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MOMENTS FOR HIRING TIME IN A WORK FORCE STRUCTURE HAVING CLUSTERS OF EGRESS AND TWO THRESHOLDS

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Abstract

In this paper, a workforce structure in which egress of employees occurs in clumps is considered. Two stochastic models are constructed using CUM and MAX hiring strategies. Analytical results for mean square deviation of hiring time and average aggregate of egress in the interval of hiring are derived under the conditions (i) total count of egress form a compound Poisson process (ii) count for egress in any clumps are independent deactivated geometric random variables and (iii) the system has two fixed positive integers as optional and mandatory thresholds. Quantitative illustrations are given to analyze the impact of leading parameters on the results.

1. Introduction

Attrition is a common phenomenon in many marketing organizations. It is a significant event in any design of work force. Work force exemplars are studied in [2] and [3] using suitable statistical technique. By assuming that deprivation of work force occurs owing to decisions, many work force

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exemplars have been studied utilizing CUM hiring strategy under different conditions for deprivation of manpower, threshold and the inter-decision times. Such models could be seen in ([1], [6], [7], [8], and [11]). Also, in ([4], [9], and [10]) mean square deviation of hiring time is obtained for a single grade workforce structure by utilizing MAX hiring strategy under different conditions. Recently in [5] the authors have obtained mean square deviation of hiring time and average cumulative egress up to hiring time for a single grade work force structure by utilizing CUM hiring strategy under the conditions (i) total count of egress form a compound Poisson process (ii) count for egress in any clumps are independent decapitated geometric random variables and (iii) the threshold is an integer valued random variable. Present paper studies the work in [5], by designing two exemplars using CUM and MAX hiring strategies respectively when the work force structure has two fixed positive integers for optional and mandatory thresholds.

2. Description of Exemplar – 1

Consider a single graded workforce structure in which clump of egress of employees occurs in $(0, \infty)$. Let $\{B(v), v > 0\}$ be a Poisson process with degree $b, b > 0$ where $B(v)$ is the count for clumps in $(0, v]$. Let X_i, the count for egresses in the i^{th} clump $i = 1, 2, 3, \ldots$, be independent decapitated geometric variables with parameter $a, a \in (0, 1)$ and are independent of $\{B(v)\}$. Let $S(v)$ be the aggregate of egresses in $(0, v]$. Let c and d be positive integers denoting the optional and mandatory thresholds for the aggregate of egresses, where $c < d$. Let q be the probability that the work force is not going for hiring when optional threshold is exceeded by aggregate of egresses. Let WCUM be the time to hiring with mean $E[W_{CUM}]$ and mean square deviation $V[W_{CUM}]$. The hiring CUM strategy states that hiring has to be done when the aggregate of egresses exceeds the mandatory threshold, but hiring may or may not be done if the aggregate of egresses exceeds the optional threshold.

3. Results for Exemplar – 1

From the recruitment policy

$$P(\mathrm{W}_{\mathrm{CUM}} > v) = (1-q)P(S(v) \leq c) + qP(S(v) \leq d) \tag{1}$$

i.e.

$$P(\mathrm{W}_{\mathrm{CUM}} > v) = (1-q)\left[\sum_{m=0}^{c} P(S(v) = m)\right] + q\left[\sum_{m=0}^{d} P(S(v) = m)\right] \tag{2}$$

Since $E[\mathrm{W}_{\mathrm{CUM}}^{\mathrm{n}}] = \int_0^\infty P(\mathrm{W}_{\mathrm{CUM}} > v)dv,\ n = 1, 2, 3, \ldots$ (3)

Applying (2) in (3)

$$E[W_{CUM}] = (1-q)\int_0^\infty \sum_{m=0}^{c} P(S(v) = m)dv + q\int_0^\infty \sum_{m=0}^{d} P(S(v) = m)dv \tag{4}$$

Since $S(v) = \sum_{i=1}^{B(v)} X_i$ and $B(v)$ is independent of X_i, 1, 2, 3, …, by conditioning upon $B(v)$ it is proved in [12] that

$$P(S(v) = m) = \begin{cases} e^{-bv} & if\ m = 0 \\ e^{-bv}\sum_{j=1}^{m} \frac{1}{j!}\binom{m-1}{j-1}(abv)^j(1-a)^{m-j}, & if\ m \neq 0 \end{cases} \tag{5}$$

