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FUZZY NEUTROSOPHIC SUPRA TOPOLOGICAL SPACES

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Abstract

In this paper, we extend the concept of fuzzy neutrosophic sets into fuzzy neutrosophic supra sets and we introduce fuzzy neutrosophic supra topological spaces and we define fuzzy neutrosophic supra closure and interior and some of their properties are investigated.

Introduction

Fuzzy sets were introduced by Zadeh [7] in 1965. The concept of intuitionistic fuzzy sets by K. Atanassov several researches were introduced on the generalizations of the notion of intuitionistic fuzzy sets. Florentin Smarandache [4, 5] developed neutrosophic set and logic of a generalization of the intuitionistic fuzzy logic and set respectively. A. A. Salama and S. A. Alblowi [1] introduced and studied neutrosophic topological spaces and its continuous function in [2].

Smarandache introduced the neutrosophic set and neutrosophic components. The sets T, I, F are not necessarily intervals but may be any real sub-unitary subsets of $]^{-}0,1^{+}[$. The neutrosophic components T, I, F represents the truth value, indeterminacy value and falsehood value respectively. In [8], Mashhour et al. introduced the concepts of supra topological spaces, supra open sets and supra closed sets. Later on ME Abd

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El-Monsef et al. [3] introduced the concept of fuzzy supra topological as a natural generalization of the notion of supra topological spaces. In this paper, we define the notion of fuzzy neutrosophic supra topological spaces with some of its properties are investigated.

1. Preliminaries

Definition 1.1 [8]. Let (X, τ) is said to be a supra topological space if it is satisfying these conditions:

1. $\phi, X \in \tau$

2. The union of any number of sets in τ belongs to τ

Each element $D \in \tau$ is called a SOS (supra open set) in (X, τ) and D^C is called a SCS (supra closed set) in (X, τ) [8].

Supra $cl(B) = \cap \{\kappa : \kappa$ is a SCS and $B \subseteq \kappa\}$.

Supra $\operatorname{int}(B) = \cup \{E : E$ is a SOS $B \supseteq E\}$ [8].

Definition 1.2 [6]. Let X be a non-empty set and P is a fuzzy neutrosophic set (FNS) is an object having the form $P = \{\langle y, T_P(y), I_P(y), F_P(y)\rangle : y \in X\}$ where the functions $T_P : X \to]^-0, 1^+[$, $I_P : X \to]^-0, 1^+[, F_P : X \to]^-0, 1^+[$ denote the degree of membership function (namely $T_P(y)$), the degree of indeterminacy function (namely $I_P(y)$), and the degree of non membership (namely $F_P(y)$) respectively of each element $y \in X$ to the set P and $^+0 \leq T_P(y) \leq I_P(y) \leq F_P(y) \leq 1^+$, for each $y \in X$.

Remark 1.3 [6]. Every fuzzy set P on a non-empty set X is obviously a FNS having the form $P = \{\langle y, T_P(y), I_P(y), F_P(y)\rangle : y \in X\}$.

A FNS $P = \{\langle y, T_P(y), I_P(y), F_P(y)\rangle : y \in X\}$ can be identified to an ordered triple $\langle y, T_P, I_P, F_P\rangle$ in $]^-0, 1^+[$ in X.

Definition 1.4 [1]. Let X be a non-empty set and FNSs P and R be in the form $P = \{\langle y, T_P(y), I_P(y), F_P(y)\rangle : y \in X\}$ and

$R = \{\langle y, T_R(y), I_R(y), F_R(y)\rangle : y \in X\}$.

We consider two definitions for subsets $(P \subseteq R)$:

$P \subseteq R \Leftrightarrow T_P(y) \leq T_R(y), I_P(y) \leq I_R(y), F_P(y) \geq F_R(y)$ for all $y \in X$,

$P \subseteq R \Leftrightarrow T_P(y) \leq T_R(y), I_P(y) \geq I_R(y), F_P(y) \geq F_R(y)$ for all $y \in X$.

