Advances and Applications in Mathematical Sciences
Volume 20, Issue 11, September 2021, Pages 2635-2644

MODELING OF DYNAMICAL SYSTEMS THROUGH MACHINE LEARNING

P. RAJENDRA, T. GUNAVARDHANA NAIDU and V. BRAHMAJIRAO

[1]Associate Professor
Department of Mathematics
CMR Institute of Technology
Bengaluru, Karnataka, India
E-mail: rajendra.padidhapu@gmail.com

[2]Assistant Professor
Department of Physics
Aditya Institute of Technology
and Management, Tekkali, Srikakulam
Andhra Pradesh, India

[3]Senior Professor
School of Biotechnology MGNIRSA
D.S.R. Foundation
Hyderabad, Telangana, India

Abstract

This review presents the key challenges of discovering dynamics from data and finding data-driven representations that make nonlinear systems amenable to linear analysis. Data-driven models drive to discover the governing equations and give laws of physics. The identification of dynamical systems through machine learning techniques succeeds in inferring physical systems. The two chief challenges are nonlinear dynamics and unknown or partially known dynamics. Machine learning is providing new and powerful techniques for both challenges. Dimensionality reduction methods are used for projecting dynamical methods in reduced form and these methods perform computational efficiency on real-world data.

2010 Mathematics Subject Classification: 37-XX.

Keywords: dynamical systems, machine learning, dimensionality reduction, dynamic mode decomposition.

Received October 14, 2020; Accepted November 10, 2020

I. Introduction

Dynamical systems provide a mathematical framework to describe the real-world problems, modeling the rich interactions among quantities that change in time. Formally, dynamical systems concern the analysis, prediction, and understanding of the behavior of systems of differential equations or iterative mappings that describe the evolution of the state of a system. Data-driven models are an emerging field of simulating and discovering dynamical systems purely from data using techniques of machine learning and data science. We have an explosion of data in climate science, neuroscience, disease modeling, and fluid dynamics. The amount of data getting from experiments, simulations, and historical records is growing at an incredible pace. Simultaneously, the algorithms in machine learning, data science, and statistics optimization techniques are getting much better. Therefore we can discover dynamical systems and characterize them purely from data. In the past, dynamical systems were essentially written down by physical laws and derived the equations from first principles using physics. But, today the systems that we want to understand like the brain, climate, or financial market. There are no first principles physics that we can write down in an easy to understand simulate and control framework. Therefore, we are trying to build data-driven techniques more powerful for these emerging classes of problems. Zhang et al. [1] proposed pre-classified reservoir computing techniques to analyze the fault of 3D printers. This method reduces the interclass separation by summing information labels of similar conditions. Because of the reservoir computing model and pre-classification strategy, the presented method achieves the maximum accuracy in the fault analysis of three-dimensional printers. Ibanez et al. [2] analyze the ability of data-driven approaches to predict the reactive extrusion in complex processes. This work is carried out based on thermo-set chase mixing steps with the polypropylene phase. The goal of this paper is to characterize the suitable processing conditions regarding the mechanical property improvements of new polypropylene materials with the help of reactive extrusion.

II. Dimensionality Reduction Methods

Singular value decomposition algorithms can be efficiently applied for data processing, high-dimensional statistics, and reduced-order modeling. SVD can be used to efficiently represent human faces, in the so-called "eigenfaces". Principal Component Analysis (PCA) is a workhorse algorithm in statistics, where dominant correlation patterns are extracted from high-dimensional data. Erichson et al. [3] introduced a spare PCA algorithm via variable projection. The algorithm allows for robust sparse PCA instead of corrupted input data and performs computational efficiency on real-world data. Erichson et al. [4] also put forward a new CANDECOMP/PARAFAC dimensionality reduction for multidimensional data. This research talks about approximation errors via oversampling. The challenges posed to big data by extracting important features from multidimensional data. Suarez et al. [5] developed an open-source pyDML that has a library of distance metric learning algorithms. pyDML is used to improve nearest neighbor algorithms and dimensional reduction. This library provides parameter tuning and visualization of classifiers. Baek et al. [6] presented a multi-choice wavelet threshold algorithm based on perception, decision, and cognition. The wavelet threshold SVMs [8] and information complexity are incorporated to assess learning models. The authors used to evaluate the available data-sets to illustrate the planned method and the results are compared with recent methods. Wei et al. [7] aim to solve the noise of internet information and compares web pages in natural language processing (NLP). Web-page visualization has been implemented based on PCA. The result shows that ML processing NLP algorithm has better performance in prediction and classification accuracy. See in Figure 1 for various dimensionality reduction methods used in the literature.

