Advances and Applications in Mathematical Sciences
Volume 20, Issue 5, March 2021, Pages 749-760

GEO/GEO/1 SWV QUEUE WITH DISCRETE TIME IN FUZZY PARAMETERS

G. KANNADASAN and N. SATHIYAMOORTHI

Department of Mathematics
Annamalai University
Annamalainagar-608 002, India
E-mail: klsk.g.21@gmail.com
n.satyamurthi@gmail.com

Abstract

This study investigates the analysis of the discrete time Geo/Geo/1 single working vacation (SWV) queue with fuzzy parameter. In this paper, we obtains some performance measure as probability of the server is in idle period, probability of server is in regular busy period, membership function of the mean queue length and membership function of the mean sojourn time. Finally, numerical results are presented to show the effects of system parameters.

1. Introduction

The queuing systems with server vacations have been well investigated and the theoretical framework whose core is the stochastic decomposition theory has been established. In the models with various vacation policies, the server completely stops service in the vacation period, but he can take the assistant work. The research work of the vacation queues has been extensively used in the computer networks, communication systems and production management et al. Details can be seen in the surveys of Doshi [3] and the monographs of Takagi [7] and Tian and Zhang [6].

Servi and Finn [5] were first introduced a class of semi-vacation policy, called the working vacation: a server does not completely stop service but serves customers at a lower rate during a vacation period. The authors studied an $M/M/1$ queue with working vacations, and gave distributions of stationary queue length and the waiting time of a customer in steady state.

2010 Mathematics Subject Classification: 68M20, 90B22, 6025K.
Keywords: Geo/Geo/1 queue; queue length; sojourn time.
Received January 8, 2020; Accepted May 22, 2020

The fuzzy queuing model with working vacation (wv), we introduced on 2017. We are working all the performance measures on fuzzy queues with working vacation.

2. The Fuzzy Model

In this section the arrival rate, service rate, the server switches service rate and working vacation time are assumed to be fuzzy numbers $\bar{\lambda}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\theta}$ respectively. Now

$$\bar{\lambda} = \{x, \mu_{\bar{\lambda}}(x); x \in S(\bar{\lambda})\},$$

$$\bar{\gamma}_1 = \{y_1, \mu_{\bar{\gamma}_1}(y_1); y_1 \in S(\bar{\gamma}_1)\},$$

$$\bar{\gamma}_2 = \{y_2, \mu_{\bar{\gamma}_2}(y_2); y_2 \in S(\bar{\gamma}_2)\},$$

$$\bar{\theta} = \{z, \mu_{\bar{\theta}}(z); z \in S(\bar{\theta})\}.$$

Where $S(\bar{\lambda}), S(\bar{\gamma}_1), S(\bar{\gamma}_2)$ and $S(\bar{\theta})$ are the universal set's of the arrival rate, service rate, the server switches service rate and working vacation time respectively.

Define $f(x, y_1, y_2, z)$ as the system performance measure related to the above defined fuzzy queuing model which depends on the fuzzy membership function $f(\bar{\lambda}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\theta})$.

Applying Zadeh's extension principle [8] the membership function of the performance measure $f(\bar{\lambda}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\theta})$ can be defined as

$$\mu_{\bar{f}(\bar{\lambda}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\theta})}(H) = \sup_{\substack{x \in S(\bar{\lambda}) \\ y_1 \in S(\bar{\gamma}_1) \\ y_2 \in S(\bar{\gamma}_2) \\ z \in S(\bar{\theta})}} \{\min\{(\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \mu_{\bar{\theta}}(z))/H$$

$$= f(x, y_1, y_2, z\}\}. \tag{1}$$

We obtain the following membership function of some performance measures.

