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SQUARE DIFFERENCE AND CUBE DIFFERENCE LABELING OF EXTENDED KUSUDAMA FLOWER GRAPH

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Abstract

Let G be a graph with p vertices and q edges. A square difference labeling of a graph G is bijection $f : V(G) \to \{0, 1, 2, \ldots, (p-1)\}$, such that the induced function $f^* : E(G) \to N$ defined by $f^*(uv) = | [f(u)]^2 - [f(v)]^2 |$ for every edge $uv \in E(G)$ are all distinct. A graph which admits square difference labeling is called a square difference labeling graph. In a graph G, the cube difference labeling is bijection $f : V(G) \to \{0, 1, 2, \ldots, (p-1)\}$, such that the incite function $f^* : E(G) \to N$ defined by $f^*(uv) = | [f(u)]^3 - [f(v)]^3 |$ for every distinct edge $uv \in E(G)$ in a graph G. A graph G which admits cube difference labeling is called a cube difference labeling graph. In this paper we analyze square difference and cube difference labeling for extended Kusudama flower graph.

I. Introduction

Graph labeling is the assignment of integers to the vertices and edges or both of a graph subject to certain conditions. The origin of graph labeling can be attributed to Rosa [2]. Graphs serves as mathematical models of many concrete real-world problems and it can be used to represent any physical situations involving discrete objects and relationships among them. In this paper, we consider only simple, finite, undirected and non-trivial graph

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$G = (V(G), E(G))$ with the vertex set $V(G)$ and the edge set $E(G)$. Gallian [1] regularly updates a dynamic survey on graph labeling and it is published by Electronic Journal of Combinatory. The square difference labeling and cube difference labeling was first introduced by J. Shiama. The square and cube difference labeling for cycle cactus, special tree and a new key graphs was proved by V. Sharon Philomena and K. Thirusangu [3]. J. Shiama [4] has obtained square difference labeling for many standard graphs like complete graphs, ladder, lattice grids and quadrilateral snakes. J. Shiama [5] has proved that cube difference labeling for some graphs like cycles, fan graphs, wheel graphs, crown graphs, helm graphs, dragon graph and shell graphs.

We define a graph as a binary relation on a set of objects. The study of graphs has recently emerged as one of the most important areas of research. Labeled graphs serve as useful models for a broad range of applications such as coding theory, circuit design, mobile telecommunication, medical field and database management. The application of square difference labeling is automatic channel allocation and used to represent global data structure.

Definition 1.1. The *Helm graph* H_n is a graph obtained from a wheel graph by attaching a pendant edge at each vertex of the n-cycle.

Definition 1.2. A *flower graph* F_n is the graph obtained from a helm graph by joining each pendant vertex to the central vertex the graph H_n.

Definition 1.3. Let v_0 be the apex vertex and $v_1, v_2, v_3, \ldots, v_{2n-1}, v_{2n}$ be consecutive $2n$ rim vertices of wheel graph $w_{2n}, n \geq 3$. Subdivide spoke edge $v_0 v_{2i-1}$ with vertex w_i and at each w_i, join two copies of path of length $2; P_2^l = v_0, u_{2i-1}, w_i$ and $P_2^r = v_0, u_{2i}, w_i$, for each $i \in [n]$. The resulting graph is called *Kusudama flower graph* KF_n.

II. Main Result

Theorem 2.1. *The extended Kusudama flower graph*, $KF_n n \geq 3$ *admits square difference labeling*.

Proof. Let $KF_n n \geq 3$ extended Kusudama flower graph be formed with n copies of F_n. Let p and q be the number of vertices and the number of edges in the Kusudama flower graph KF_n respectively. The graph KF_n is described as follows: Denote the vertices of Kusudama flower graph KF_n as $\{v_0, v_1, v_2, \dots v_{2n}, u, u_1, u_2, \dots, u_{nn}\}$. The edges between the vertices v_i and v_{n+i} are represented as E_1 for $1 \leq i \leq n$. The edges lies between the vertices v_{n+i} and v_{n+1} are denoted as E_2 for $1 \leq i \leq n-1$. The edge between the vertex v_i and v_1 are represented as E_3. The edges between the vertices v_0 and v_{n+i} are expressed as E_4 for $1 \leq i \leq n$. The edges between the vertices v_0 and u_i are mentioned as E_5 for $1 \leq i \leq n^2$. The edges between the vertices v_i and $u_{ni\text{-}(n\text{-}1)}$ are defined as E_6 for $1 \leq i \leq n$. The edges between the vertices v_i and $u_{ni\text{-}(n\text{-}j)}$ are represented as E_7 for $1 \leq i \leq n$, $2 \leq j \leq n-1$. The edges between the vertices v_i and u_{ni} are denoted as E_8 for $1 \leq i \leq n$. The edges between the vertices u_i and u_j are denoted as E_9 for $1 \leq i \leq n^2 - 1$, $2 \leq j \leq n^2$.

