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NON TRIVIAL INTEGRAL SOLUTIONS OF TERNARY QUADRATIC DIOPHANTINE EQUATION

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Abstract

The ternary homogeneous equation instead of an infinite cone given by $x^2 + y^2 - 6x + 10y = 34(z^2 - 1)$ is analyzed for its non-zero distinctive integer points. Few dissimilar patterns of integer points satisfying the infinite cone under deliberation are obtained.

I. Introduction

An enormous compact of also arises from the revise of the solutions in integers of a polynomial equation $f(x_1, x_2, \ldots, x_n) = 0$, called the Diophantine equation. They have slighter number equations than indefinite variables and grip integers that work properly for all equations. Quadratic Diophantine equationsare awfully significant in number theory. There are more than a few Diophantine equationsthat have no solutions, trivial solutions, finitely or infinitely several solutions.

In this manuscript concerns with fascinating ternary quadratic equation

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$x^2 + y^2 - 6x + 10y = 34(z^2 - 1),\ x,\ y,\ z, \in Z - \{0\}$ representing an infinite cone for determining its infinitely many non-zero lattice points.

II. Method of Analysis

The ternary quadratic Diophantine equation studied for its non-zero distinct integer solutions is given by

$$x^2 + y^2 - 6x + 10y = 34(z^2 - 1)$$

$$x^2 - 6x + 9 + y^2 + 10y + 25 = 34z^2$$

$$(x-3)^2 + (y+5)^2 = 34z^2. \tag{1}$$

Take $\alpha = x - 3,\ \beta = y + 5.$

$$\alpha^2 + \beta^2 = 34z^2 \tag{2}$$

However, we have supplementary patterns of solutions which are illustrated as follows:

Pattern 1. Let $\alpha = 3u + 5v$ and $\beta = 5u - 3v.$ Then $34(u^2 + v^2) = 34z^2$ and the equation (2) becomes $u^2 + v^2 = z^2.$

We obtain $u = k(m^2 - n^2),\ v = 2kmn,\ z = k(m^2 + n^2),$ for some integers $k,\ m$ and n with $m > n,$ hence the solution is

$$x = k(3m^2 - 3n^2 + 10mn) + 3$$

$$y = k(5m^2 - 5n^2 - 6mn) - 5$$

$$z = k(m^2 + n^2).$$

Pattern 2. Let $\alpha = 3u - 5v$ and $\beta = 5u + 3v.$ Then $34(u^2 + v^2) = 34z^2$ and the equation (2) becomes $u^2 + v^2 = z^2.$

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$$z = k(m^2 + n^2).$$

Pattern 3. Assume

$$z = z(a, b) = a^2 + b^2, \text{ where } a, b > 0 \tag{3}$$

and write

$$34 = 3^2 + 5^2 \text{ as } 3^2 + 5^2 = (3 + 5i)(3 - 5i). \tag{4}$$

Substituting (3) and (4) in (2) and employing the development of factorization, Write

$$\alpha + i\beta = (3 + 5i)(a + ib)^2.$$

Equating the real and imaginary parts in the on top of equation, we acquire

$$\alpha = 3(a^2 - b^2) - 10ab$$

$$\beta = 5(a^2 - b^2) + 6ab, \text{ here } a > b \text{ and } a, b \in Z - \{0\}.$$

Hence the solution is

$$x = 3(a^2 - b^2) - 10ab + 3$$

$$y = 5(a^2 - b^2) + 6ab - 5.$$

Pattern 4. We can also write

$$34 = 3^2 + 5^2 \text{ as } 3^2 + 5^2 = (5 + 3i)(5 - 3i). \tag{5}$$

Substituting (3) and (5) in (1) and employing the technique of factorization, write $\alpha + i\beta = (5 + 3i)(a + ib)^2$.

Equating the real and imaginary parts in the on top of equation, we get

$$x = 5(a^2 - b^2) - 6ab + 3$$

$$y = 3(a^2 - b^2) + 10ab - 5$$

$$z = a^2 + b^2$$

which represents the dissimilar integer points on the cone (1).

Pattern 5. Equation (4) can also be written in the subsequent technique:

$$34 = 3^2 + 5^2 = (-3 + 5i)(-3 - 5i).$$

Proceeding as above, we get

$$x = -3(a^2 - b^2) - 10ab + 3$$

$$y = 5(a^2 - b^2) - 6ab - 5$$

$$z = a^2 + b^2,\ a^2 > b^2 \text{ and } a, b \in Z - \{0\}.$$

Pattern 6. Equation (4) can also be written in the subsequent technique:

$$34 = 3^2 + 5^2 = (-5 + 3i)(-5 - 3i).$$

Proceeding as on top of, we get

$$x = -5(a^2 - b^2) - 6ab + 3$$

$$y = 3(a^2 - b^2) - 10ab - 5$$

$$z = a^2 + b^2,\ a^2 > b^2 \text{ and } a, b \in Z - \{0\}.$$

Pattern 7. Equation (1) can be written as

$$\frac{\alpha + 3z}{5z + \beta} = \frac{5z - \beta}{\alpha - 3z} = \frac{p}{q}, \text{(say)}, q \neq 0. \tag{6}$$