$$\mu_{\overline{P_1}}(B) = \sup_{\substack{x \in S(\bar{\lambda}) \\ y_1 \in S(\bar{\gamma}_1) \\ y_2 \in S(\bar{\gamma}_2) \\ z \in S(\bar{\theta})}} \{\min\{(\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \mu_{\bar{\theta}}(z))/B\}\}, \tag{2}$$

where, $B = \dfrac{(y_1z - xy_1z)^2 \times \left[\dfrac{(y_1 - xy_1) - (x - xy_1)}{(y_1 - xy_1)}\right] \times (a_1)}{(xy_1 - x^2y_1) \times (a_2) + (a_3) + y_1 + x^2 z}$

where, $a_1 = \left[\dfrac{2x - 2xy_2z + 3(1 - z - x + xz) - A}{(2x - 2xz)}\right]$

$$a_2 = \left[\frac{2xz - 2zy_2z + (1 - z - x + xz)y_2(2x - 2xy_2) - A + (2x - 2xy_2)(x - xz)}{(2x - 2xy_2)}\right]$$

$$a_3 = \left[\frac{(2xz - 2x^2z)(1 - y_2) - (x - xz) \cdot A}{(2x - 2xy_2)}\right]$$

$$\mu_{\overline{P_2}}(C) = \sup_{\substack{x \in S(\bar{\lambda}) \\ y_1 \in S(\bar{\gamma}_1) \\ y_2 \in S(\bar{\gamma}_2) \\ z \in S(\bar{\theta})}} \{\min\{(\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \mu_{\bar{\theta}}(z))/C\}\}, \tag{3}$$

where,

$$C = \frac{(xz - x^2z)(e)\left[\dfrac{(2x(1 - y_2)) - A + 2x^2z - 2x^2zy_2}{2x(1 - y_2)}\right]}{(xy_1 - x^2y_1)\left[\dfrac{(2x - 2xy_2) - A}{(2x - 2xy_2)}\right]\left[\dfrac{y_2 - xy_2 - x + xy_1}{(y_1 - xy_1)}\right](f) + (g)y_1 + x^2z}$$

where,

$$e = \left[\frac{4x - 2x^2 - 4xy_2 - 4xz - 4xz + 2x^2y_2 + 4xy_2z + ((2x^2y_2 - 2x^2y_2z) - A)y_2}{2x - 2xy_2}\right]$$

$$f = \left[\frac{(2xz - 2xzy_2)((2x - 2xy_2) - A) + (2x^2 - 2x^2z)(1 - y_2)}{(2x - 2xy_2)}\right]$$

$$g = \left[\frac{(z - zx)(2xz - 2xy_2z + 2y_2 - 2y_2z - 2xy_2 + 2xy_2z)(2x - 2xy_2) - A}{(2x - 2xy_2)}\right]$$

$$\mu_{\overline{E(L)}}(D) = \sup_{\substack{x \in S(\bar{\lambda}) \\ y_1 \in S(\bar{\gamma}_1) \\ y_2 \in S(\bar{\gamma}_2) \\ z \in S(\bar{\theta})}} \{\min\{(\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \mu_{\bar{\theta}}(z))/D\}\}, \tag{4}$$

where,

$$D = \frac{\left(\frac{x - xy_1}{y_1 - x}\right) + (m) \times (xy_1 - x^2 y_1)\left[\frac{(2x - 2xy_2) - A}{(2x - 2xy_2)}\right] + (x - xz)\left[\frac{(2x - 2xz) - A}{A}\right] \times (l)}{\frac{(2x - 2xz)A}{2x - 2xz}}$$

where,

$$l = \left[\frac{xy_2 - xy_1 y_2 + (z - zy_2)(y_1 - xy_1)}{(y_1 - xy_1)}\right] + (x^2 - x^2 z)\left[\frac{(3x - 3xz) + Ax}{2x - 2xz}\right]\left[\frac{(2x - 2xz) - A}{A}\right]$$

$$(y_1 - y_2)$$

$$m = \left\{(y_1 x - x^2 y_1)\left(\frac{(2x - 2xy_2) - A}{(2x - 2xy_2)}\right)\left(\frac{y_1 - x}{y_1 - xy_1}\right) \times (n) + (z - zx)\left[\frac{(2x - 2xy_2) - A}{2x - 2xy_2}\right] \times (o)\right\}$$

