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BEST PRICING AND ORDERING POLICY FOR DETERIORATING OR DECAYING ITEMS WITH MAXIMUM LIFESPAN UNDER ACCEPTABLE DELAYS IN PAYMENTS

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Abstract

The products like fruits, vegetables, medicines, etc. not only deteriorate or decay continuously as a result of obsolescence, Evaporation, malfunction, etc. But they also have expiry dates. Numerous researchers have studied Economic Order Quantity (EOQ) inventory models for perishable goods, but few of them have considered the maximum life expectancy of deteriorating items. In this paper, an EOQ inventory model is formulated to maximize the salesman's profit. We have taken the products which deteriorate or decay continuously and have a maximum life. In addition, the salesman is allowed an acceptable delay in payments to settle the account. Mathematical formulation of the model is clarified through numerical examples. Finally, the sensitivity investigation is performed with respect to different parameters.

1. Introduction and literature review

In the traditional EOQ inventory models, researchers supposed that the salesmen pay to the suppliers for the purchased goods at the time of goods delivery. But in practice, the suppliers apply trade credit policy to enhance

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their sales and offer a delay period to the salesmen to adjust the account for the purchased goods. The classical EOQ model was introduced by Harris [5]. Haley and Higgins [4] were first established the relation between inventory policy and trade credit policy in EOQ model. Goyal [3] presented an EOQ model to calculate the economic order quantity when the supplier provides a definite credit period to the salesman to settle the account. Aggarwal and Jaggi [1] extended EOQ model by considering deterioration rate and allowing the trade credit period. Degradation is a process that reduces the usefulness or usefulness of a product from its natural state. Ouyang et al. [6] Developed an EOQ model for perishable items under allowable delay in cash discount and payment scheme. The government [7] introduced an inventory model for timely delays in payment delays. Wang et al. determined the best commercial credit period for sellers and the longest life cycle in the supply chain for perishable goods. [11]. Shah and Chaudhry [8] also developed an inventory model for the three players when the demand for the product depends on the duration of the credit. Singh and Singh [10] introduced an excellent inventory policy for perishable goods. By allowing delay in payments and partial backlogging.

Wu et al. [12] developed an inventory model for perishable items with expiration dates. By considering selling price and credit period dependent demand, Singh and Kumar [9] presented an optimal ordering policy for deteriorating items which have an expiry date. Considering an expiration date dependent demand, discounted cash flow approach was presented by Yang [13].

In the inventory modelling, the prices of items play an important function to put an order. The optimal price of items leads the retailers in making pricing policy to boost the demand of the items. None of the above reviewed papers take the maximum life of the deteriorating or decaying items with selling price dependent demand into consideration. In this study, an inventory model is formulated for the salesman to obtain optimal selling price and order quantity by including the following relevant features: the product deteriorates or decays continuously and also has a maximum life; the demand of the product depends on the price and supplier of the product offers delay period to attract the salesman. To justify the model, numerical experiments are presented. Finally, the concavity of the profit function with respect to the price is discussed graphically and future insights are outlined.

2. Assumptions and Notations

1. There is only one supplier and only one salesman and they trade in the same thing.

2. The demand rate $D(p)$ of the items is taken as $D(p) = a - bp,$ where a and bare nonnegative. To ensure that the demand is non-negative, we have taken the selling price p of the item such that $0 < p \leq \frac{a}{b}$.

3. This inventory model deals with an item which deteriorates or decays and has a particular expiration date.

4. To make the problem more realistic, we have taken (this assumption was taken by Sarkar [7] as well as Chen and Teng [2]) the deterioration or decay rate as $\theta(t) = \frac{1}{1 + m - t}$, $0 \leq t \leq T \leq m,$ where m is maximum life of the item. When time t tends to the maximum life m, the deterioration or decay rate $\theta(t)$ tends to 1 i.e. when $t = m,$ the item deteriorates completely.

5. Replenishment is instantaneous and shortage is not permitted.

6. Salesman is allowed an acceptable delay period for payment of all his dues, but if the salesman does not pay all his dues at the end of the credit period, interest will be activated by the supplier and the rest Obligations are paid by the retailer with interest at the end of the cycle.