This equation is equal to the following two equations:

$$q\alpha - p\beta + (3q - 5p)z = 0,$$

$$p\alpha + q\beta - (3p + 5q)z = 0.$$

By the system of cross multiplication, we get the integral solutions of (1) to be

$$x = 3p^2 + 2q^2 + 5pq + 3$$

$$y = -5p^2 + 5q^2 + 6pq - 5$$

$$z = p^2 + q^2.$$

Pattern 8. Equation (1) can be written as

$$\frac{5z + \beta}{\alpha + 3z} = \frac{\alpha - 3z}{5z - \beta} = \frac{p}{q}, \ (say), \ q \neq 0.$$

This equation is consequent to the following two equations:

$$p\alpha - q\beta + (3p - 5q)z = 0,$$

$$q\alpha + p\beta - (3q + 5p)z = 0.$$

By the way of cross multiplication, we get the integral points of (1) to be

$$x = q(3q + 5p) - p(3p - 5q) + 3$$

$$y = p(3q + 5p) + q(3p - 5q) - 5$$

$$z = p^2 + q^2.$$

Note 1. Equation (2) can be written as

$$\frac{\alpha - 3z}{5z + \beta} = \frac{5z - \beta}{\alpha + 3z} = \frac{p}{q}, \ (say), \ q \neq 0.$$

Proceeding as on top of, we get

$$x = -p(3p - 5q) + q(3q + 5p) + 3$$

$$y = -p(3q + 5p) - q(3p - 5q) - 5$$

$$z = q^2 + p^2.$$

Note 2. Applying the above technique to the case,

$$\frac{\alpha - 3z}{5z - \beta} = \frac{5z + \beta}{\alpha + 3z} = \frac{p}{q}, \text{(say)}, \ q \neq 0$$

we get hold of the solution of (1) as

$$x = p(3p - 5q) - q(3q + 5p) + 3$$

$$y = -p(3q + 5p) - q(3p - 5q) - 5$$

$$z = -q^2 - p^2.$$

Pattern 9. Equation (2) can be written as

$$(3^2 + 5^2)z^2 - \beta^2 = \alpha^2 * 1 \tag{7}$$

Assume

$$\alpha = (3^2 + 5^2)a^2 - b^2, \text{ where } a, b > 0 \tag{8}$$

Write 1 as

$$1 = \frac{(\sqrt{(3^2 + 5^2)} + 3)(\sqrt{(3^2 + 5^2)} - 3)}{5^2}. \tag{9}$$

Using (8) and (9) in (7) and applying the manner of factorization,

$$\sqrt{(3^2 + 5^2)}z + \beta = \frac{(\sqrt{(3^2 + 5^2)}a + b)^2(\sqrt{(3^2 + 5^2)} + 3)}{5}.$$

Equating the rational and irrational factors, we get hold of

$$x = (3^2 + 5^2)a^2 - b^2 + 3$$

$$y = \frac{1}{5}[3((3^2 + 5^2)a^2 + b^2) + 2(3^2 + 5^2)ab] - 5$$

$$z = \frac{1}{5}[(3^2 + 5^2)a^2 + b^2 + 6ab].$$

Because our concentration centers on decision integral solutions, substitute a by $5A$ and b by $5B$ in the on top of equations. Consequently the equivalent solutions to (1) are given by

$$x = 5^2[(3^2 + 5^2)A^2 - B^2] + 3$$

$$y = 5[3(34A^2 + B^2) - 2(34)AB] - 5$$

$$z = 5[(34)A^2 + B + 6AB].$$

Note 3. Equation (9) can be written as

$$1 = \frac{(-\sqrt{(3^2 + 5^2)} + 3)(-\sqrt{(3^2 + 5^2)} - 3)}{5^2}.$$

Proceeding as beyond, we get hold of

$$x = 5^2[(34)A^2 - B^2] + 3$$

$$y = 5[3((34)A^2 + B^2) - 2(3^2 + 5^2)AB] - 5$$

$$z = 5[6AB - (34)A^2 - B^2].$$

Pattern 10. Instead of (8) we can also write 1 as

$$1 = \frac{(\sqrt{(3^2 + 5^2)} + 5)(\sqrt{(3^2 + 5^2)} - 5)}{3^2}.$$

Thus the analogous solutions to (1) are agreed for the choice $a = 3A,\ b = 3B$ by

$$x = 3^2[(34)A^2 - B^2] + 3$$

$$y = 3[5((34)A^2 + B^2) + 2(34)AB] - 5$$

$$z = 3[(34)A^2 + B^2 + 10AB].$$

Note 4. Equation (8) can be write as

$$1 = \frac{(-\sqrt{(3^2 + 5^2)} + 5)(-\sqrt{(3^2 + 5^2)} - 5)}{3^2}.$$

Proceeding as on top of, we get

$$x = 3^2[(34)A^2 - B^2] + 3$$

$$y = 3[5((34)A^2 + B^2) - 2(34)AB] - 5$$

$$z = 3[10AB - (34)A^2 - B^2].$$

III. Conclusion

The ternary quadratic Diophantine equations are prosperous in diversity. One possibly will search for further choices of Diophantine equations to discover their consequent integer solutions.

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