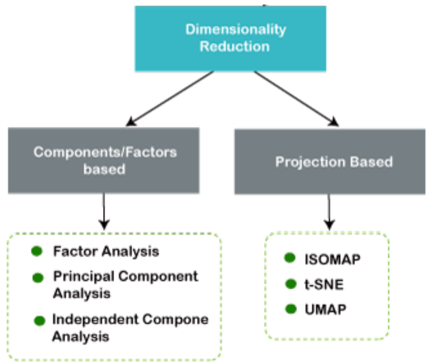

Figure 1. Dimensionality reduction methods.

III. Dynamic Mode Decomposition

The Dynamic Mode Decomposition (DMD) is a new framework to extract features from high dimensional data. The DMD is applied to complex systems in fluid dynamics, disease modeling, neuroscience, plasma physics, robotics, and video modeling. Erichson et al. [9] introduced a compression DMD model for modeling of background. The key principle of compressed DMD algorithm scales with matrix rank instead of the actual matrix. The result proves that the background quantified by a recall, precision which helps the algorithm's computational performance. Erichson et al. [10] also presented a randomized DMD algorithm for computing low-rank matrices and eigenvalues. The algorithm takes about the modular probability framework. This approach provides features to extract from big data with the intrinsic rank of a matrix. Bai et al. [11] used DMD for compressive system identification via the Koopman operator. This operator is introduced in 1931 but has experienced renewed interest recently because of the increasing availability of measurement data and advanced regression algorithms. This research allows us to identify order reduced models from limited data. The results demonstrate the extraction of the main features that are well characterized. Kaptanoglu et al. [12] developed a reduced-order model of characterizing magnetized plasmas with DMD. The magnetic features of simulation and experimental data-sets are analyzed. This algorithm provides new insights into the plasma structure. Fujii et al. [13] proposed a new algorithm via multitask learning that incorporated information of labels into supervised DMD. The authors researched the empirical performance by utilizing synthetic data-sets and validated their algorithm that can extract the label-specific structures. The supervised DMD method shows enhanced accuracy compared to conventional DMD methods. Vigneswran et al. [14] presented a method to extract the features using DMD. ImageNet data samples are used to perform experiments. The extracted features using DMD with a random kitchen walk approach performs better results. Brunton et al. [15] explored finite-dimensional linear representation by restricting the Koopman operator and investigated the choice of observable functions. Finally, the authors demonstrated the advantages of nonlinear observable sub-spaces via Koopman operator. Yu et al. [16] proposed a low-rank DMD model for the prediction of traffic flow. The low-rank DMD predicts the traffic flow via the state transmission matrix. The result shows that the low-rank DMD performs better on modern methods. Takeishi et al. [17] proposed a data-driven approach via "learning Koopman invariant sub-spaces" principle from observed data. The authors introduced ANN to evaluate the data-driven DMD performance using nonlinear dynamical systems and estimated a set of parametric functions. See in Figure 2 for the accuracy of various Dynamic Mode Decomposition methods versus their training frequencies.