Definition 1.5 [1]. Let $\{P_j : j \in J\}$ be an arbitrary family of FNSs in X, where $P_j = \{\langle y, T_{P_j}(y), I_{P_j}(y), F_{P_j}(y)\rangle : y \in X\}$ then

1. $\cap P_j$ may be defined as two types:

$\cap P_j = \{\langle y, \wedge_{j\in J}, T_{P_j}(y), \wedge_{j\in J}, I_{P_j}(y), \vee_{j\in J} F_{P_j}(y)\rangle : y \in X\}$

$\cap P_j = \{\langle y, \wedge_{j\in J}, T_{P_j}(y), \wedge_{j\in J}, I_{P_j}(y), \vee_{j\in J} F_{P_j}(y)\rangle : y \in X\}$.

2. $\cup P_j$ may be defined as two types:

$\cup P_j = \{\langle y, \vee_{j\in J}, T_{P_j}(y), \vee_{j\in J}, I_{P_j}(y), \wedge_{j\in J} F_{P_j}(y)\rangle : y \in X\}$

$\cup P_j = \{\langle y, \vee_{j\in J}, T_{P_j}(y), \vee_{j\in J}, I_{P_j}(y), \wedge_{j\in J} F_{P_j}(y)\rangle : y \in X\}$.

Definition 1.6 [1]. Let $P = \{\langle y, T_P(y), I_P(y), F_P(y)\rangle : y \in X\}$ be a FNSs in X, then the complement of the set $P(CO(P))$ may be defined as three kinds of complements.

1. $CO(P) = \{\langle y, F_P(y), I_P(y), T_P(y)\rangle : y \in X\}$

2. $CO(P) = \{\langle y, F_P(y), 1 - I_P(y), T_P(y)\rangle : y \in X\}$

3. $CO(P) = \{\langle y, 1 - T_P(y), 1 - I_P(y), 1 - F_P(y)\rangle : y \in X\}$

Definition 1.7 [1]. Let X be a non-empty set and the FNSs P and R be in the form $P = \{\langle y, T_P(y), I_P(y), F_P(y)\rangle : y \in X\}$ and $R = \{\langle y, T_R(y), I_R(y), F_R(y)\rangle : y \in X\}$ on X

1. $P = R \Leftrightarrow P \subseteq R$ and $R \subseteq P$.

2. $P - R = \{\langle y, T_P(y) \wedge F_R(y), I_P(y) \wedge (1 - I_R(y)), F_P(y) \vee T_R(y)\rangle : y \in X\}$

3. $[\ ]P = \{\langle y, T_P(y), I_P(y), 1 - T_P(y)\rangle : y \in X\}$.

4. $\langle\ \rangle P = \{\langle y, 1 - T_P(y), I_P(y), F_P(y)\rangle : y \in X\}$.

5. $O_N = \langle y, 0, 0, 1\rangle$ and $1_N = \langle y, 1, 1, 0\rangle$.

2. Fuzzy Neutrosophic Supra Topological Spaces

Definition 2.1. A fuzzy neutrosophic supra topology (FNST) a non-empty set X is a family τ^{μ} of fuzzy neutrosophic supra subsets in X satisfying the following axioms.

(a) $0_N, 1_N \in \tau^{\mu}$

(b) $\cup G_i \in \tau^{\mu}, \forall\{G_i : i \in J\} \subseteq \tau^{\mu}$

In this case the pair (X, τ^{μ}) is called a Fuzzy neutrosophic supra topological space (FNSTS) and any fuzzy neutrosophic supra set in τ^{μ} is known as fuzzy neutrosophic supra open set (FNSOS) in X. The element of τ^{μ} are called open fuzzy neutrosophic supra sets.

The complement of FNSOS in the FNSTS (X, τ^{μ}) is called fuzzy neutrosophic supra closed set (FNSCS).

Example 2.2. Let $X = \{y\}$ and consider the family $\tau^{\mu} = \{0_N, 1_N, P, Q, R\}$ where $P = \langle y, 0.3, 0.4, 0.7\rangle$, $Q = \langle y, 0.2, 0.6, 0.8\rangle$, $R = \langle y, 0.3, 0.6, 0.7\rangle$. Then (X, τ^{μ}) is called FNSTS on X.

Definition 2.3. Let $(X, \tau^{\mu})(X, \tau^{\mu}_{*})$ be two FNSTSs on X. Then τ^{μ} is said to be contained in τ^{μ}_{*} (in symbols, $\tau^{\mu} \subseteq \tau^{\mu}_{*}$), if $P \in \tau^{\mu}_{*}$ for each $P \in \tau^{\mu}$. In this case, we also say that τ^{μ} is coarser than τ^{μ}_{*}.