Notations	Description
a, b	: Demand parameters
h	: Holding cost per unit time
c	: Purchase cost per unit time including ordering cost
p	: Selling price of the item per unit
d	: Deterioration cost per unit per unit time
$I(t)$	: Inventory level at any time t
Q	: Order quantity per cycle
m	: Maximum lifetime or expiration time of the item
T	: Length of replenishment cycle

M	: Length of credit period
I_e	: Rate of interest earned by salesman
I_c	: Rate of interest charged to salesman
Z	: The salesman's profit per unit time

3. Mathematical Inventory Model Formulation

The salesman places an order of size Q units for an item and receives it at time $t = 0$. The inventory level is decreased gradually due to the cumulative effects of demand and decay of the item. It becomes zero at time $t = T$. The behavior of the inventory level over time is shown in figure1.

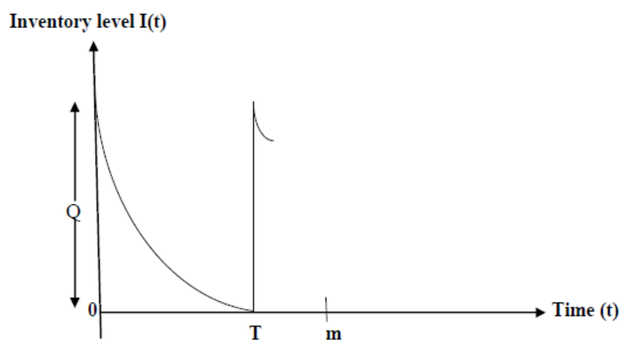

Figure 1. The behavior of the inventory level over time.

The Salesman's total profit per cycle compromises the following components: (i) the purchasing cost of the deteriorating item including ordering cost, (ii) the sale revenue, (iii) the holding cost, (iv) the deteriorating cost, (v) the interest charged by the supplier on the unpaid amount if any, and (vi) the interest obtained on the sale revenue during credit period. The main purpose of this inventory model is to find the optimal price and order quantity per cycle such that the total profit of salesman per cycle is maximized.

The inventory level is governed by the following differential equation:

$$I'(t) = -\theta(t)I(t) - D(p),\ 0 \le t \le T \tag{1}$$

with the boundary condition $I(T) = 0$. Therefore, we have

$$I(t) = (a - bp)(1 + m - t)\log\left(\frac{1 + m - t}{1 + m - T}\right),\ 0 \le t \le T \tag{2}$$

The salesman's order quantity per cycle is

$$Q = I(0) = (a - bp)(1 + m)\log\left(\frac{1 + m}{1 + m - T}\right) \tag{3}$$

Therefore, the salesman's purchasing cost including ordering cost is

$$PC = cQ = c(a - bp)(1 + m)\log\left(\frac{1 + m}{1 + m - T}\right) \tag{4}$$

The retailer's sales revenue is

$$SR = p\int_0^T D(p)dt = p(a - bp)T \tag{5}$$

The retailer's holding cost is

$$HC = h\int_0^T I(t)dt$$

$$= \frac{h(a - bp)}{4}\left\{(1 + m)^2\left(2\log\left(\frac{1 + m}{1 + m - T}\right) - (1 + m - T)\right) + (1 + m - T)^3\right\} \tag{6}$$

The retailer's deterioration cost is

$$DC = d\int_0^T I(t)dt$$

$$= \frac{h(a - bp)}{4}\left\{(1 + m)^2\left(2\log\left(\frac{1 + m}{1 + m - T}\right) - (1 + m - T)\right) + (1 + m - T)^3\right\} \tag{7}$$

Based on the value of the credit period M offered by the supplier to the salesman and the time length of the replenishment cycle T, there are the following two possible cases $T \le M \le m$ and $M \le T \le m$. Now, we will discuss each case in details in the next sections:

Case I: When $T \le M \le m$

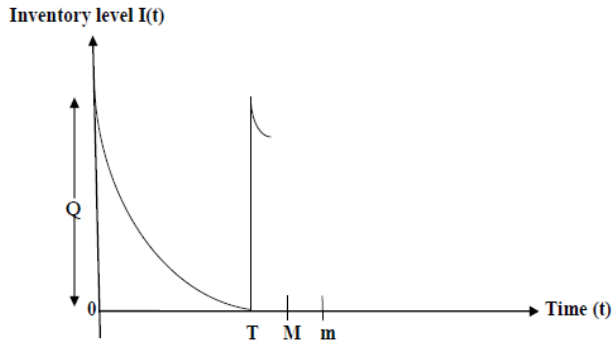

Figure 2. When $T \leq M \leq m$.