From (5) and using the result $\sum_{l=0}^{m-1}\binom{m-1}{l}\left(\frac{a}{1-a}\right)^l = \left(1 + \frac{a}{1-a}\right)^{m-1}$ it can be shown that

$$\int_0^\infty \sum_{m=0}^{c} P(S(v) = m)dv = \frac{1}{b}[1 + ac] \tag{6}$$

Applying (6) in (4)

$$E[W_{CUM}] = \frac{1}{b}[1 + ac + qa(d-c)] \tag{7}$$

We now determine $E[W_{CUM}^2]$. From (2) and (3)

$$E[W_{CUM}^2] = 2(1-q)\int_0^\infty v \sum_{m=0}^{c} P(S(v) = m)dv + 2q\int_0^\infty v \sum_{m=0}^{d} P(S(v) = m)dv \tag{8}$$

From (5) we can show that

$$\int_0^\infty v \sum_{m=0}^{c} P(S(v) = m)dv = \frac{1}{b^2}\left[1 + \sum_{m=1}^{c}(1-a)^m \sum_{j=1}^{m}(j+1)\binom{m-1}{j-1}\left(\frac{a}{1-a}\right)^j\right] \tag{9}$$

Writing $(j+1)$ as $(j-1+2)$ and using the results

$$\sum_{m=1}^{c}(1-a)^m \sum_{j=1}^{m}(j+1)\binom{m-1}{j-1}\left(\frac{a}{1-a}\right)^j = \frac{c(c-1)a^2}{2}$$

And

$$\sum_{m=1}^{c}(1-a)^m \sum_{j=1}^{m}\binom{m-1}{j-1}\left(\frac{a}{1-a}\right)^j = 2ac$$

in (9) we get

$$\int_0^\infty v \sum_{m=0}^{c} P(S(t) = m)dv = \frac{1}{b^2}\left[1 + \frac{c(c-1)a^2}{2} + 2ac\right] \tag{10}$$

From (8), (10) it can be shown that

$$E[W_{CUM}^2] = \frac{1}{b^2}[2 + c(c-1)(1-q)a^2 + qd(d-1)a^2 + 4a((1-q)c + qd)] \tag{11}$$

From (7) and (11) it can be shown that

$$Var[W_{CUM}] = \frac{1}{b^2}\begin{bmatrix}(2-(1+ac)^2 - q(d-c)(2+2ac+q(d-c))) + \\ 4a((1-q)c+qd) + a^2(c(c-1)(1-q) + qd(d-1))\end{bmatrix}$$

If $q(d-c)(2+2ac+q(d-c)) + (1+ac)^2 \leq 2$ (12)

(7) and (12) give the mean and mean square deviation of time to hiring.

Result for $E[S(W_{CUM})]$ is obtained as follows. By Wald's lemma, we get

$$E[S(W_{CUM})] = E[X]E[B(W_{CUM})] \tag{13}$$

Since $B(v)$ is a Poisson process with degree b, $E[X] = 1/a$ and from (13) the average aggregate of egresses in the interval of hiring is given by

$$E[S(W_{CUM})] = \frac{b}{a} E[W_{CUM}] \tag{14}$$

where $E[W_{CUM}]$ is given by (7).

4. Description of Exemplar -2

In this model, $M(v) = \underset{1 \leq i \leq B(v)}{Max} X_i$ represents the maximum number of egress in $B(v)$ clumps. The MAX strategy for hiring in this exemplar states that hiring is optional when maximum number of egresses exceeds the optional threshold, but hiring has to be done when this maximum exceeds the mandatory threshold. The rest of the assumptions and notations are as in exemplar-1.

