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PELL LABELING AND MEAN SQUARE SUM LABELING FOR THE EXTENDED DUPLICATE GRAPH OF QUADRILATERAL SNAKE

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Abstract

In this paper, we investigate the extended duplicate graph of quadrilateral snake graph admits Pell labeling and mean square sum labeling.

1. Introduction

Most graph labeling methods trace their origin to one introduced by Rosa [2] in 1967. E. Sampathkumar [3] introduced the concept of duplicate graph. Shaima [5] introduced a new labeling technique called the Pell labeling. The concept of mean square sum labeling was introduced by C. Jayasekaran, S. Robinson Chellathurai and M. Jaslin Melbha [6] and they investigated the mean square sum labeling of several standard graphs. Thirusangu et al. [4], have introduced the concept of extended duplicate graph.

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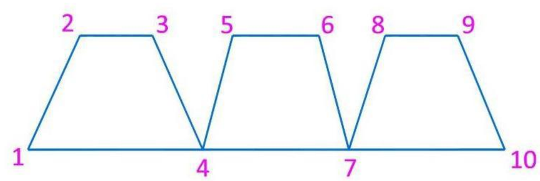
2. Preliminaries

We give the following definitions and examples which are useful for this paper.

Definition 2.1. Quadrilateral snake graph:

A quadrilateral snake QS_m is obtained from a path $u_1, u_2, u_3, \cdots u_n$ by joining u_i and u_{i+1} to two new vertex v_i and w_i respectively and then joining v_i and w_i, $1 \leq i \leq n-1$, where 'm' is the number of edges of the path. In general, a quadrilateral snake has $3m+1$ vertices and 4m edges.

QUADRILATERAL SNAKE GRAPH



Definition 2.2. Duplicate graph:

Let G be a simple graph with vertex set V and edge set E. The duplicate graph of G is $DG = (V_1, E_1)$ where the vertex set V_1 is the union of V and V' and the intersection of V and V' is ϕ and $f : V \to V'$ is bijective. The edge set E_1 of DG is defined as the edge $ab \in E$ if and only if both edges ab' and $a'b$ are in E_1.

Definition 2.3. Extended duplicate graph of Quadrilateral snake:

Let $DG = (V_1, E_1)$ be a duplicate graph of the quadrilateral snake graph $G(V, E)$ Extended duplicate graph of quadrilateral snake graph is obtained by adding the edge $v_2 v_2'$ to the duplicate graph and it is denoted by $EDG(QS_m)$. Clearly it has $6m+2$ vertices and $8m+1$ edges, where 'm' is the number of edges.

Definition 2.4. Pell labeling:

Let G be a graph with p vertices. If there exist a mapping

$f : V(G) \rightarrow \{0, 1, 2, 3, \cdots, p-1\}$ such that the induced function $f^* : E(G) \rightarrow N$ given by $f^*(uv) = f(u) + 2f(v)$, for every $uv \in E(G)$ are all distinct, where $u, v \geq 0$, then the function f is called a Pell labeling. A graph which admits Pell labeling is called Pell graph.

Definition 2.5. Mean square sum labeling:

Let $G(V, E)$ be a graph. A bijection $f : V \rightarrow \{0, 1, 2, 3, \ldots, p-1\}$. Then G is said to be a mean square sum labeling if the induced function $f^* : E(G) \rightarrow N$ given by $f^*(uv) = [(f(u))^2 + (f(v))^2]/2$, for every $uv \in E(G)$ is injective.

3. Main Results

3.1. Pell Labeling

In this section we present an algorithm and prove the existence of Pell labeling for the EDG of quadrilateral snake QS_m, $m \geq 1$.

Algorithm: 3.1.1.