In this case, the credit period M offered to the salesman is greater than the cycle time T. The salesman receives the goods from the supplier at $t = 0$; once the salesman gets the goods, he starts selling it and deposits the sale revenue in interest generating account to obtain interest on it during the time interval $[0, M]$. Since the salesman settles the account with the supplier at time $t = M$ and $T \leq M \leq m$. Therefore, the salesman has sold all the stock and has sufficient money to pay all his/her payment at time $t = M$. So, the interest are paid by salesman to the supplier will be zero i.e.

$$IC_1 = 0 \tag{8}$$

and the interest gained by the salesman is

$$IE_1 = pI_e\left\{\int_0^T D(p)tdt + (M - T)\int_0^T D(p)dt\right\} = pI_e(a - bp)\left(\frac{2MT - T^2}{2}\right) \tag{9}$$

Hence, the salesman's profit per unit time is

$$Z_1 = \frac{1}{T}\{SR - PC - HC - DC - IC_1 + IE_1\} \tag{10}$$

Case II. When $M < T \leq m$

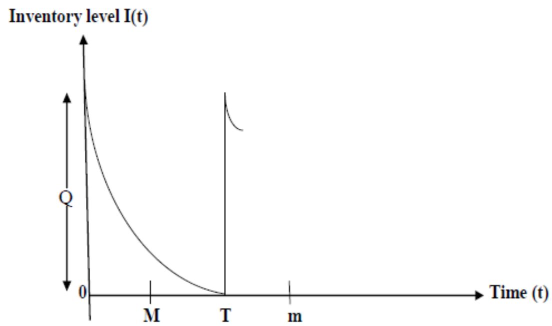

Figure 3. When $M < T \leq m$.

In this case, the credit period M offered to the salesman is less than the cycle time T. The salesman receives the goods from the supplier at $t = 0$; once the salesman gets the goods, he/she starts selling it and deposits the sale revenue in interest generating account to obtain interest on it. The interest earned by the salesman on this sales revenue during the time period $[0, M]$ is

$$IE_2 = pI_e \int_0^M D(p)t\,dt = \frac{1}{2} pI_e(a - bp)M^2 \tag{11}$$

Since the salesman settles the account with the supplier at time $t = M$ and $M < T \leq m$. Therefore, the salesman does not have sufficient money in his/her account to pay all his/her payment at time $t = M$. So, the supplier also permits another period $(T - M)$ to the salesman to pay the unpaid amount at $t = T$ and charges interest on this unpaid amount during the period $[M, T]$. Here, the interest paid by the salesman will be

$$IC_2 = pI_c \int_M^T I(t)dt$$

$$IC_2 = \frac{pI_c(a-bp)}{4}\left[(1+m-T)^3 - (1+m-M)^2\left\{(1+m-T) - 2\log\left(\frac{1+m-M}{1+m-T}\right)\right\}\right] \tag{12}$$

Hence, the salesman's profit per unit time is

$$Z_2 = \frac{1}{T}\{SR - PC - HC - DC - IC_2 + IE_2\} \tag{13}$$

4. Numerical Analysis

Example 1. Let $a = 1000$, $b = 5$, $c = Rs\ 50$ per unit, $d = Rs\ 7$ per unit per year, $h = Rs\ 5$ per unit per year, $I_e = 8\%$ per rupee per year, $T = 3$ years and $M = 4$ years. With the help of Mathematica 11.2, we find the optimal result as:

$p = Rs\,103.48$ per unit, $Q = 2007.06$ units per cycle and $Z = Rs\,978162.01$.

