Advances and Applications in Mathematical Sciences
Volume 19, Issue 11, September 2020, Pages 1189-1203

A NEW METHOD TO DISCUSSING THE MANUFACTURING DEFECTS IN EOQ/EPQ INVENTORY MODELS WITH SHORTAGES USING FUZZY TECHNIQUES

A. NAGOOR GANI and U. MOHAMMED RAFI

P.G. and Research Department of Mathematics
Jamal Mohamed College (Autonomous)
Tiruchirappalli-620020, India
E-mail: ganijmc@yahoo.co.in

Research Scholar
P.G and Research Department of Mathematics
Jamal Mohamed College (Autonomous)
Tiruchirappalli-620020, India
E-mail: rafithowheed@gmail.com

Abstract

In this research article, a new approach to find EOQ/EPQ inventory models with shortages involving the manufacturing defects by using Arithmetic geometric mean (AGM) inequality and Cauchy-Bunyakovsky-Schwarz (CBS) inequality under fuzzy environment. Usually we all are assuming that there is no defect in all types of inventory models but everyone knows that, no one is 100% perfect. Therefore, we included the defects herewith. This paper solicited to find the optimal lot size, optimal shortage level, and the optimal total relevant cost per unit time without using any derivatives. Both the demand rate and shortage cost are not stable so here we take these two as a trapezoidal fuzzy number. An example is taken into consideration with this method and results are calculated using this systematic process.

1. Introduction

When talking about deterministic inventory models, the highest valuable findings have concerned the economic and production order quantity models

2010 Mathematics Subject Classification: 11D09, 11D99.

Keywords: inventory, Economic Order Quantity (EOQ), Economic Production Quantity (EPQ), Arithmetic Geometric Mean (AGM) inequality, Cauchy-Bunyakovsky-Schwarz (CBS) inequality, cost comparison optimization, shortage level, defuzzification, graded mean integration representation method, trapezoidal fuzzy number.

Received June 14, 2020; Accepted August 21, 2020

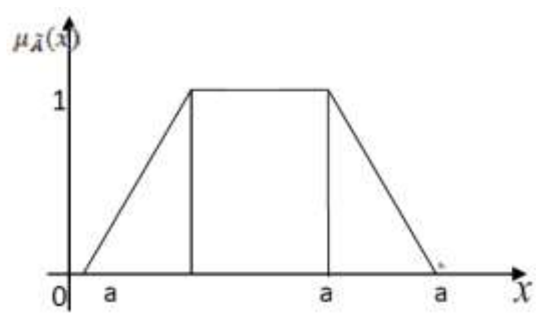
with and without shortages. The best solution can be achieved using differential calculus for these models and the same has been discussed in several previous articles. But many students lack of knowledge in differential calculus and in past many authors have given attempts to derive the best solution for the EOQ/EPQ models without differential calculus, and one suitable way is using algebraic approach. First, Grubbstrom [9] used algebraic method and gave a standard EOQ formula and was extended by Grubbstrom [8] to arrive at the EOQ model with shortages. Cardenas Barron [4] enhanced this method to EPQ model with shortages, [15] tried to improve the algebraic method to solve the EOQ and EPQ models with shortages and Chang [5] enhanced the method of Ronald [15], by replacing their experienced algebraic skill, to determine the optimal solution for these models. Recently, an optimization approach: the arithmetic geometric mean inequality and Cauchy-Bunyakovsky-Schwarz inequality, which proposed by Cardenas Barron [3]. This approach is simpler than the algebraic approach, presented by Grubbstrom [8] and Cardenas Barron [4], for driving the EOQ and EPQ models with shortages. The EOQ and EPQ models with shortages assume that units purchased or produced are of 100% good quality. But in a real world scenario, it's not possible to produce or purchase 100% good quality products and it's the fact 100% inspection policy was assumed by Huang [10] and known percentages of defective items were removed prior to store or use at the end of the screening process. Additionally, he applied the algebraic method from Grubbstrom [8] and Cardenas Barron [4] to drive the best optimal lot size and optimal shortage level for the EOQ and EPQ models with shortages and defective items. However, Tu et al. [17] noticed that this algebraic method had the same problem as Grubbstrom [8] and Cardenas Barron [4]. They later used the simple method in Cardenas Barron [3], the AGM and CBS inequalities, to get the optimal solutions for the semodels. Although the method is simpler than the algebraic methods in Grubbstrom [8] and Cardenas Barron [4], but it is difficult to set the expected variables to satisfy the CBS inequality. In this paper, we use the algebraic method and the AGM inequality to derive the optimal lot size and the optimal shortage level for the EOQ and EPQ models with shortages and defective items.