IV. Machine Learning of Dynamical Systems

Machine learning is currently being used to extract useful patterns and coherent structures in high-dimensional dynamics of complex systems. Weinan et al. [18] proposed a machine learning algorithm via high dimensional nonlinear dynamical systems. The supervised learning algorithm is developed by consideration of regression problems. This paper concludes that continuous dynamical systems are an alternative way to develop machine learning techniques. Brutnton et al. [19] proposed the discovery of a multi-scale model for materials with the help of data-driven methods. The authors developed a python based program for sparse identification of high dimensional dynamical systems called pySINDy. This research demonstrated how to use pySINDy to simulate canonical dynamical systems and perform numerical studies of nonlinear tracking control Regazzoni et al. [20] presented a new data-driven method called "Model order reduction" based on machine learning that is applied to nonlinear dynamical systems arising in ODE. This model formulates the dimensional reduction problem as a probability of maximum likelihood to minimize errors in input-output data pairs. Lee et al. [21] critically examined the main advantages of machine learning. The authors particularly discussed RNN and its role in decision making and control problems. This paper also presented the advantages and disadvantages of these methods for the field of energy and process systems engineering. Boots et al. [22] proposed an online spectral algorithm that uses SVD to scale the partial observable nonlinear complex dynamical systems. The authors demonstrated a high bandwidth video mapping and illustrated the behaviors of dynamical systems. Wolfe et al. [23] presented the algorithms for learning the predictive state representation model. The Monte Carlo and Temporal difference algorithms were developed to model dynamic systems. The performance and results of these algorithms compared with existing algorithms. Song et al. [24] extended the Hilbert space embeddings and estimated a kernel to handle conditional distributions. The authors presented a nonparametric method for dynamic system models via conditional embedding and verified the effectiveness of the model in a variety of dynamical systems.

V. Data- Driven Models of Dynamical Systems

The recent innovations in data-driven models for PDE systems have been highlighted by many re-searchers. Davoudi et al. [25] proposed a vision-based inspection of data-driven models for surface observations of concrete beams. The image dataset containing 862 has been included in a database for reinforced concrete beams. A supervised ML builds predictive models that are useful for estimating internal shear and moment loads. The authors estimated the accuracy of reinforced concrete beams. Figure 2 visualizes two important problems of regression and classification that arise in data-driven methods.

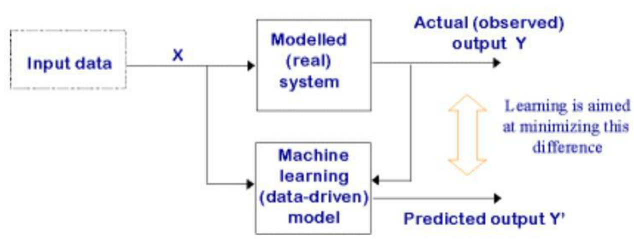

Figure 2. Driven framework used to develop machine learning algorithms.

Sun et al. [26] provided a physics-constrained deep neural network and developed a surrogate model for fluid dynamics. This model is not dependent on any simulation-based data and it shows good results on the flow field in between deep learning and numerical simulations. Wu et al. [27] presented a data-driven time-dependent PDE framework in modal space using DNNs. The finite-dimensional model is accomplished by training DNN based on ResNet using specified data. The predictive accuracy of models for different PDEs including Burger's equation is presented to illustrate the error analysis and effectiveness. Pai [28] points out that the frequency-time study is appropriate for parameter and nonparametric of dynamical systems. The Hilbert-Huang transform is the alliance of Hilbert transform and mode decomposition chosen empirically. The Hilbert-Huang transform provides piece decomposition and precise time-frequency analysis compared with Fourier and Wavelet transforms. The conjugate pair decomposition is used for online frequency tracking. The planned method could provide accurate identification of various nonlinear dynamical systems. Dsilva et al. [29] focus on applying machine learning methods to spot components in a group of multi-scale data. The authors presented an approach to utilize local geometry and noise dynamics. The analysis of data-driven reduction for multiscale dynamical systems recovers the underlying slow variables. Schulze et al. [30] presented a data-driven insight for dynamical systems with delay. This approach is validated by different examples. The result shows that the need for preserving the delay formation in the model of dynamical systems. Giannakis et al. [31] developed a scheme for dynamic mode decomposition and forecasting of ergodic unobserved component modeling systems ([32], [33]). This scheme is based on Perron-Frobenius and Koopman groups on an orthogonal basis. The authors established the connection between Laplace-Beltrami and Koopman operators to provide an analysis of diffusion-mapped coordinates for system dynamics.