5. Results for Exemplar -2

By the univariate MAX policy of recruitment

$$P(W_{MAX} > v) = P(M(v) \leq c) + qP(c < M(v) \leq d)$$

(i.e.)

$$P(W_{MAX} > v) = (1 - q)P(M(v) \leq c) + qP(M(v) \leq d) \tag{15}$$

In (15)

$$P(M(v) \leq c) = P[\underset{1 \leq i \leq B(v)}{Max} X_i \leq c]$$

Conditioning upon $B(v)$ and using the independent of $B(v)$ and X_i, 1, 2, 3, … we get

$$P(M(v) \leq c) = \sum_{j=0}^{\infty} P[\underset{1 \leq i \leq j}{Max} X_i \leq c] P(B(v) = j) \tag{16}$$

Since $\underset{1 \leq i \leq 0}{Max} X_i \leq c$ is an empty set, without loss of generality j can vary from 0 to ∞.

If $F(c) = P(X_i \leq c) = P(X \leq c)$, $i = 1, 2, 3, \ldots$ then

$$P[\underset{1 \leq i \leq j}{Max} X_i \leq c] = \sum_{j=0}^{\infty} (F(c))^j \tag{17}$$

From (16) and (17) it is shown that

$$P(M(v) \leq c) = e^{-bv[1-F(c)]} \tag{18}$$

$$1 - F(c) = P(X > c) = a \sum_{k=i+1}^{\infty} (1-a)^{k-1} \tag{19}$$

From (18) and (19) we get

$$P(M(v) \leq c) = e^{-bv(1-a)^c} \tag{20}$$

From (15) and (20) we get

$$P(W_{MAX} > v) = (1-q)e^{-bv(1-a)^c} + qe^{-bv(1-a)^d} \tag{21}$$

From (3), (21) and on simplification we get

and $$E[W_{MAX}] = \frac{1}{b}\left[\frac{1-q}{(1-a)^c} + \frac{q}{(1-a)^d}\right] \tag{22}$$

$$E[W_{MAX}^2] = \frac{2}{b^2}\left[\frac{1-q}{(1-a)^{2c}} + \frac{q}{(1-a)^{2d}}\right] \tag{23}$$

Result for mean square deviation of time to hiring are given by (22) and (23).

6. Numerical Illustration

In this section impact of leading parameters on $E[W_{CUM}]$, $E[W_{MAX}]$, $E[W_{CUM}]$ and $V[W_{MAX}]$ is analyzed. Effect of 'q', 'a' and 'b' when $c = 10$ and $c = 13$ are tabulated below.

Table 1. IMPACT OF 'q', 'a' and 'b' FOR EXEMPLAR – 1.

p	a	b	$E[W_{CUM}]$	$V[W_{CUM}]$
0.45	0.2	0.7	1.814	31.590
0.5	0.2	0.7	1.857	32.102
0.55	0.2	0.7	1.900	32.586
0.4	0.45	0.7	2.200	78.236
0.4	0.5	0.7	2.286	90.898
0.4	0.55	0.7	2.371	104.636
0.4	0.2	0.4	3.100	95.090
0.4	0.2	0.5	2.481	60.857
0.4	0.2	0.6	2.067	42.262

Table 2. IMPACT OF 'q', 'a' and 'b' FOR EXEMPLAR – 2.

p	a	b	$E[W_{MAX}]$	$V[W_{MAX}]$
0.45	0.2	0.7	19.011	441.02
0.5	0.2	0.7	19.645	466.33
0.55	0.2	0.7	20.279	490.84
0.4	0.1	0.7	4.706	23.263
0.4	0.15	0.7	9.080	92.424
0.4	0.2	0.7	18.377	414.90
0.4	0.2	0.5	25.728	813.21
0.4	0.2	0.6	21.440	564.73
0.4	0.2	0.7	18.377	414.90

7. Findings

From the above tables it is observed that for both exemplars

- When 'q' the probability for not going for hiring increases, the mean time to hiring increases.
- It is significant to note that count for egresses in each clump and time taken for exceeding the threshold level have opposite monotonicity. Therefore, increase in 'a' will lead to an increase in mean time for hiring.
- The monotonicity of count for clumps and time to hiring are opposite to each other. Therefore, increase in 'b' will lead to decrease in mean time to hiring.

8. Conclusion

The present paper has studied the problem of hiring in a single grade workforce structure with a provision of an optional threshold by considering clump of egresses of employees. The analytical results for expected time to hiring are consistent with reality in the context of impact of leading parameters on mean time hiring.

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