2. Preliminaries

Definition 2.1. A fuzzy set $\widetilde{A}$ is defined by $\widetilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0, 1]\}$. In the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$ belong to the closed interval [0, 1] called Membership function.

2.2. Trapezoidal Fuzzy Number. A trapezoidal fuzzy number $\widetilde{A}$ is denoted as, $\widetilde{A} = (a_1, a_2, a_3, a_4)$ and is defined by the member ship function as,

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x - a_1)}{(a_2 - a_1)} & \text{for } a_1 \le x \le a_2 \\ 1 & \text{for } a_2 \le x \le a_3 \\ \dfrac{(a_4 - x)}{(a_4 - a_3)} & \text{for } a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases}$$



2.3 The Fuzzy Arithmetic Operations on Trapezoidal Fuzzy Number under Function Principle. Let $\widetilde{A} = (a_1, a_2, a_3, a_4)$ and $\widetilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers with the condition that $a_1 \le a_2 \le a_3 \le a_4$ and $b_1 \le b_2 \le b_3 \le b_4$. If $a_1 = a_2 = a_3 = a_4$ and $b_1 = b_2 = b_3 = b_4$ then $\widetilde{A}$ and $\widetilde{B}$ are crisp numbers.

Then the fuzzy arithmetic operations under function principle is given by,

Addition

$\widetilde{A}+\widetilde{B}=(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$, where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all any real numbers.

Subtraction

$\widetilde{A}-\widetilde{B}=(a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$, where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all any real numbers.

Multiplication

$\widetilde{A}\cdot\widetilde{B} = (\min(a_1b_1, a_1b_4, a_4b_1, a_4b_4), \min(a_2b_2, a_2b_3, a_3b_2, a_3b_3), \max(a_2b_2, a_2b_3, a_3b_2, a_3b_3), \max(a_1b_1, a_1b_4, a_4b_1, a_4b_4))$. If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all nonzero positive real numbers, then $\widetilde{A}\cdot\widetilde{B} = (a_1\cdot b_1, a_2\cdot b_2, a_3\cdot b_3, a_4\cdot b_4)$

Scalar Multiplication:

Let, $\lambda \in R$, then $\lambda\widetilde{A} = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4)$; $\lambda \geq 0$ $\lambda\widetilde{A} = (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1)$; $\lambda < 0$.

Division:

$$\frac{\widetilde{A}}{\widetilde{B}} = \left(\min\left(\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_4}\right), \min\left(\frac{a_2}{b_2}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_3}{b_3}\right), \max\left(\frac{a_2}{b_2}, \frac{a_2}{b_3}, \frac{a_3}{b_3}\right),\right.$$

$$\left.\max\left(\frac{a_1}{b1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}\right)\right)$$

If are $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 all nonzero positive real numbers, then $\frac{\widetilde{A}}{\widetilde{B}} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$.

2.4. Graded Mean Integration Representation Method

Defuzzification is a process of transforming fuzzy values to crisp values. Defuzzification method shave been widely studied for some years and were applied to fuzzy systems. The major idea behind these methods was to obtain a typical value from a given set according to some specified characters. Defuzzification method provides a correspondence from the set of all fuzzy sets into the set of all real numbers.

Let $\widetilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number, then the defuzzified value $\widetilde{A}$ using the graded mean integration representation method [4] is given by $P(\widetilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$.

This method is also used to ranking the triangular fuzzy numbers to choose which is minimum and maximum.

3. Derivation of the Optimal Lot Size and the Shortage Level for EOQ/EPQ Models with Shortages with Manufacturing Defects

This section provided a method to derive both the optimal lot size and the optimal shortage level for the EOQ/EPQ models with shortages including the manufacturing defects discussed. Taking into two inequalities the Arithmetic Geometric Mean (AGM) inequality and Cauchy-Bunyakovsky-Schwarz (CBS) inequality.