VI. Way Forward

Data-driven methods are revolutionizing in science and technology and most of these methods are applicable to model the complex dynamical systems. These formulation models are facilitated in the future to encompass an incredible range of phenomena, including those observed in classical mechanical systems, electrical circuits, turbulent fluids, climate science, finance, ecology, social systems, neuroscience, epidemiology, and nearly every other system that evolves in time. In the future, these techniques will continue to gain greater relevance, because there are investigations that are working in the process of integrating data, which will allow data of different types to be used and origins, allowing discoveries to be made about the relationships and interactions between the different dynamical systems.

References

[1] Shaohui Zhang, Xiang Duan, Chuan Li and Ming Liang, Pre-classified reservoir computing for the fault diagnosis of 3D printers, Mechanical Systems and Signal Processing 146 (2021) 106961. https://doi.org/10.1016/j.ymssp.2020.106961.

[2] R. Ibañez et al., On the Data-Driven Modeling of ctive Extrusion, Fluids 5 (2020), 94.

[3] Erichson and N. Benjamin et al., Sparse Principal Component Analysis via Variable Projection, SIAM J. Appl. Math., doi:10.1137/18m1211350 80 (2020), 977-1002.

[4] N Benjamin Erichson, Krithika Manohar et al., Randomized CP tensor decomposition, Machine Learning: Science and Technology, https://doi.org/10.1088/2632-2153/ab8240, 1 (2) (2020).

[5] Juan Luis Suarez, Salvador Garca and Francisco Herrera., pyDML: A Python Library for Distance Metric Learning", Journal of Machine Learning Research 21(2020), 1-7.

[6] S. H. Baek, A. Garcia-Diaz and Y. Dai, Multi-Choice Wavelet Thresholding Based Binary Classification Method. Methodology, https://doi.org/10.5964/meth.2787 16(2) (2020), 127-146.

[7] Ziyi Wei, Zhiyong Zhang, WeiWei Gu and Nan Fang, Visualization Classification and Prediction Based on Data Mining", Journal of Physics: Conference Series, Vol 1550, Machine Learning, Intelligent data analysis and Data Mining. https://doi.org/10.1088/1742-6596/1550/3/032122

[8] P. Rajendra, A. Subbarao and G. Ramu et al., Prediction of drug solubility on parallel computing architecture by support vector machines, Netw Model Anal Health Inform Bioinforma 7(13) (2018).

[9] N. B. Erichson, S. L. Brunton and J. N. Kutz, Compressed dynamic mode decomposition for background modeling, J Real-Time Image Proc https://doi.org/10.1007/s11554-016-0655-2 16 (2019), 1479-1492.

[10] N. Erichson and Benjamin et al., Randomized Dynamic Mode Decomposition, SIAM J. Applied Dynamical Systems doi:10.1137/18M1215013 18 (2019), 1867-1891.

[11] Zhe Bai and Eurika Kaiser et al., Dynamic Mode Decomposition for Compressive System Identification, https://doi.org/10.2514/1.J057870 58(2) (2020), 561-574.

[12] A. A. Kaptanoglu, K. D. Morgan, C. J. Hansen and S. L. Brunton., Characterizing

magnetized plasmas with dynamic mode decomposition, Physics of Plasmas, 27, 032108, 2020. https://doi.org/10.1063/1.5138932

[13] Keisuke Fujii and Yoshinobu Kawahara, Supervised dynamic mode decomposition via multitask learning", Pattern Recognition Letters, https://doi.org/10.1016/j.patrec.2019.02.010 122 (2019), 7-13.

[14] K. Rahul-Vigneswaran, S. Sachin-Kumar, N. Mohan and K. P. Soman, Dynamic Mode Decomposition based feature for Image Classification, TENCON 2019 – 2019 IEEE Region 10 Conference (TENCON), Kochi, India, doi: 10.1109/TENCON.2019.8929663 (2019), 745-750.

[15] SL. Brunton, BW. Brunton, JL. Proctor and JN. Kutz, Koopman Invariant Subspaces and Finite Linear Representations of Nonlinear Dynamical Systems for Control, PLoS ONE 11(2): e0150171, 2016.

[16] Y. Yu, Y. Zhang, S. Qian, S. Wang, Y. Hu and B. Yin, A Low Rank Dynamic Mode Decomposition Model for Short-Term Traffic Flow Prediction, IEEE Transactions on Intelligent Transportation Systems.

