5. Discussion and Conclusion

As mentioned earlier, density of population in Bhutan is 20 per square kilometer and that of Nepal is 203 per square kilometre with population 2.97×10^7 in comparison to the population of Bhutan which is 7.80×10^5. This demographic disparity, affected the spread of Covid-19 in the two countries accordingly.

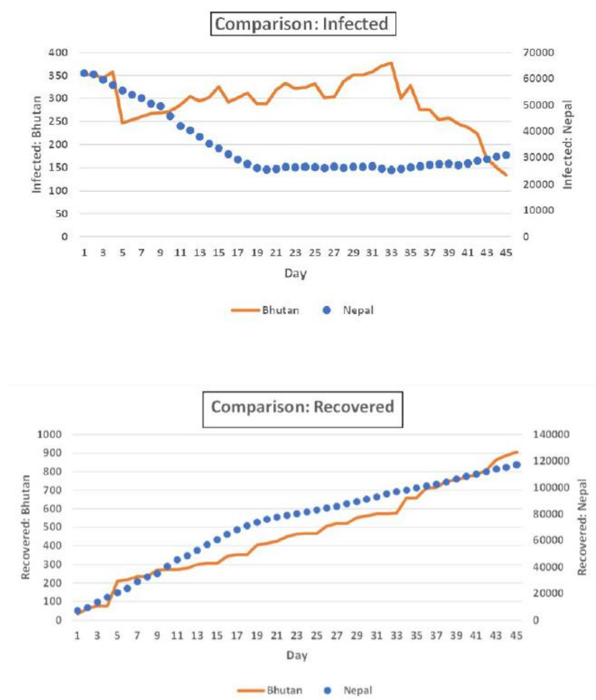

Figure 4. Comparisons: Infected and Recovered.

When we compare estimated number of infected in two countries obtained through RMsDTM, number of infected are lesser in Bhutan as shown in the Figure 4, which is showing these numbers of the two countries by adjusting the scales. Also, when we compare number of recovered in the two countries from the graph, we see that Bhutan has less number of recovered cases than that of Nepal.

Further, to analyse the data, we determined number of relatively infected of both the countries. For this, for each country, we considered the ratio of active infected with number of susceptibles. We plotted the graph for relative infected of both countries, as given in Figure 5.

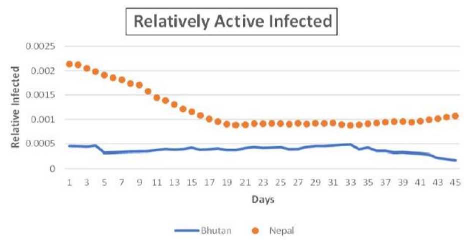

Figure 5. Relative Infected.

Through Figure 5, we can compare number of relatively infected in Bhutan and Nepal. We observe that throughout the considered period, number of relatively infected is lesser in Bhutan. For Bhutan, this number remains almost same during the considered period but decreases towards the end. This behaviour is consistent with the values of β and γ for different times given in Table 1. In Nepal, the number of relatively infected is higher in the initial period and then reduces to almost half. This reduction in the number is due to improvement in recovery rate in that period, as visible in Table 3. There is slight increase in number of relatively infected in Nepal again at the end of the considered period.

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