The AGM inequality: Let the positive real numbers be $a_1, a_2, a_3, \ldots, a_n$ such that, $\frac{a_1 + a_2 + a_3 + \ldots + a_n}{n} \geq (a_1, a_2, a_3 \ldots a_n)^{1/n}$ the equation holds only if $a_1 = a_2 = a_3 = \ldots = a_n$.

The CBS inequality: Let $a_1 + a_2 + a_3 + \ldots + a_n$ and $b_1 + b_2 + b_3 + \ldots + b_n$ be any two sets of real numbers, then $(a_1^2 + a_2^2 + a_3^2 + \ldots + a_n^2)$ $(b_1^2 + b_2^2 + b_3^2 + \ldots + b_n^2) \geq (a_1 b_1 + a_2 b_2 + a_3 b_3 + \ldots + a_n b_n)^2$ with equality if and only if the two sets of numbers proportional $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \ldots = \frac{a_n}{b_n}$.

Notations: The following notations are used throughout the entire paper.

q Lot size per order (Production rate) inclusive of defective items

q_1 Inventory level including defective items

q_2 Shortage level including defective items

D Demand rate per unit time for perfect items

K Production rate for perfect items per unit time $(k > D)$

C_1 Inventory carrying cost (holding cost) per order

C_2 Shortage cost per item

C_3 Setup cost (ordering cost) per order

E The fixed inspection cost incurred with each lot

e Unit inspection cost

r The known percentage of defective items in $q(0 < r < 1)$.

3.1. The EOQ model with shortages and defects

According to Huang [10], we know the total relevant cost per unit time can be written as

$$TC(q_1, q_2) = \frac{\tilde{D}}{(1-r)(q_1+q_2)}\left[C_3 + E + (q_1+q_2)e + \frac{(1-r)^2 q_1^2 C_1}{2\tilde{D}} + \frac{(1-r)^2 q_2^2 \tilde{C}_2}{2\tilde{D}}\right] \quad (1)$$

Where $\tilde{D} = (d_1, d_2, d_3, d_4)$ and $\tilde{C}_2 = (C_{21}, C_{22}, C_{23}, C_{24})$.

Let Substitute $q = q_1 + q_2$ sub in (1) we get,

$$TC(q, q_2) = \frac{\tilde{D}}{(1-r)q}\left[C_3 + E + qe + \frac{(1-r)^2(q-q_2)^2 C_1}{2\tilde{D}} + \frac{(1-r)^2 q_2^2 \tilde{C}_2}{2\tilde{D}}\right]. \quad (2)$$

Here we should apply the CBS inequality, so we to change the equation (2) as

$$TC(q, q_2) = \frac{(C_3+E)\tilde{D}}{(1-r)q} + \frac{\tilde{D}e}{1-r} + \frac{(1-r)(q-q_2)^2 C_1}{2q} + \frac{(1-r)q_2^2 \tilde{C}_2}{2q}$$

$$= \frac{(C_3+E)\tilde{D}}{(1-r)q} + \frac{\tilde{D}e}{1-r} + \frac{(1-r)q}{2}\left[\left(1 - \frac{q_2}{q}\right)^2 C_1 + \left(\frac{q_2}{q}\right)^2 \tilde{C}_2\right]$$

$$= \frac{(C_3 + E)\widetilde{D}}{(1-r)\,q} + \frac{\widetilde{D}e}{1-r} + \frac{(1-r)q_1}{2}\left[\left[\left[\sqrt{C_1}\left(1 - \frac{q_2}{q}\right)\right]^2 + \left[\sqrt{\widetilde{C}_2}\left(\frac{q_2}{q}\right)\right]^2\right]\right.$$

$$\left.\left[\left(\frac{\sqrt{\widetilde{C}_2}}{\sqrt{C_1 + \widetilde{C}_2}}\right)^2\left(\frac{\sqrt{C_1}}{\sqrt{C_1 + \widetilde{C}_2}}\right)^2\right]\right] \tag{3}$$

Applying the CBS inequality in (3) we get,

$$= \frac{(C_3 + E)}{(1-r)q} + \frac{\widetilde{D}e}{1-r} + \frac{(1-r)q}{2}\left[\frac{\sqrt{C_1\widetilde{C}_2}}{\sqrt{C_1 + \widetilde{C}_2}}\left(1 - \frac{q_2}{q}\right) + \frac{\sqrt{C_1\widetilde{C}_2}}{\sqrt{C_1 + \widetilde{C}_2}}\left(\frac{q_2}{q}\right)\right]^2. \tag{4}$$