$$n = \left[\frac{2xy_2 z + y_2 - y_2 z - xy_2 + xy_2 z(2x - 2xy_2) - A + 2x^2 - 2x^2 y_2(1 - z)}{(2x - 2xy_2)}\right]$$

$$o = \left\{\left[\frac{(2xz - 2xy_2 z + y_2 - y_2 z - xy_2 + xy_2 z((2x - 2xy_2) - A)y_1 + 2x^3 z - 2x^3 y_2 z)}{(2x - 2xy_2)}\right]\right\}^{-1}$$

$$\mu_{\overline{EW}}(E) = \sup_{\substack{x \in S(\bar{\lambda}) \\ y_1 \in S(\bar{\gamma}_1) \\ y_2 \in S(\bar{\gamma}_2) \\ z \in S(\bar{\theta})}} \{\min\{(\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}_1}(y_1), \mu_{\bar{\gamma}_2}(y_2), \mu_{\bar{\theta}}(z))/E\}\}, \tag{5}$$

where,

$$E = \left[\frac{(y_1 - xy_1)}{y_1^2 - xy_1}\right] + \left[\frac{2x^3 - 2x^3 z - Ay_1 - Ay_2}{A}\right] \times \left[\frac{2x^2 - 2x^2 y_2 + A \cdot x}{2x^2 - 2x^2 y_2 - A \cdot x}\right] \times m.$$

Using the fuzzy analysis technique explained we can find the membership of $\overline{P_1}$, $\overline{P_2}$, $\overline{E(L)}$ and $\overline{E(W)}$ as a function of the parameter α, thus the α-cut approach can be used to develop the membership function of $\overline{P_1}$, $\overline{P_2}$, $\overline{E(L)}$ and $\overline{E(W)}$.

3. Performance of Measure

Probability of the server is in idle period

By Zadeh's extension principle $\mu_{\overline{P_1}}(B)$ is the superimum of minimum over $\{(\mu_{\bar{\lambda}}(x), \mu_{\overline{\gamma_1}}(y_1), \mu_{\overline{\gamma_2}}(y_2), \mu_{\bar{\theta}}(z))\}$ to satisfying $\mu_{\overline{P_1}}(B) = \alpha$, $0 < \alpha \leq 1$. We consider the following four cases

Case (i) $\mu_{\bar{\lambda}}(x) = \alpha$, , $\mu_{\overline{\gamma_1}}(y_1) \geq \alpha$, $\mu_{\overline{\gamma_2}}(y_2) \geq \alpha$, $\mu_{\bar{\theta}}(z) \geq \alpha$.

Case (ii) $\mu_{\bar{\lambda}}(x) \geq \alpha$, $\mu_{\overline{\gamma_1}}(y_1) = \alpha$, $\mu_{\overline{\gamma_2}}(y_2) \geq \alpha$, $\mu_{\bar{\theta}}(z) \geq \alpha$.

Case (iii) $\mu_{\bar{\lambda}}(x) \geq \alpha$, $\mu_{\overline{\gamma_1}}(y_1) \geq \alpha$, $\mu_{\overline{\gamma_2}}(y_2) = \alpha$, $\mu_{\bar{\theta}}(z) \geq \alpha$.

Case (iv) $\mu_{\bar{\lambda}}(x) \geq \alpha$, , $\mu_{\overline{\gamma_1}}(y_1) \geq \alpha$, $\mu_{\overline{\gamma_2}}(y_2) \geq \alpha$, $\mu_{\bar{\theta}}(z) = \alpha$.