Proposition 2.4. *Let $\{\tau^{\mu}_{j} : j \in J\}$ be a family of FNSTS in X. Then $\cap_{j \in J} \tau^{\mu}_{j}$ is FNST on X. Furthermore, $\cap_{j \in J} \tau^{\mu}_{j}$ is the coarsest FNST on X containing all $\tau^{\mu'}_{j}$s.*

Proof. Obvious.

Definition 2.5. The complement of P (CO(P)) of FNSOS P is called a FNSCS in X.

Example 2.6. Let $X = \{y\}$ and $\tau^{\mu} = \{0_N, 1_N, P\}$ is a FNSTS and $P = \{\langle y, 0.3, 0.5, 0.7\rangle\}$ is a FNSOS in X.

We know that any number of τ^{μ} is called FNSOS in X if $P \in \tau^{\mu}$ [1].

Then CO(G) is called FNSCS in X [1]

Then $CO(P) = \{\langle y, 0.7, 0.5, 0.3\rangle\}$

$\therefore$ CO(P) is FNSCS in X.

Now, we define FNSI and FNSC operations in FNSTS.

Definition 2.7. Let (X, τ^{μ}) be FNSTS and $P = \langle y, T_P, I_P, F_P\rangle$ be a FNSS in X. Then the fuzzy neutrosophic supra interior (FNSI) and fuzzy neutrosophic supra closure (FNSC) of P are defined by $FNScl(P) = \cap\{Q : Q$ is a FNSCS in X and $P \subseteq Q\}$,

$FNS\ \text{int}(P) = \cup\{R : R$ is a FNSOS in X and $R \subseteq P\}$.

Now that $cl(P)$ is a FNSCS & $\text{int}(P)$ is a FNSOS in X, Further,

1. P is a FNSCS in X iff $cl(P) = P$

2. P is a FNSOS in X iff $\text{int}(P) = P$

Example 2.8. Let $X = \{y\}$ and $\tau^{\mu} = \{0_N, 1_N, P, R, G\}$ is a FNSTS Where $P = \{\langle y, 0.3, 0.5, 0.8\rangle\}$ and $R = \{y, \langle 0.2, 0.5, 0.8\rangle\}$.

Let $G = \{\langle y, 0.4, 0.5, 0.6\rangle\}$ is a FNSOS in X.

Then $FNS\ \text{int}(G) = \cup\{P, R : P, R$ are FNSOSs in X and $P \subset G, R \subset G\}$.

$FNS\ \text{int}(G) = \{y, \langle 0.3, 0.5, 0.8\rangle\} = P$.

$\therefore P$ is FNSI of G.

Here FNSCSs are complement of FNSOSs

And $CO(P) = \{y, \langle 0.7, 0.5, 0.2\rangle\}$ and $CO(R) = \{y, \langle 0.8, 0.5, 0.2\rangle\}$ is FNSCSs.

Then $FNScl(G) = \cap\{CO(P), CO(R) : CO(P), CO(R)$ are FNSCSs in X and $G \subset CO(P), G \subset CO(R)\}$

$FNScl(G) = \{y, \langle 0.7, 0.5, 0.2\rangle\} = CO(P)$.

$\therefore$ $CO(P)$ is FNSC of G.

Proposition 2.9. *Let (X, τ^{μ}) be a FNSTS over X. Then the following properties hold.*

1. $FNScl(CO(P)) = CO(FNS \text{ int}(P))$

2. $FNS \text{ int}(CO(P)) = CO(FNScl(P))$.

Proof. Obvious.

Proposition 2.10. *Let P and Q be two fuzzy neutrosophic supra sets in X then the following properties hold.*

1. $P \cup Q = P$ *iff* $Q \subseteq P$

2. $P \subseteq Q \Leftrightarrow Q^C \subseteq P^C$

3. $(P^C)^C = P$

4. $P \cup 0_N = P, P \cup 1_N = 1_N \; P \cap 0_N = 0_N \; P \cap 1_N = P$

5. $P/Q = Q^C/P^C$.

Proof. Obvious.

Proposition 2.11. *Let A_j, s and B be fuzzy neutrosophic supra sets in X $(j \in J)$ then* (i) $A_j \subseteq B$ *for each* $j \in J \Rightarrow \cup A_j \subseteq B$ *and* (ii) $B \subseteq A_j$ *for each* $j \in J \Rightarrow B \subseteq \cap A_j$.