Since $(a_1^2 + a_2^2)(b_1^2 + b_2^2) \geq (a_1b_1 + a_2b_2)^2$ with equality if and only if $\frac{a_1}{b_1} = \frac{a_2}{b_2}$

$$\frac{\sqrt{C_1}\left(1 - \frac{q_2}{q}\right)}{\left(\frac{\sqrt{\widetilde{C}_2}}{\sqrt{C_1 + \widetilde{C}_2}}\right)} = \frac{\sqrt{\widetilde{C}_1}\left(\frac{q_2}{q}\right)}{\left(\frac{\sqrt{C_1}}{\sqrt{C_1 + \widetilde{C}_2}}\right)}. \tag{5}$$

If and only if, the two sets of numbers are proportional by CBS inequality. Thus equation (4) becomes

$$TC(q) = \frac{(C_3 + E)\widetilde{D}}{(1-r)q} + \frac{\widetilde{D}e}{1-r} + \frac{(1-r)q}{2}\left[\frac{\sqrt{C_1\widetilde{C}_2}}{\sqrt{C_1 + \widetilde{C}_2}}\left(1 - \frac{q_2}{q} + \frac{q_2}{q}\right)\right]^2.$$

Solving we get,

$$TC(q) = \frac{(C_3 + E)\widetilde{D}}{(1-r)q} + \frac{\widetilde{D}e}{1-r} + \frac{(1-r)q}{2}\left\{\frac{C_1\widetilde{C}_2}{C_1 + \widetilde{C}_2}\right\}. \tag{6}$$

Here we have to apply AGM inequality because the product of these two functions is a constant function.

$$TC(q) = \frac{\frac{2(C_3 + E)\widetilde{D}}{(1-r)q} + (1-r)q\left(\frac{C_1\widetilde{C}_2}{C_1 + \widetilde{C}_2}\right)}{2} + \frac{\widetilde{D}e}{1-r} \geq \sqrt{2(C_3 + E)\widetilde{D}\left(\frac{C_1\widetilde{C}_2}{C_1 + \widetilde{C}_2}\right)}$$

$$+\frac{\widetilde{D}e}{1-r}. \tag{7}$$

Since $\frac{a+b}{2} \geq \sqrt{ab}$ with equality if and only if $a = b$

$$\frac{2(C_3+E)\widetilde{D}}{(1-r)q} = (1-r)q\left(\frac{C_1\widetilde{C}_2}{C_1+\widetilde{C}_2}\right). \tag{8}$$

Holds $TC(q)$ has a minimum, on solving equation (8) we get the optimal ordering quantity is given by

$$q^2 = \frac{1}{(1-r)^2}\frac{2(C_3+E)\widetilde{D}}{C_1}\left(\frac{C_1+\widetilde{C}_2}{\widetilde{C}_2}\right)$$

$$q^* = \frac{1}{1-r}\sqrt{\frac{2(C_3+E)\widetilde{D}}{C_1}\left(\frac{C_1+\widetilde{C}_2}{\widetilde{C}_2}\right)}. \tag{9}$$

And the required optimal shortage level is given by

$$q_2^* = \frac{\widetilde{C}_2 q^*}{C_1+\widetilde{C}_2} = \frac{1}{1-r}\sqrt{\frac{2(C_3+E)\widetilde{D}}{C_1}\left(\frac{\widetilde{C}_2}{C_1+\widetilde{C}_2}\right)}$$

From (7), the optimal total inventory cost is given by

$$TC(q^*) = \sqrt{2(C_3+E)\widetilde{D}\left(\frac{C_1\widetilde{C}_2}{C_1+\widetilde{C}_2}\right)} + \frac{\widetilde{D}e}{1-r}.$$

Where $\widetilde{D} = (d_1, d_2, d_3, d_4)$ and $\widetilde{C}_2 = (C_{21}, C_{22}, C_{23}, C_{24})$.