For case (i) the lower and upper bound of α-cuts of $\overline{P_1}$ can be obtained through the corresponding parametric non-linear programs,

$$[P_1]_\alpha^{L_1} = \min_{\Omega} \{[B]\} \text{ and } [P_1]_\alpha^{U_1} = \max_{\Omega} \{[B]\}$$

Similarly, we can calculate the lower and upper bounds of the α-cuts of $\overline{P_1}$ for the case (ii), (iii) and (iv).

By considering the cases, simultaneously the lower and upper bounds of the α-cuts of $\overline{P_1}$ can be written as

$$[P_1]_\alpha^{L} = \min_{\Omega} \{[B]\} \text{ and } [P_1]_\alpha^{U} = \max_{\Omega} \{[B]\}$$

such that, $x_\alpha^L \leq x \leq x_\alpha^U$, $y_{1\alpha}^L \leq y_1 \leq y_{1\alpha}^U$, $y_{2\alpha}^L \leq y_2 \leq y_{2\alpha}^U$, $z_\alpha^L \leq z \leq z_\alpha^U$. If both $(P_1)_\alpha^L$ and $(P_1)_\alpha^U$ are invertible with respect to α, the left and right shape function, $L(B) = [(P_1)_\alpha^L]^{-1}$ and $R(B) = [(P_1)_\alpha^U]^{-1}$ can be derived from which the membership function $\mu_{\overline{P_1}}(B)$ can be constructed as

$$\mu_{\overline{P_1}}(B) = \begin{cases} L(B), (P_1)_{\alpha=0}^L \leq B \leq (P_1)_{\alpha=0}^U \\ 1, (P_1)_{\alpha=0}^L \leq B \leq (P_1)_{\alpha=1}^U \\ R(B), (P_1)_{\alpha=1}^L \leq B \leq (P_1)_{\alpha=0}^U \end{cases}. \quad (6)$$

In the same way we get the following results.

Probability of the server is in regular busy period

$$\mu_{\overline{P_2}}(C) = \begin{cases} L(C), \ (P_2)^L_{\alpha=0} \le C \le (P_2)^U_{\alpha=0} \\ 1, \ (P_2)^L_{\alpha=0} \le C \le (P_2)^U_{\alpha=1} \\ R(C), \ (P_2)^L_{\alpha=1} \le C \le (P_2)^U_{\alpha=0} \end{cases}. \quad (7)$$

Membership function of the mean queue length

$$\mu_{\overline{E(Q)}}(D) = \begin{cases} L(D), \ (E(Q))^L_{\alpha=0} \le D \le (E(Q))^U_{\alpha=0} \\ 1, \ (E(Q))^L_{\alpha=0} \le D \le (E(Q))^U_{\alpha=1} \\ R(D), \ (E(Q))^L_{\alpha=1} \le D \le (E(Q))^U_{\alpha=0} \end{cases}. \quad (8)$$

Membership function of the mean sojourn time

$$\mu_{\overline{E(W)}}(E) = \begin{cases} L(E), \ (E(W))^L_{\alpha=0} \le E \le (E(W))^U_{\alpha=0} \\ 1, \ (E(W))^L_{\alpha=0} \le E \le (E(W))^U_{\alpha=1} \\ R(E), \ (E(W))^L_{\alpha=1} \le E \le (E(W))^U_{\alpha=0} \end{cases}. \quad (9)$$

4. Numerical Study

Probability of the server is in idle period

Suppose the arrival rate $\overline{\lambda}$, the service rate $\overline{\gamma_1}$, the server switches service rate $\overline{\gamma_2}$ and working vacation time $\overline{\theta}$ are assumed to be trapezoidal fuzzy numbers described by $\overline{\lambda} = [41, 42, 43, 44]$, $\overline{\gamma_1} = [71, 72, 73, 74]$, $\overline{\gamma_2} = [91, 92, 93, 94]$ and $\overline{\theta} = [31, 32, 33, 34]$ per mins respectively. Then

$$\lambda(\alpha) = \min_{x \in S(\overline{\lambda})} \{x \in S(\overline{\lambda}), G(x) \ge \alpha\}, \ \max_{x \in S(\overline{\lambda})} \{x \in S(\overline{\lambda}), G(x) \ge \alpha\},$$

where

$$G(x) = \begin{cases} x - 41, \ 41 \le x \le 42 \\ 1, \ 42 \le x \le 43 \\ 44 - x, \ 43 \le x \le 14 \end{cases}$$