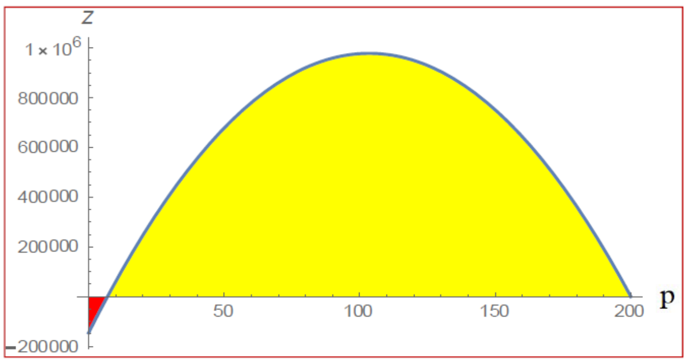

Figure 4. When $T \le M \le m$: Concavity of the Salesman's profit function Z with respect to p.

Example 2. Using the same data as used in Example 1, except the values of $I_c = 10\%$ per rupee per year and $M = 2$ years and with the help of Mathematica 11.2, we find the optimal result as:

$p = Rs\,116.26$ per unit, $Q = 1741.32$ units per cycle and $Z = Rs\,157592.07$.

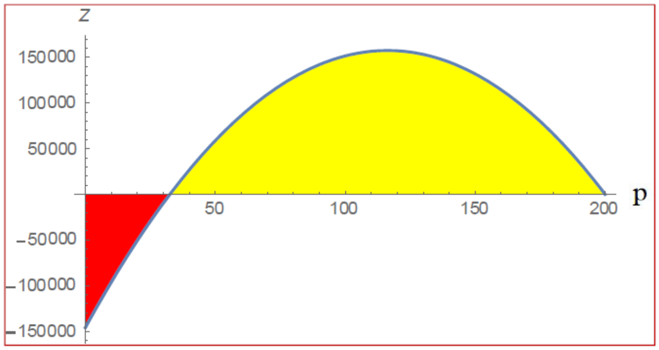

Figure 5. When $M < T \leq m$: Concavity of the Salesman's profit function Z with respect to p.

5. Sensitivity Analysis

In this inventory model, there are numerous parameters which influence the decision of the retailer. Here, we choose five parameters which are directly connected to our model, to check the sensitivity investigation on the optimal results.

Using the data which are used in Example 1, we investigate the sensitivity investigation on the optimal results with respect to each selected parameter and outcomes are shown below.

Parameters	Value of Parameters	p	Q	Z
a	600	63.48	1175.30	335406.26
	800	83.48	1519.19	614784.13
	1000	103.48	2007.06	978162.01
	1200	123.48	2422.97	1455398.70
	1400	143.48	2838.85	1956917.73
b	3	170.14	2036.11	1677652.93
	4	128.48	2021.55	1240407.46
	5	103.48	2007.06	978162.01
	6	86.81	1992.69	803416.53

	7	74.91	1978.09	678671.06
T	2.4	102.96	1487.12	1101712.82
	2.7	103.20	1736.12	1040061.66
	3.0	103.48	2007.06	978162.01
	3.3	103.80	2318.87	915959.86
	3.6	104.21	2633.14	853379.21
M	3.2	105.01	1975.26	658719.87
	3.6	104.11	1993.98	818390.79
	4.0	103.48	2007.06	978162.01
	4.4	103.02	2016.64	1137993.71
	4.8	102.67	2023.92	1297664.73
m	3	103.61	2672.50	975480.75
	4	103.48	2212.84	979634.08
	5	103.48	2007.06	978162.01
	6	103.64	1887.36	974830.34
	7	103.85	1807.63	970645.01

The sensitivity analysis reveals that:

1. The retailer earns more profit if the demand parameter a as well as credit period M is increasing.

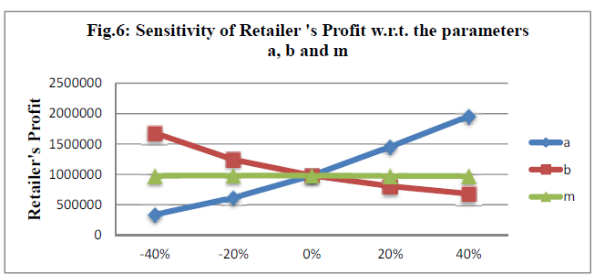

Figure 6. Sensitivity of Retailer‘s Profit w.r.t. the parameters a, b and m.