The EPQ model with Shortages and Manufacturing Defects

According to Huang [10], we know the Total relevant cost per cycle is

$$TC = C_3 + E + (q_1+q_2)e + \frac{(1-r)^2\left[q\left(1-\frac{\widetilde{D}}{k}\right)-q_2\right]^2 C_1}{2\widetilde{D}\left(1-\frac{\widetilde{D}}{k}\right)} + \frac{(1-r)^2 q_2^2\widetilde{C}_2}{2\widetilde{D}\left(1-\frac{\widetilde{D}}{k}\right)}. \tag{10}$$

Then the total relevant cost per unit time is

$$TC(q_1, q_2) = \frac{\widetilde{D}}{(1-r)(q_1+q_2)}\left[C_3 + E + (q_1+q_2)e + \frac{(1-r)^2\left[q\left(1-\frac{\widetilde{D}}{k}\right)-q_2\right]^2 C_1}{2\widetilde{D}\left(1-\frac{\widetilde{D}}{k}\right)}\right.$$

$$\left.+\frac{(1-r)^2 q_2^2 \widetilde{C}_2}{2\widetilde{D}\left(1-\frac{\widetilde{D}}{k}\right)}\right]. \quad (11)$$

Where $\widetilde{D} = (d_1, d_2, d_3, d_4)$ and $\widetilde{C}_2 = (C_{21}, C_{22}, C_{23}, C_{24})$.

For convenience, Let $\rho = 1 - \frac{\widetilde{D}}{k}$ then

$$(1-r)\left[q\left(1-\frac{\widetilde{D}}{k}\right)-q_2\right]^2 = (1-r)q_1$$

$q_1 = q\rho - q_2$ then

$$q = \frac{q_1 + q_2}{\rho}. \quad (12)$$

Substitute in (11) we get,

$$TC(q, q_2) = \frac{\widetilde{D}}{(1-r)q}\left[C_3 + E + qe + \frac{(1-r)^2(q\rho - q_2)^2 C_1}{2\widetilde{D}\rho} + \frac{(1-r)^2 q_2^2 \widetilde{C}_2}{2\widetilde{D}\rho}\right]. \quad (13)$$

Equation (13) can be rewritten as

$$TC(q, q_2) = \frac{(C_3+E)}{(1-r)q}\frac{\widetilde{D}e}{1-r} + \frac{(1-r)q}{2}\left[\left(1-\frac{q_2}{q\rho}\right)^2 C_1 + \left(\frac{q_2}{q\rho}\right)\widetilde{C}_2\right]$$

$$= \frac{(C_3+E)\widetilde{D}}{(1-r)q} + \frac{\widetilde{D}e}{1-r} + \frac{(1-r)q_1}{2}\left[\left[\left[\sqrt{C_1}\left(1-\frac{q_2}{q\rho}\right)\right]^2 + \left[\sqrt{\widetilde{C}_2}\left(\frac{q_2}{q\rho}\right)\right]^2\right]\right.$$

$$\left.\left[\left(\frac{\sqrt{\widetilde{C}_2}}{\sqrt{C_1+\widetilde{C}_2}}\right)^2\left(\frac{\sqrt{C_1}}{\sqrt{C_1+\widetilde{C}_2}}\right)^2\right]\right] \quad (14)$$

Applying CBS inequality in equation (14) we get,

$$TC(q, q_2) \geq \frac{(C_3+E)\tilde{D}}{(1-r)q} + \frac{\tilde{D}e}{1-r} + \frac{(1-r)q_1}{2}\left[\frac{\sqrt{C_1\tilde{C}_2}}{\sqrt{C_1+\tilde{C}_2}}\left(1-\frac{q_2}{q\rho}\right) + \frac{\sqrt{C_1\tilde{C}_2}}{\sqrt{C_1+\tilde{C}_2}}\left(\frac{q_2}{q\rho}\right)\right]^2$$

Since $(a_1^2 + a_2^2)(b_1^2 + b_2^2) \geq (a_1 b_1 + a_2 b_2)^2$ with equality if and only if $\frac{a_1}{b_1} = \frac{a_2}{b_2}$

$$\frac{\sqrt{C_1}\left(1-\frac{q_2}{q\rho}\right)}{\left(\frac{\sqrt{\tilde{C}_2}}{\sqrt{C_1+\tilde{C}_2}}\right)} = \frac{\sqrt{\tilde{C}_2}\left(\frac{q_2}{q\rho}\right)}{\left(\frac{\sqrt{C_1}}{\sqrt{C_1+\tilde{C}_2}}\right)}. \tag{15}$$