That is, $\lambda(\alpha) = [41 + \alpha, \ 44 - \alpha]$, $\gamma_1(\alpha) = [71 + \alpha, \ 74 - \alpha]$,

$$\gamma_2(\alpha) = [91 + \alpha, \ 94 - \alpha] \text{ and } \theta(\alpha) = [31 + \alpha, \ 34 - \alpha].$$

It is clear that, when $x = x_\alpha^U$, $y_1 = y_{1\alpha}^U$, $y_2 = y_{2\alpha}^U$ and $z = z_\alpha^U$, H attains its maximum value and when $x = x_\alpha^L$, $y_1 = y_{1\alpha}^L$, $y_2 = y_{2\alpha}^L$ and $z = z_\alpha^L$, H attains its minimum value.

From the generated for the given input values of $\overline{\lambda}$, $\overline{\gamma_1}$, $\overline{\gamma_2}$ and $\overline{\theta}$.

(i) For fixed values of x, y_1 and y_2, B decreases as z increase.

(ii) For fixed values of y_1, y_2 and z, B decreases as x increase.

(iii) For fixed values of y_2, z and x, B decreases as y_1 increase.

(iv) For fixed values of z, x and y_1, B decreases as y_2 increase.

The smallest value of P_1 occurs, when x-takes its lower bound, i.e., $x = 41 + \alpha$ and y_1, y_2 and z, take their upper bounds given by $y_1 = 74 - \alpha$, $y_2 = 94 - \alpha$ and $z = 34 - \alpha$ respectively. And the maximum value of P_1 occurs when $x = 44 - \alpha$, $y_1 = 71 + \alpha$, $y_2 = 91 + \alpha$, $z = 31 + \alpha$. If both $[P_1]_\alpha^L$ and $[P_1]_U^\alpha$ are invertible with respect to α' then, the left shape function $L(B) = [(P_1)_\alpha^L]^{-1}$ and right shape function $R(B) = [(P_1)_\alpha^U]^{-1}$ can be obtained and from which the membership function $\mu_{\overline{P_1}}(B)$ can be constructed as

$$\mu_{\overline{P_1}}(B) = \begin{cases} L(B), & B_1 \le B \le B_2 \\ 1, & B_2 \le B \le B_3 \\ R(B), & B_3 \le B \le B_4 \end{cases}. \tag{10}$$

The values of B_1, B_2, B_3 and B_4 as obtained from (10) are

$$\mu_{\overline{P_1}}(B) = \begin{cases} L(B), & 0.0759 \le B \le 0.1216 \\ 1, & 0.1216 \le B \le 0.1975 \\ R(B), & 0.1975 \le B \le 0.3481 \end{cases}.$$

In the same way we get the following results.

Probability of the server is in regular busy period

$$\mu_{\overline{P_2}}(C) = \begin{cases} L(C), & 0.0214 \le C \le 0.2752 \\ 1, & 0.2752 \le C \le 0.4133, \\ R(C), & 0.4133 \le C \le 0.8628 \end{cases}. \tag{11}$$