The total number of vertices in Kusudama flower graph $KF_n = n(n+2)+1$.

The total number of edges in Kusudama flower graph $KF_n = n(3n+2)$.

Without loss of generality we initiate the labeling from the apex vertex v_0 and proceed in the clockwise direction.

Define the vertex labels as follows,

Let $V(KF_n) = \{v_0\} \cup \{v_i : 1 \leq i \leq 2n\} \cup \{u_i : 1 \leq i \leq n^2\}$

Define the edge labels as follows,

Let

$$E(KF_n) = \{v_i v_{n+i} : 1 \leq i \leq n\} \cup \{v_{n+i} v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_n v_1\}$$

$$\cup \{v_0 u_i : 1 \leq i \leq n^2\} \cup \{v_i u_j / 1 \leq i \leq n, 1 \leq j \leq n^2\}$$

$$\cup \{u_i u_j / 1 \leq i \leq n^2 - 1, 2 \leq j \leq n^2\}.$$

The vertex labeling for the graph Kusudama flower graph KF_n is defined as follows,

$f(v_0) = n(n + 2).$

$f(v_i) = 2i - 2 \quad 1 \leq i \leq n.$

$f(v_{n+i}) = 2i - 1 \quad 1 \leq i \leq n.$

$f(u_i) = i - 1 + 2n \quad 1 \leq i \leq n^2.$

We know clearly the vertex labels are distinct.

The edge labels are derived as follows, f is called square difference labeling if $f^*(uv) = | [f(u)]^2 - [f(v)]^2 |$ for every distinct edge $uv \in E(G)$ in a graph G, where $u, v \geq 0$.

The Kusudama flower graph KF_n is shown in figure 1.

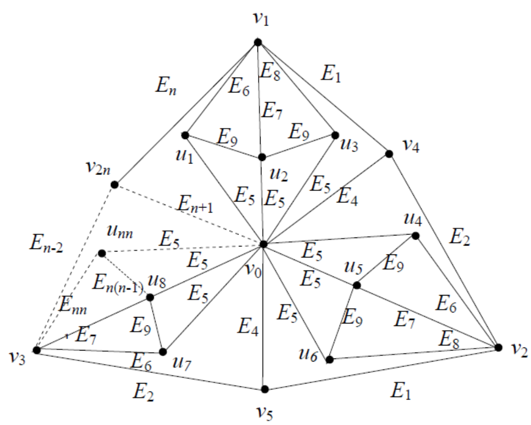

Figure 1. Kusudama flower graph KF_n.

Define the edge labels be

$E_1 = \{v_i v_{n+i} : 1 \leq i \leq n\}$

$E_2 = \{v_{n-i} v_{i+1} : 1 \leq i \leq n - 1\}$

$E_3 = \{v_n v_1\}$

$E_4 = \{v_0 v_{n+i} / 1 \leq i \leq n\}$

$E_5 = \{v_i v_{n+i} / 1 \leq i \leq n^2\}$

$E_6 = \{v_i u_{ni(n-1)} / 1 \leq i \leq n\}$

$E_7 = \{v_i u_{ni(n-j)} / 1 \leq i \leq n, 2 \leq j \leq n-1\}$

$E_8 = \{v_i u_{ni} / 1 \leq i \leq n\}$

$E_9 = \{u_i u / 1 \leq i \leq n^2 - 1, 2 \leq j \leq n^2\}$

Compute the edge labels of Kusudama flower graph KF_n as follows,

$f^*(v_i v_{n+i}) = |\, [f(v_i)]^2 - [f(v_{n+i})]^2 \,| = 4i - 3 \;\; 1 \leq i \leq n.$

$f^*(v_{n+i} v_{i+1}) = |\, [f(v_{n+i})]^2 - [f(v_{n+i})]^2 \,| = 4i - 1 \;\; 1 \leq i \leq n-1$