If and only if the two sets of numbers are proportional by CBS inequality

Thus equation (15) becomes,

$$TC(q) = \frac{(C_3+E)\tilde{D}}{(1-r)q} + \frac{\tilde{D}e}{1-r} + \frac{(1-r)q}{2}\left\{\frac{\sqrt{C_1\tilde{C}_2}}{\sqrt{C_1+\tilde{C}_2}}\left(1-\frac{q_2}{q\rho}+\frac{q_2}{q\rho}\right)\right\}^2$$

$$= \frac{(C_3+E)\tilde{D}}{(1-r)q} + \frac{\tilde{D}}{1-r} + \frac{(1-r)q}{2}\left\{\frac{C_1\tilde{C}_2}{C_1+\tilde{C}_2}\right\}. \tag{16}$$

Here we have to apply the AGM inequality because the product of these two function is constant.

$$TC(q) = \frac{\frac{2(C_3+E)\tilde{D}}{(1-r)q} + (1-r)q\left(\frac{C_1\tilde{C}_{2}\rho}{C_1+\tilde{C}_2}\right)}{2} + \frac{\tilde{D}e}{1-r}$$

$$\geq \sqrt{2(C_3+E)\tilde{D}\left(\frac{C_1\tilde{C}_{2}\rho}{C_1+\tilde{C}_2}\right)} + \frac{\tilde{D}e}{1-r}. \tag{17}$$

Since $\frac{a+b}{2} \geq \sqrt{ab}$ with equality if and only if $a = b$

$$\frac{2(C_3+E)\tilde{D}}{(1-r)q} + (1-r)q\left(\frac{C_1\tilde{C}_{2}\rho}{C_1+\tilde{C}_2}\right). \tag{18}$$

Holds $TC(q)$ has a minimum, on solving equation (18) we get, the optimal production quantity is given by

$$q^2 = \frac{1}{(1-r)^2} \frac{2(C_3 + E)\widetilde{D}}{C_1 \rho} \left(\frac{C_1 + \widetilde{C}_2}{\widetilde{C}_2} \right)$$

$$q^* = \frac{1}{1-r} \sqrt{\frac{2(C_3 + E)\widetilde{D}}{C_1 \rho} \left(\frac{C_1 + \widetilde{C}_2}{\widetilde{C}_2} \right)}.$$

And the optimal shortage level is given by

$$q_2^* = \frac{\widetilde{C}_2 q^*}{C_1 + \widetilde{C}_2} = \frac{1}{1-r} \sqrt{\frac{2(C_3 + E)\widetilde{D}\rho}{C_1} \left(\frac{\widetilde{C}_2}{C_1 + \widetilde{C}_2} \right)}.$$

From equation (17), the optimal total inventory cost is given by

$$TC(q^*) = \sqrt{2(C_3 + E)\widetilde{D} \left(\frac{C_1 \widetilde{C}_2 \rho}{C_1 + \widetilde{C}_2} \right)} + \frac{\widetilde{D}e}{1-r}$$

Where $\widetilde{D} = (d_1, d_2, d_3, d_4)$ and $\widetilde{C}_2 = (C_{21}, C_{22}, C_{23}, C_{24})$.

4. Illustrate Examples

4.1. Problem on EOQ with shortages and defects:

A pen retailer purchases around 9000 pockets of pens per year. Each pocket has 10 pens. The ordering cost per order Rs. 150 and the holding cost is Rs. 30 per year. Assuming that the shortage cost is Rs.20 per pocket per year also assuming that the manufacturing defects are 5% in purchasing items. Provided the fixed inspection cost Rs. 200 and Rs. 2 for the unit inspection cost. You have to suggest the retailer the optimal ordering quantity, optimal shortage level and the optimal total inventory cost.

Solution:

Given that,

$\widetilde{D} = (8400{,}8800920096\,00)$, $C_1 = Rs.\,30/item/year$, $\widetilde{C}_2 = (14{,}18{,}22{,}26)$,

$C_3 = Rs.500/setup\, k = 3000 units/year$, $E = Rs.\,300$,
$e = Rs.\,5/each\,defective\,items$, $r = 5\% = 0.05$.