[17] Naoya Takeishix, Yoshinobu Kawaharay and Takehisa Yairi, Learning Koopman Invariant Subspaces for Dynamic Mode Decomposition, NIPS'17: Proceedings of the 31st International Conference on Neural Information Processing Systems, December (2017), 1130-1140.

[18] E. Weinan, A Proposal on Machine Learning via Dynamical Systems, Commun. Math. Stat., doi: 10.1007/s40304-017-0103-z [19]. Steven L Brunton, J Nathan Kutz., Methods for data-driven multi-scale model discovery for materials, J. Phys.: Mater, 2 (2019), 044002. 5 (2017), 1-11.

[20] F. Regazzoni, L. Dedè and A. Quarteroni, "Machine learning for fast and reliable solution of time-dependent differential equations", Journal of Computational Physics 397 (2019), 108-852.

[21] Jay H. Lee, Joohyun Shin and Matthew J. Realff, Machine learning: Overview of the recent progresses and implications for the process systems engineering field", Computers and Chemical Engineering, https://doi.org/10.1016/j.compchemeng.2017.10.008. 114 (2018), 111-121.

[22] Byron Boots and Geoffrey J. Gordon, An Online Spectral Learning Algorithm for Partially Observable Nonlinear Dynamical Systems, AAAI'11: Proceedings of the Twenty-Fifth AAAI Conference on Artificial Intelligence, Doi/10.5555/2900423.2900469. (2011), 293-300.

[23] Britton Wolfe, Michael R. James and Satinder Singh, Learning predictive state representations in dynamical systems without reset, ICML '05: Proceedings of the 22nd international conference on Machine learning, https://doi.org/10.1145/1102351.1102475. August (2005), 980-987.

[24] Le Song, Jonathan Huang, Alex Smola and Kenji Fukumizu, Hilbert Space Embeddings of Conditional Distributions with Applications to Dynamical Systems, ICML '09: Proceedings of the 26th Annual International Conference on Machine Learning,

https://doi.org/10.1145/1553374.1553497 June (2009) 961-968.

[25] Rouzbeh Davoudia, Gregory R. Millera and J. Nathan Kutz, Data-driven vision- based inspection for reinforced concrete beams and slabs: Quantitative damage and load estimation, Automation in Construction 96 (2018) 292–309. https://doi.org/10.1016/j.autcon.2018.09.024

[26] Luning Suna and Han Gaoa et al., Surrogate modeling for fluid flows based on physics-constrained deep learning without simulation data, Comput. Methods Appl. Mech. Engrg. 361 (2020), 112-732.

[27] Wu. Kailiang, Xiu. Dongbin, Data-driven deep learning of partial differential equations in modal space, Journal of Computational Physics, 408(2020)109307. https://doi.org/10.1016/j.jcp.2020.109307

[28] P. Frank Pai, Time–frequency analysis for parametric and non-parametric identification of nonlinear dynamical systems, Mechanical Systems and Signal Processing 36(2) (2013), 332-353.

[29] Carmeline J. Dsilva et al., Data-Driven Reduction for a Class of Multiscale Fast- Slow Stochastic dynamical Systems, Siam J. Applied Dynamical Systems 15(3) 1327-1351.

[30] Philipp Schulze, Benjamin Unger, Data-driven interpolation of dynamical systems with delay, systems and Control Letters, https://doi.org/10.1016/j.sysconle.2016.09.007. 97 (2016), 125-131.

[31] Dimitrios Giannakis, Data-driven spectral decomposition and forecasting of ergodic dynamical systems, Applied and Computational Harmonic Analysis, https://doi.org/10.1016/j.acha.2017.09.001. 47(2) (2019), 338-396.

[32] K. V. Narasimha Murthy, R. Saravana and P. Rajendra, Unobserved component modeling for seasonal rainfall patterns in Rayalaseema region, India 1951-2015, Meteorol Atmos Phys 131 (2019), 1387-1399.

[33] Murthy, Narasimha, R. Saravana, P. Rajendra, Critical comparison of north east monsoon rainfall for different regions through analysis of means technique, Mausam. 69 (2018), 411-418.