$f^*(v_n v_1) = |\, [f(v_n)]^2 - [f(v_1)]^2 \,| = (2n-1)^2$

$f^*(v_0 v_{n+i}) = |\, [f(v_0)]^2 - [f(v_{n+i})]^2 \,| = [n(n+2)]^2 - [2i-1]^2 \;\; 1 \leq i \leq n.$

$f^*(v_0 u_{ni-(n-1)}) = |\, [f(v_0)]^2 - [f(u_i)]^2 \,| = [n(n+2)]^2 - [2i-2]^2 \;\; 1 \leq i \leq n$

$f^*(v_i u_{ni-(n-j)}) = |\, [f(v_i)]^2 - [f(u_{ni-(n-j)})]^2 \,| = [(ni+n+1)]^2$

$- [2i-2]^2 \;\; 1 \leq i \leq n, 2 \leq j \leq n-1$

$f^*(v_i u_{ni}) = |\, [f(v_i)]^2 - [f(u_{ni})]^2 \,| = [(ni+2n-1)]^2 - [2i-2]^2 \;\; 1 \leq i \leq n$

$f^*(v_i u_j) = |\, [f(u_i)]^2 - [f(u_j)]^2 \,| = (4n-2) + i + j \;\; 1 \leq i \leq n^2 - 1, 2 \leq j \leq n^2.$

The edge labels of the graph Kusudama flower graph KF_n are distinct. Hence *f* admits square difference labeling.

Therefore Kusudama flower graph KF_n is a square difference graph.

Theorem 2.2. *The extended Kusudama flower graph*, $KF_n n \geq 3$ *admits cube difference labeling*.

Proof. Let $KF_n n \geq 3$ extended Kusudama flower graph be a formed with n copies of F_n. Let p and q be the number of vertices and the number of edges in the Kusudama flower graph KF_n respectively. The graph KF_n is described as follows: Denote the vertices of Kusudama flower graph KF_n as $\{v_0, v_1, v_2, \ldots, v_{2n}, u, u_1, u_2, \ldots, u_{nn}\}$. The edges between the vertices v_i and v_{n+i} are represented as E_1 for $1 \leq i \leq n$. The edges between the vertices v_{n+i} and v_{n+1} are denoted as E_2 for $1 \leq i \leq n$. The edge between the vertex v_n and v_1 are expressed as E_3 for $1 \leq i \leq n$. The edges between the vertices v_0 and v_{n+i} are mentioned as E_4 for $1 \leq i \leq n$. The edges between the vertices v_0 and u_i are defined as E_5 for $1 \leq i \leq n^2$. The edges between the vertices v_0 and u_i are signified as E_6 for $1 \leq i \leq n^2$. The edges between the vertices v_i and $v_{ni-(n-1)}$ are defined as E_7 for $1 \leq i \leq n$, $2 \leq j \leq n-1$. The edges between the vertices v_i and u_{ni} are represented as E_8 for $1 \leq i \leq n$. The edges between the vertices u_i and u_j are denoted as E_9 for $1 \leq i \leq n^2 - 1$, $2 \leq j \leq n^2$.

The total number of vertices in Kusudama flower graph $KF_n = n(n+2)+1$.

The total number of edges in Kusudama flower graph $KF_n = n(n+2)+1$

Without loss of generality we initiate the labeling from the apex vertex v_0 and proceed in the clockwise direction.

The vertex labels are determined as follows,

Let $V(KF_n) = \{v_0\} \cup \{v_i : 1 \leq i \leq 2n\} \cup \{u_i : 1 \leq i \leq n^2\}$

Define the edge labels as follows,

Let

$$E(KF_n) = \{v_i v_{n+i} : 1 \leq i \leq n\} \cup \{v_{n+i} v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_n v_1\}$$

$$\cup \{v_0 v_{n+i} : 1 \leq i \leq n\} \cup \{v_i u_j / 1 \leq i \leq n, 1 \leq j \leq n^2\}$$

$$\cup \{u_i u_i / 1 \leq i \leq n^2 - 1, 2j \leq n^2\}.$$

The vertex labeling for the graph Kusudama flower graph KF_n is defined as follows,

$f(v_0) = n(n+2)$

$f(v_{n+i}) = 2i - 1 \quad 1 \le i \le n.$

$f(u_i) = 2n + i - 1 \quad 1 \le i \le n^2.$

$f(u_i) = 2n + i - 1 \quad 1 \le i \le n^2.$

The vertex labels are distinct.