i. The Optimal Ordering Quantity

$$q^{*} = \frac{1}{1-r}\sqrt{\frac{2(C_3+E)\widetilde{D}}{C_1}\left(\frac{C_1+\widetilde{C}_2}{\widetilde{C}_2}\right)}$$

$$= \frac{1}{1-0.05}\sqrt{\frac{2(150+200)(8400,8800,9200,9600)}{30}}\sqrt{\frac{30+(14,18,22,26)}{(14,18,22,26)}}$$

$$= (606.24,\ 704.56,\ 828.94,\ 996.39)$$

The defuzzified value is $\frac{2(a_2+a_3)+a_4}{6}$

$$\frac{606.24+2(704.56+828.94)+996.39}{6} = 778.27.$$

ii. The optimal shortage level

$$q_2^{*} = \frac{\widetilde{C}_2 q^{*}}{C_1+\widetilde{C}_2} = \frac{1}{1-r}\sqrt{\frac{2(C_3+E)\widetilde{D}}{C_1}\left(\frac{\widetilde{C}_2}{C_1+\widetilde{C}_2}\right)}$$

$$= \frac{1}{1-0.05}\sqrt{\frac{2(150+200)(8400,8800,9200,9600)}{30}}\sqrt{\frac{(14,18,22,26)}{30+(14,18,22,26)}}$$

$$= (233.01,280.63,330.18,382.97)$$

The defuzzified value is $\frac{a_1+2(a_2+a_3)+a_4}{6}$

$$\frac{233.01+2(280.63+330.18)+382.97}{6} = 306.27.$$

iii. The optimal total inventory cost

$$TC(q^{*}) = \sqrt{2(C_3+E)\widetilde{D}\left(\frac{C_1\widetilde{C}_2}{C_1+\widetilde{C}_2}\right)} + \frac{\widetilde{D}e}{1-r}$$

$$= \sqrt{2(150+200)(8400,8800,9200,9600)(30)\left(\frac{(14,18,22,26)}{30+(14,18,22,26)}\right)}$$

$$+ \frac{(8400,8800,9200,9600)2}{1-0.05}$$

$$= (2432.94,26524.40,28778.52,31125.07).$$

The defuzzified value is $\frac{a_1 + 2(a_2 + a_3) + a_4}{6}$

$$\frac{2432.94 + 2(26524.40,28778.52) + 31125.07}{6} = 27625.975.$$

4.2. Problem on EOQ with shortages and defects:

Consider the Inventory problem with demand rate 24000 units per year and the production rate is 30000 units per year and the setup cost is Rs. 500 per setup the holding cost is Rs. 2 per unit per year and the shortage cost is Rs. 30 per unit per year. Also consider the defects are involved, provided the fixed inspection cost is Rs.300 and unit inspection cost is Rs. 5 per item. Assuming that, 5% of defects are there. Find the optimal lot size and the optimal total cost per year.

Solution:

Given that,

$$\widetilde{D} = (21000,23000,25000,27000),\ C_1 = Rs.\,2/item/year,\ \widetilde{C}_2 = (24,28,32,36),$$

$$C_3 = Rs.\,500/seup,\ k = 30000\,unit/year,\ E = Rs.\,300,$$

$$e = Rs.\,5/each\ defective\ items,\ r = 5\% = 0.05.$$

i. The Optimal Ordering Quantity

$$q^* = \frac{1}{1-r}\sqrt{\frac{2(C_3 + E)\widetilde{D}}{C_1\rho}\left(\frac{C_1 + \widetilde{C}_2}{\widetilde{C}_2}\right)}$$

$$= \frac{1}{1-0.05}\sqrt{\frac{2(500+300)(21000,\ 23000,\ 25000,\ 27000)}{2(0.3,\ 0.2333,\ 0.1667,\ 0.1)}}\sqrt{\frac{2+(24,28,32,36)}{(24,28,32,36)}}$$

$$= (11594.71,\ 10707.87,\ 10739.82,\ 11238.91)$$

The defuzzified value is $\frac{a_1 + 2(a_2 + a_3) + a_4}{6}$

$$= \frac{11594.71 + 2(10707.87 + 10739.82) + 11238.91}{6} = 10954.83.$$

ii. The optimal total inventory cost

$$TC(q^*) = \sqrt{2(C_3 + E)\tilde{D}\left(\frac{C_1\tilde{C}_2\rho}{C_1 + \tilde{C}_2}\right)} + \frac{\tilde{D}}{1 - r}$$

$$= \sqrt{\begin{array}{l} 2(500 + 300)(21000,\ 23000,\ 25000,\ 27000)(2)\left(\frac{(24,28,32,36)}{2 + (24,28,32,36)}\right) \\ (0.3,\ 0.2333,\ 0.1667,\ 0.1) + \frac{(21000,\ 23000,\ 25000,\ 27000)}{1 - 0.05} \end{array}}$$

$$= (121810.404, 132943.896, 143506.025, 153042.784).$$

The defuzzified value is $\frac{a_1 + 2(a_2 + a_3) + a_4}{6}$.