Procedure: (Pell labeling for $EDG(QS_m)$, $m \geq 1$)

$$V \leftarrow \{v_1, v_2, \ldots, v_{3m}, v_{3m+1}, v'_1, v'_2, \ldots, v'_{3m}, v'_{3m+1}\}$$

$$E \leftarrow \{e_1, e_2, \ldots, e_{4m}, e_{4m+1}, e'_1 e'_2, \ldots, e'_{4m}\}$$

$v_1 \leftarrow 0, v'_1 \leftarrow 1$

for $i = 0$ to $(m-1)/2$

for $j = 0$ to 1

$v_{2+6i+2j} \leftarrow 3 + 4j + 12i$

$v'_{2+6i+2j} \leftarrow 2 + 4j + 12i$

end for

end for

for $i = 0$ to $(m-2)/2$

for $j = 0$ to 1

$v_{5+6i+2j} \leftarrow 8 + 12i + 4j$

$v'_{5+6i+2j} \leftarrow 9 + 12i + 4j$

end for

end for

for $i = 0$ to $(m-1)/2$

$v_{3+6i} \leftarrow 4 + 12i$

$v'_{3+6i} \leftarrow 5 + 12i$

end for

for $i = 0$ to $(m-2)/2$

$v_{6+6i} \leftarrow 11 + 12i$

$v_{6+6i} \leftarrow 10 + 12i$

end for

end procedure

Theorem 3.1.2. *The extended duplicate graph of quadrilateral snake* QS_m, $m \geq 1$ *admits Pell labeling.*

Proof. Let QS_m, be the quadrilateral snake graph. Let $EDG(QS_m)$ be the extended duplicate graph of quadrilateral snake and has $6m+2$ vertices and $8m+1$ edges.

Using the algorithm 3.1.1, the vertices function $f : G \rightarrow \{0, 1, 2, 3, \ldots (p-1)\}$ as follows:

The vertices v_1 receive the label 0 and v_1' receive the label 1;

For $0 \leq i \leq (m-1)/2$ and $0 \leq j \leq 1$, the vertices $v_{2+2j+6i}$ receive the label $3 + 4j + 12i$ and $v'_{2+2j+6i}$ the vertices $v'_{2+2j+6i}$ receive the label $2 + 4j + 12$;

For $0 \leq i \leq (m-2)/2$ and $0 \leq j \leq 1$, the vertices $v_{5+2j+6i}$ receive the label $8 + 4j + 12i$ and the vertices $v'_{5+2j+6i}$ receive the label $9 + 4j + 12i$;

For $0 \leq i \leq (m-1)/2$ the vertices v_{3+6i} receive the label $4 + 12i$ and the vertices v'_{3+6i} receive the label $5 + 12i$;

For $0 \leq i \leq (m-2)/2$ the vertices v_{6+6i} receive the label $11 + 12i$ and the vertices v'_{6+6i} receive the label $10 + 12i$.

Thus, the entire $6m + 2$ vertices are labeled.

To obtain the labels for edges, we define the induced function $f^*(uv) = f(u) + 2f(v)$, for every $uv \in E(G)$ is bijective.

The induced function yields the label 7 for the edge e_{3m+1};

For $0 \leq i \leq (m-1)/2$ and $0 \leq j \leq 1$, the edges $e_{1+3j+8i}$ receive the label $4 + 12j + 36i$; the edges $e'_{1+3j+8i}$ receive the label $5 + 12j + 36i$ the edges e_{2+j+8i} receive the label $12 + j + 36i$ and the edges e'_{2+j+8i} receive the label $9 - j + 36i$;

For $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$, the edges $e_{5+3j+8i}$ receive the label $25 + 12j + 36i$; the edges e_{6+j+8i} receive label $33 - 5j + 36i$; the edges $e'_{5+3j+8i}$ receive label $20 + 12j + 36i$ and the edges e'_{6+j+8i} receive label $24 + 5j + 36i$.

Thus the entire $8m + 1$ edges are labeled and satisfied the required condition.

Hence the extended duplicate graph of quadrilateral snake graph QS_m, $m \geq 1$ admits Pell labeling.