2. The salesman earns little more profit if the maximum life m is decreasing.

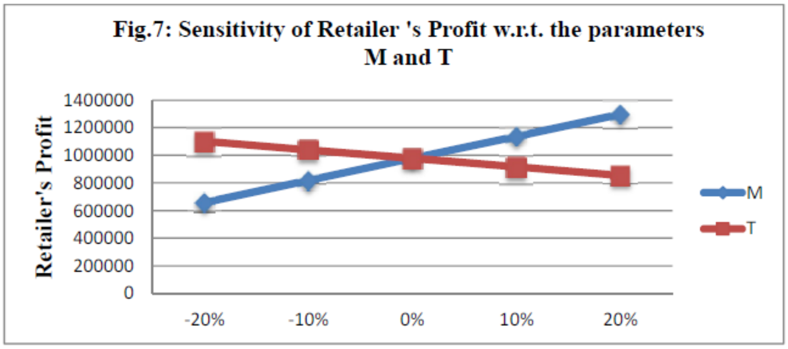

Figure 7. Sensitivity of Retailer's Profit w.r.t. the parameters.

3. The retailer also earns more profit if the demand parameter b as well as replenishment cycle time T is decreasing.

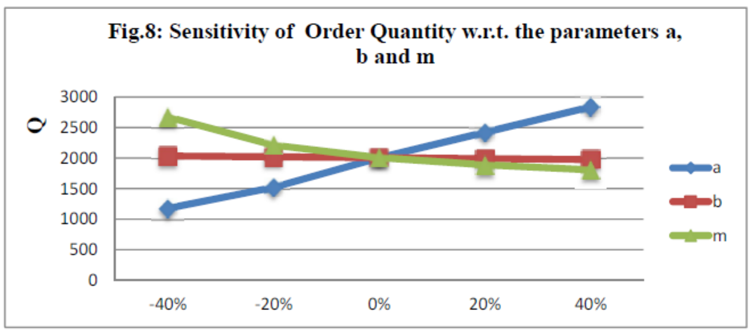

Figure 8. Sensitivity of Order Quantity w.r.t. the parameters a, b and m.

4. The salesman orders more quantity if the demand parameter a as well as T is increasing.

5. The retailer orders less quantity if the demand parameter b as well as the maximum life m is increasing.

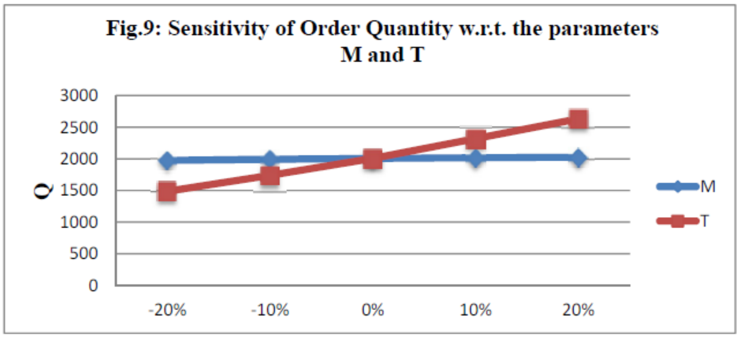

Figure 9. Sensitivity of Order Quantity w.r.t. the parameters M and T

6. The salesman also orders more quantity if credit period M is increasing.

6. Conclusions

In this article, we have developed an inventory model for salesmen to obtain the best selling price and order quantity of goods by incorporating the following relevant features:

1. Product deteriorates or decays continuously and has a maximum life.

2. Demand is taken as the selling price of the product.

3. The supplier offers acceptable delays in payments to attract salesmen.

Finally, numerical investigations and sensitivities are performed to explain the model.

This inventory model can be extended to allow for a reduction in the inflation scenario or to allow for partial backlogging.

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