$$\frac{121810.404 + 2(132943.896 + 143506.025) + 153042784}{6} = 137958.838.$$

5. Conclusion

In this paper, we offer both basic inequalities arithmetic geometric mean inequality and Cauchy-Bunyakovsky-Schwarz inequality, to improve Huang's [10] algebraic procedure to find the optimal ordering quantity and the optimal production quantity with shortages and defective items. Using this improved approach presented in this paper, we can find the optimal ordering quantity and allowable optimal short a ges level without using any derivatives and calculus functions. This should also mean that the simple CBS and AGM methods are more accessible to ease the learning of basic theories for younger students who lack the knowledge of calculus.

References

[1] R.E. Bellman and L. A. Zadeh, Decision making in a fuzzy environment, Management Science 17 (1970), 141-164.

[2] L. E. Cardenas-Borran, A Simple method to compute economic order quantities: some observations, Applied Mathematical Modelling 34 (2010), 1684-1688.

[3] L. E. Cardenas-Barron, An easy method to derive EOQ and EPQ inventory models with backorders, Computer and Mathematics with Applications 59 (2010), 948-952.

[4] L. E. Cardenas-Barron, The economic production quantity (EPQ) with shortage derived algebraically, Int. J. Product. Econom. 70 (2001), 289-292.

[5] S. K. J. Chang, J.P.C. Chuang and H.J. Chen, Short comments on technical note-The EOQ and EPQ models with shortages derived without derivatives, Int. J. Product. Econom. 97 (2005), 241-243.

[6] S.H. Chen, Operations on fuzzy numbers with function principle, Tamkang Journal of Management Sciences 6(1) (1985), 13-26.

[7] D. Dubois and H. Prade, Operations of Fuzzy Number's, Internat. J. Systems Sci. 9(6) (1978), 613-626.

[8] R. W. Grubbstrom and A. Erdem, The EOQ with backlogging derived without derivatives, Int. J. Product. Econom. 59 (1999), 529-530.

[9] R. W. Grubbstrom, Material requirements planning and manufacturing resource planning, International Encyclopedia of Business and Management, Routledge, London, 1996.

[10] Y. F. Huang, The deterministic inventory models with shortage and defective items derived without derivatives, Journal of Statistical Management Systems 6 (2003), 171-180.

[11] S. Minner, A note on how to compute economic order quantity without derivatives by cost comparisons, International Journal of Production Economics 105 (2007), 293-296.

[12] A. Nagoor Gani and U. Mohammed Rafi, The Arithmetic Geometric Mean (AGM) inequality approach to compute EOQ/EPQ under fuzzy environment, International Journal of Pure and Applied Mathematics 118(6) (2018), 361-370.

[13] A. Nagoor Gani and U. Mohammed Rafi, Multi inequality approach to evaluate EOQ and EPQ inventory models with shortages in fuzzy nature, American International Journal of Research in Science, Technology, Engineering and Mathematics.

[14] A. Nagoor Gani and U. Mohammed Rafi, A simplistic method to work out the EOQ/EPQ with shortages by applying algebraic method and Arithmetic Geometric Mean inequality in fuzzy atmosphere, Bulltin of Pure and Applied Sciences 38E(1) (Math & Stat.) (2019), 348-355.

[15] R. Ronald, G. K. Yang and P. Chu, The EOQ and EPQ models with shortages derived without derivatives, Int. J. Product. Econom. 92 (2004), 197-200.

[16] J. T. Teng, A Simple method to compute economic order quantities, European Journal of Operational Research 194 (2009), 351-353.

[17] Y. C. Tu, Y. F. Huang, W. K. Chen and H. F. Chen, Using simple method to derive EOQ and EPQ models with shortage and imperfect quality, Journal of Statistical Management Systems 32 (2011), 1333-1340.

[18] H. M. Wee, W. T. Wang and C. J. Chung, A modified method to computer economic order quantities without derivatives by cost-difference comparisons, European Journal of Operational Research 194 (2009), 336-338.