Example 3.1.3. A graph and its Pell labeling is shown in figure 1 and figure 2

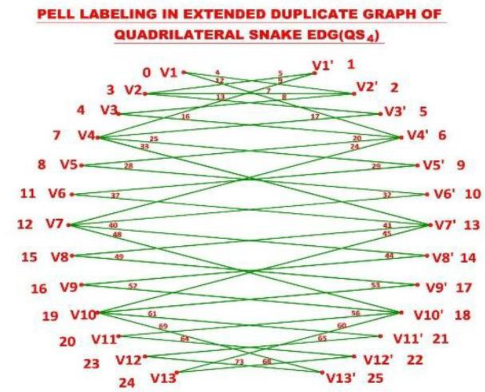


FIG.1:EDG(QS_4)

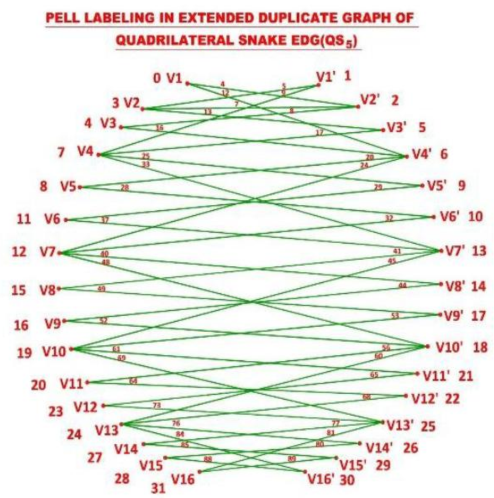


FIG.2:EDG(QS_5)

3.2. Mean Square Sum Labeling

Here we present an algorithm and prove the existence of mean Square sum labeling for the EDG of Quadrilateral snake QS_m, $m \geq 1$.

Algorithm 3.2.1.

Procedure: (Mean square sum labeling for $EDG(QS_m)$, $m \geq 1$)

$$V \leftarrow \{v_1, v_2, \ldots, v_{3m}, v_{3m+1}, v'_1, v'_2, \ldots, v'_{3m}, v'_{3m+1}\}$$

$$E \leftarrow \{e_1, e_2, \ldots, e_{4m}, e_{4m+1}, e'_1, e'_2, \ldots, e'_{4m}\}$$

$v_1 \leftarrow 0$, $v'_1 \leftarrow 1$

for $i = 0$ to $(m-1)/2$

for $j = 0$ to 1

$v_{2+6i+2j} \leftarrow 3 + 4j + 12i$

$v'_{2+6i+2j} \leftarrow 2 + 4j + 12i$

end for

end for

for $i = 0$ to $(m-2)/2$

for $j = 0$ to 1

$v_{5+6i+2j} \leftarrow 8 + 12i + 4j$

$v'_{5+6i+2j} \leftarrow 9 + 12i + 4j$

end for

end for

for $i = 0$ to $(m-1)/2$

$v_{3+6i} \leftarrow 4 + 12i$

$v'_{3+6i} \leftarrow 5 + 12i$

end for

for $i = 0$ to $(m-2)/2$

$v_{6+6i} \leftarrow 11 + 12i$

$v_{6+6i} \leftarrow 10 + 12i$

end for

end procedure

Theorem 3.2.2. *The extended duplicate graph of quadrilateral snake* QS_m, $m \geq 1$ *admits mean square sum labeling.*

Proof. Let QS_m, be the quadrilateral snake graph. Let $EDG(QS_m)$ be the extended duplicate graph of quadrilateral snake and has $6m+2$ vertices and $8m+1$ edges.

Defining the set of vertices and edges are

$$V(G) = \{v_1, v_2, v_3, \dots, v_{3m}, v_{3m+1}, v'_1, v'_2, \dots, v'_{3m}, v'_{3m+1}\}$$

$$E(G) = \{e_1, e_2, e_3, \dots, e_{4m}, e_{4m+1}, e'_1 e'_2, \dots, e'_{4m}\}.$$

Using the algorithm 3.2.1, the vertices function $f : G \to \{0, 1, 2, 3, \dots, (p-1)\}$ as follows: The vertices v_1 receive the label 0 and v'_1 receive the label 1;

For $0 \le i \le (m-1)/2$ and $0 \le j \le 1$, the vertices $v_{2+2j+6i}$ receive the label $3+4j+12i$ and the vertices $v'_{2+2j+6i}$ receive the label $2+4j+12i$;

For $0 \le i \le (m-2)/2$ and $0 \le j \le 1$, the vertices $v_{5+2j+6i}$ receive the label $8+4j+12i$ and the vertices $v'_{5+2j+6i}$ receive the label $9+4j+12i$;

For $0 \le i \le (m-1)/2$ the vertices v_{3+6i} receive the label $4+12i$ and the vertices v'_{3+6i} receive the label $5+12i$;

For $0 \le i \le (m-2)/2$ the vertices v_{6+6i} receive the label $11+12i$ and the vertices v'_{6+6i} receive the label $10+12i$.