The edge labels are given as follows, f is called cube difference labeling if $f^*(uv) = |\,[f(u)]^3 - [f(v)]^3\,|$ for every $uv \in E(G)$ are all distinct where $u, v \ge 0$.

The Kusudama flower graph KF_n is shown in figure 2.

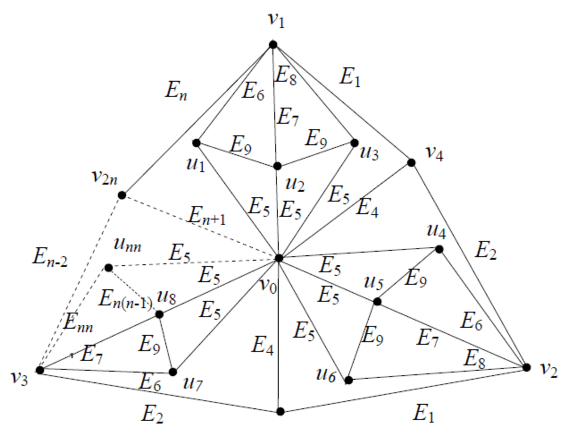

Figure 2. Kusudama flower graph KF.

Define the edge labels be

$E_1 = \{v_i v_{n+1} : 1 \le i \le n\}$

$E_2 = \{v_i v_{n+1} : 1 \le i \le n-1\}$

$E_3 = \{v_n v_1\}$

$E_4 = \{v_0 v_{n+i} / 1 \le i \le n\}$

$E_5 = \{v_0 u_{ni} / 1 \le i \le n^2\}$

$E_6 = \{v_i u_{in-(n-1)}/1 \le i \le n\}$

$E_7 = \{v_i u_{in-(n-j)}/1 \le i \le n,\ 2 \le j \le n-1\}$

$E_8 = \{v_i u_{ni}/1 \le i \le n\}$

$E_9 = \{u_i u_j/1 \le i \le n^2 - 1,\ 2 \le j \le n^2\}$

Compute the edge labels of Kusudama flower graph KF_n as follows,

$f^*(v_i v_{n+i}) = |\ [f(v_1)]^3 - [f(v_{n+i})]^3\ | = (2i-2)^3 - (2i-1)^3\ \ 1 \le i \le n.$

$f^*(v_{n+i} v_{i+1}) = |\ [f(v_{n+i})]^3 - [f(v_{i+1})]^3\ | = (2i-1)^3 - (2i)^3\ \ 1 \le i \le n-1$

$f^*(v_n v_1) = |\ [f(v_n)]^3 - [f(v_1)]^3\ | = (2n-1)^3$

$f^*(v_0 v_{n+i}) = |\ [f(v_0)]^3 - [f(v_{n+i})]^3\ | = [n(n+2)]^3 - [2i-1]^3 1 \le i \le n.$

$f^*(v_0 u_i) = |\ [f(v_0)]^3 - [f(u_i)]^3\ | = [n(n+2)]^3 - [2n+i-1]^3\ \ 1 \le i \le n^2.$

$f^*(v_i u_{ni-(n-1)}) = |\ [f(v_i)]^3 - [f(u_{ni-(n-1)})]^3\ | = [n(i+1)]^3 - [2i-2]^3\ \ 1 \le i \le n$

$f^*(v_i u_{ni-(n-j)}) = |\ [f(v_i)]^3 - [f(u_{i-(n-j)})]^3\ | = [ni+n+1]^3$

$-[2i+2]^3\ \ 1 \le i \le n,\ 2 \le j \le n-1$

$f^*(v_i u_{ni-(n-j)}) = |\ [f(v_i)]^3 - [f(u_{ni})]^3\ | = [ni+n+1]^3 - [2i-2]^3\ \ 1 \le i \le n,$

$f^*(u_i u_j) = |\ [f(u_i)]^3 - [f(u_j)]^3\ | = (4n-2)+i+j\ \ 1 \le i \le n^2 - 1,\ 2 \le j \le n^2$

The edge labels of the graph Kusudama flower graph KF_n are distinct. Hence *f* admits cube difference labeling.

Therefore Kusudama flower graph KF_n is a cube difference graph.

III. Conclusion

It is very fascinating to study graphs which admit square difference and cube difference labeling. In this paper the Kusudama flower graph KF_n

admits square difference and cube difference labeling. Square difference labeling can be applied in areas of network security and channel assignment process. The application of square difference labeling is automatic channel allocation and used to represent global data structure.

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