Membership function of the mean queue length

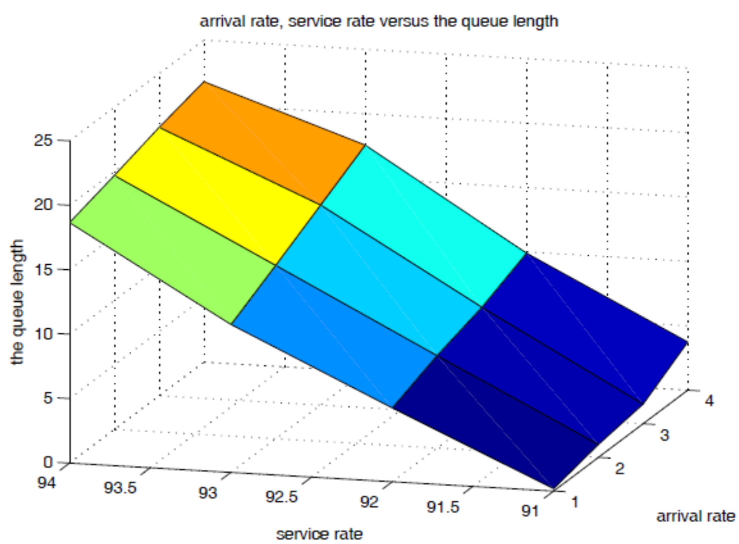
$$\mu_{\overline{E(Q)}}(D) = \begin{cases} L(D), & 0.1798 \le D \le 1.5183 \\ 1, & 1.5193 \le D \le 5.7284 \\ R(D), & 5.7284 \le D \le 21.7734 \end{cases}. \tag{12}$$

Membership function of the mean sojourn time

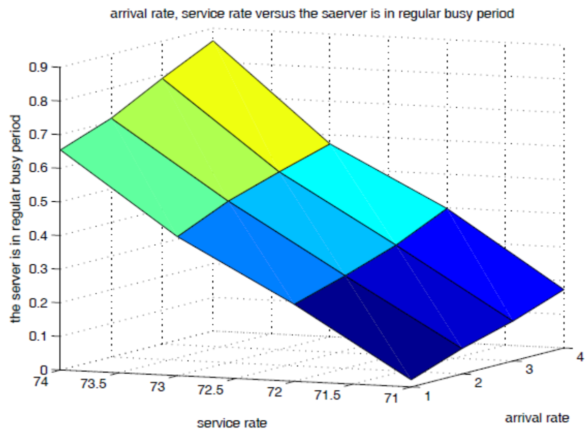
$$\mu_{\overline{E(W)}}(E) = \begin{cases} L(E), & 0.3021 \le E \le 0.4341 \\ 1, & 0.4341 \le E \le 0.6078 \\ R(E), & 0.6078 \le E \le 0.9720 \end{cases}. \tag{13}$$

Further by fixed the vacation rate by a crisp value $\overline{\theta} = 32.4$ and $\overline{\gamma_2} = 71.3$ taking arrival rate $\overline{\lambda} = [41, 42, 43, 44]$ service rate $\overline{\gamma_1} = [71, 72, 73, 74]$ both trapezoidal fuzzy numbers the values of the regular busy period are generated and are plotted in the figure 1, it can be observed that as $\overline{\lambda}$ increases the server is in idle period increases for the fixed value of the service rate, where as for fixed value of arrival rate, the server is in idle period decreases as service rate increases. Similar conclusion can be obtained for the case $\overline{\theta} = 32.6$, $\overline{\gamma_2} = 72.3$. Again for fixed values of taking $\overline{\lambda} = [41, 42, 43, 44]$, $\overline{\gamma_1} = [71, 72, 73, 74]$ the graphs of the regular busy period are drawn in figure 2 respectively, these figure show that as arrival rate increases that the sojourn time also increases, while the sojourn time decreases as the service rate increases in both the case.