Thus, the entire $6m+2$ vertices are labeled.

To obtain the labels for edges, we define the induced function $f^* : E(G) \to N$ given by $f^*(uv) = ([f(u)]^2 + [f(v)]^2)/2$, for every $uv \in E(G)$ is bijective.

The induced function yields the label 7 for the edge e_{4m+1};

(i) $f^*(v_i, v'_{i+1}) = 2(2i^2 - 2i + 1)$ *for* $i = 1, 3, 5 \cdots (2n-1)$, $n \in N$.

(ii) $f^*(v'_i, v_{i+1}) = 2(2i^2 - 2i + 1)$ *for* $i = 2, 4, 6 \cdots (2n)$, $n \in N$.

(iii) $f^*(v'_i, v_{i+1}) = 4i^2 + 1$ *for* $i = 1, 3, 5, \dots, (2n-1)$, $n \in N$.

(iv) $f^*(v_i, v'_{i+1}) = 4i^2 + 1 \quad for\ i = 2, 4, 6 \ldots (2n), n \in N.$

(v) $f^*(v'_{3i\text{-}2}, v_{3i+1}) = 18(2i^2 - 2\text{i} + 1) \quad for\ i = 1, 3, 5 \cdots (2n-1), n \in N.$

(vi) $f^*(v'_{3i-2}, v_{3i+1}) = 18(2i^2 - 2\text{i} + 1) \quad for\ i = 2, 4, 6, 8 \ldots (2n), n \in N.$

(vii) $f^*(v'_{3i-2}, v_{3i+1}) = 36i^2 - 24i + 13 \quad for\ i = 1, 3, 5, 7 \ldots (2n-1), n \in N.$

(vii) $f^*(v_{3i-2}, v'_{3i+1}) = 36i^2 - 24i + 13 \quad for\ i = 2, 4, 6 \cdots 2n, n \in N.$

Thus the entire $8m + 1$ edges are labeled and satisfied the required condition.

Hence the extended duplicate graph of quadrilateral snake graph $QS_m, m \geq 1$ admits mean square sum labeling.

Example 3.2.3. A graph and its Mean square sum labeling is shown in figure 3.

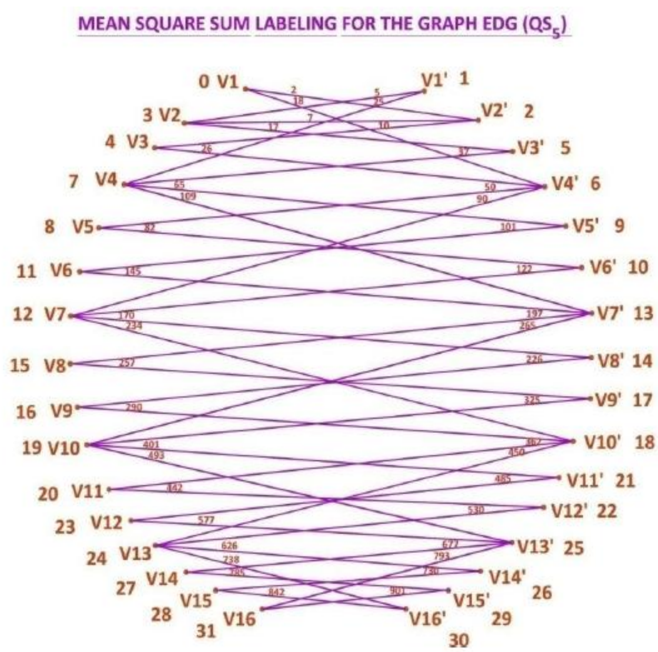


Figure 3.

4. Conclusion

In this paper, we have presented algorithms and investigate the extended duplicate graph of quadrilateral snake graph $(QS_m, m \geq 1)$ admits Pell labeling and mean square sum labeling.

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