Proposition 2.12. *Let (X, τ^{μ}) and (X, τ^{μ}_*) be two FNSTS. Denote $\tau^{\mu} \cap \tau^{\mu}_* = \{A : A \in \tau^{\mu}$ and $A \in \tau^{\mu}_*\}$ then $\tau^{\mu} \cap \tau^{\mu}_*$ is a FNSTS.*

Proof. Obviously $0_N, 1_N \in \tau^{\mu} \cap \tau_*^{\mu}$.

Let $A_1, A_2 \in \tau^{\mu} \cap \tau_*^{\mu} \Rightarrow A_1, A_2 \in \tau^{\mu}, A_1, A_2 \in \tau_*^{\mu}$ τ^{μ} and τ_*^{μ} are FNSTS on X.

Then $A_1 \cap A_2 \in \tau^{\mu}$ and $A_1 \cap A_2 \in \tau_*^{\mu} \Rightarrow A_1 \cap A_2 \in \tau^{\mu} \cap \tau_*^{\mu}$.

Let $\{A_j : j \in J\} \subseteq \tau^{\mu} \cap \tau_*^{\mu} \Rightarrow A_j \in \tau^{\mu}$ and $A_j \in \tau_*^{\mu}$ $\forall j \in J$.

Since τ^{μ} and τ_*^{μ} are FNSTS on X.

$\bigcup \{A_j : j \in J\} \in \tau^{\mu}$ and $\bigcup \{A_j : j \in J\} \in \tau_*^{\mu} \Rightarrow \bigcup\{A_j : j \in J\} \in \tau^{\mu} \cap \tau_*^{\mu}$.

Therefore $\tau^{\mu} \cap \tau_*^{\mu}$ is a FNSTS.

Remark 2.13. $\tau^{\mu} \cup \tau_*^{\mu}$ is not a FNSTS can be seen by the following example.

Example 2.14. Let $X = \{g, h\}$, $\tau^{\mu} = \{0_N, 1_N, G\}$ and $\tau_*^{\mu} = \{0_N, 1_N, H\}$

Where $G = \{\langle g, 0.2, 0.6, 0.8\rangle, \langle h, 0.5, 0.4, 0.5\rangle\}$, $H = \{\langle g, 0.6, 0.2, 0.4\rangle, \langle h, 0.3, 0.4, 0.7\rangle\}$.

Here $\tau^{\mu} \cup \tau_*^{\mu} = \{0_N, 1_N, G, H\}$. Since $G \cup H \notin \tau^{\mu} \cup \tau_*^{\mu}$, $\tau^{\mu} \cup \tau_*^{\mu}$ is not a FNSTS.

Proposition 2.15. *Let* (X, τ^{μ}) *be a FNSTS. If P and Q be two FNS sets in X. Then the FNS* $\operatorname{int}(P)$ *operator satisfies the following properties hold*:

(a) *FNS* $\operatorname{int}(P) \subseteq P$,

(b) $P \subseteq Q \Rightarrow$ *FNS* $\operatorname{int}(P) \subseteq$ *FNS* $\operatorname{int}(Q)$,

(c) *FNS* $\operatorname{int}($*FNS* $\operatorname{int}(P)) =$ *FNS* $\operatorname{int}(P)$,

(d) *FNS* $\operatorname{int}(P \cap Q) =$ *FNS* $\operatorname{int}(P) \cap$ *FNS* $\operatorname{int}(Q)$,

(e) *FNS* $\operatorname{int}(1_N) = 1_N$.

Proof. Obvious.

Proposition 2.16. *Let* (X, τ^{μ}) *be a FNSTS. If P and Q be two FNS sets in X. Then the* $FNScl(P)$ *operator satisfies the following properties hold*:

(a) $P \subseteq FNScl(P)$,

(b) $P \subseteq Q \Rightarrow FNScl(P) \subseteq FNScl(Q)$,

(c) $FNScl(FNScl(P)) = FNScl(P)$,

(d) $FNScl(P \cup Q) = FNScl(P) \cup FNScl(Q)$,

(e) $FNScl(0_N) = 0_N$.

Proof. Obvious.

Conclusion

In this paper, we introduced the fuzzy neutrosophic supra topological spaces with some definitions and examples. We have introduced Fuzzy neutrosophic supra closure and interior and some definitions and properties are investigated.

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