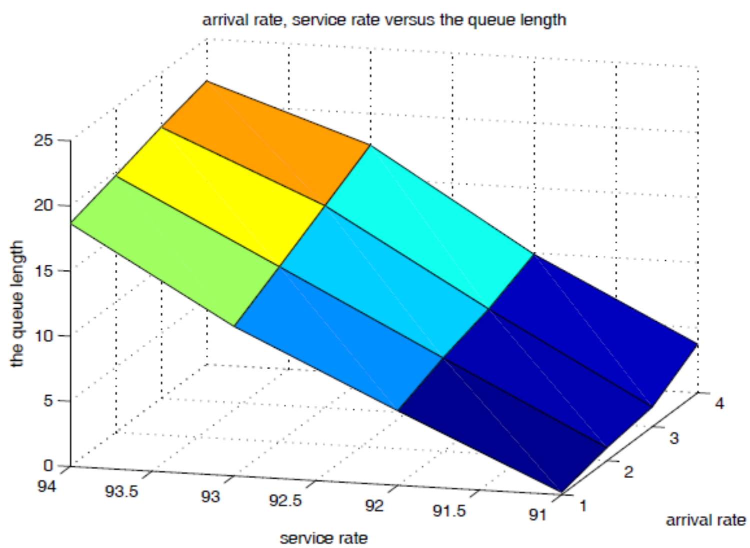
It is also observed from the data generated that the membership value of the server is in idle period is 0.41, the membership value of the server is in regular busy period 0.46, the queue length 21.7, and the vacation time 0.97 when the ranges of arrival rate, service rate, and the vacation rate lie in the intervals (42.3, 44.1), (71.4, 73.6), (92.7, 93.4) and (31.8, 43) respectively.



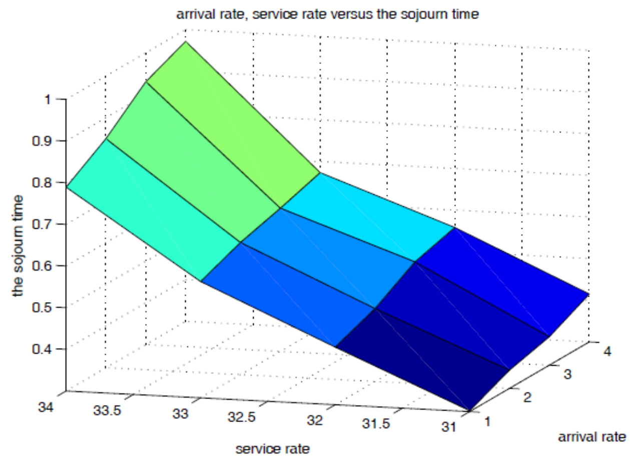
Arrival rate, service rate versus the probability of the server is in idle period



Arrival rate, service rate versus the probability of the server is in regular busy period



Arrival rate, service rate versus membership function of the mean queue length



Arrival rate, service rate versus membership function of the mean sojourn time

5. Conclusion

In this paper, we have studied "The Geo/Geo/1 SWV queue with discrete time in fuzzy parameters". We have obtained the probability of the server is in idle period, probability of the server is in regular busy period, membership function of the mean queue length and membership function of the mean sojourn time. We have obtained numerical results to all the performance measure for this fuzzy queues.

References

[1] A. S. Alfa, Vacation models in discrete time, Queueing Systems, 44 (2003), 5-30.

[2] Y. Baba, Analysis of a GI/M/1 queue with multiple working vacations. Operations Research Letters 33 (2005), 201-209.

[3] B. T. Doshi, Single server queues with vacations, in: H. Takagi (Ed.). Stochastic Analysis of the Computer and Communication Systems. North-Holland Elsevier, Amsterdam (1990), 217-264.

[4] T. Meisling, Discrete time queueing theory. Operation Research, 6 (1958), 96-105.

[5] L. D. Servi and S. G. Finn, M/M/1 queue with working vacations (M/M/1/WV). Performance Evaluation, 50 (2002), 41-52.

[6] N. Tian and Z. G. Zhang, The discrete time GI/Geo/1 queue with multiple vacations, Queueing Systems 40 (2002), 283-294.

[7] D. Wu and H. Takagi, M/G/1 queue with multiple working vacation. Performance Evaluation 63(7) (2006), 654-681.

[8] L. A. Zadeh, Fuzzy sets as a basis a theory of possibility, Fuzzy sets and Systems 1 (1978), 3-28.