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LOCAL NECKS OF SINGLE VALUED NEUTROSOPHIC AUTOMATA

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Abstract

The purpose of this paper is to study the characterizations of lock necks of single valued neutrosophic automaton (SVNA). We define local neck, monogenically directable, strongly directable, trap-directable, common directing word, uniformly monogenically (strongly) directable, uniformly monogenically (strongly) directable, locally direct able and ω-relation of single valued neutrosophic automaton. Further, we describe the properties of local necks of a SVNA. We show that if local exists in SVNA, then it is sub automaton of S and some equivalent conditions of S.

1. Introduction

In 1965 [23] Lofti A. Zadeh suggested Fuzzy's theory of the set, which generalizes conventional set theory. The fuzzy set is a simple mathematical tool for representing the inherent vagueness, uncertainty, and imprecision in everyday life. W.G. used the fuzzy concept in automata in 1967, [22]. Later, many researchers applied the fuzzy concept in a variety of fields, and it has a wide range of applications. Doostfatemeh introduced general fuzzy automata in [4].

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Intuitionist fuzzy set, bipolar fuzzy set, vague set in different fields were therefore developed and applied. In 1998 [20], the concept of neutrosophy and neutrosophic set was further developed by F. Smarandache for the generalisation of all of the above-mentioned set. Subsequently, in [21], Wang et al. introduced single valued and interval valued neutrosophical sets. Neutrosophical systems have recently had significant applications in various fields, especially in decision making issues relating to multiple criteria.

A single valued and interval neutroophical finite automates were introduced by Tahir Mahmoud et al. in [16]. Later on, neutrosophic general finite automata and neutrosophic composite finite automaton [14, 15] have been introduced by J. Kavikumar et al. Directable automata are also known as synchronizable and reset automata. It has numerous applications in a variety of fields. Many authors have contributed to the advancement of directable automata and generalized directable automata, among other things. T. Petkovic et al. [3] introduced and studied directable automata with necks, trap-directable, trapped, monogenically directable automata, and trap-directable automata. Further, it is also studied by *Z*. Popovic et al. in [18] and [19]. M. Bogdanovic et al. [2] studied directable automata, and their generalizations.

Decompositons automata and transition semigroups were studied by T. Petkovic et al. [17]. Also, necks and local necks of automata were discussed in [1] Consequently, necks of fuzzy automata was introduced and discussed in [5] and local necks of fuzzy automaton was discusses in [6]. Further directable fuzzy automata, μ-direct able fuzzy automata were discussed in [7, 8]. Least directing congruence on fuzzy automata was discussed in [9]. Generalized direct able fuzzy automata and their characterizations was discussed in [11]. Also generalized products of Δ-synchronized fuzzy automata, generalized products of direct able fuzzy automata were studied in the papers [12, 13]. Applications related to γ-synchronized fuzzy automata were studied in [10].

We define local neck, monogenically direct able, strongly directable, trap-directable, common directing word, uniformly monogenically (strongly) directable, uniformly monogenically (strongly) direct able, locally direct able and ω-relation of single valued neutrosophic automaton. Further, we describe the properties of local necks of a SVNA. We show that if local exists in SVNA, then it is sub automaton of S and some equivalent conditions of S.

2. Preliminaries

2.1. Neutrosophic Set [20]. Let E be the universal set. A neutrosophic set (NS) T in E is classified by a truth χ_T, indeterminacy ψ_T and a falsity membership τ_T, where χ_T, ψ_T, and τ_T are real standard or non-standard subsets of $]0^-, 1^+[$.

$T = \{\langle e, (\chi_T(e), \psi_T(e), \tau_T(e)) \rangle, e \in E, \chi_T, \psi_T, \tau_T \in]0^-, 1^+[\}$ and $0^- \leq \sup \chi_T(e) + \sup \psi_T(e) + \sup \tau_T(e) \leq 3^+$. We use $[0, 1]$ instead of $]0^-, 1^+[$.

2.2 Single Valued Neutrosophic Set [21]. Let E be the universe of discourse. A single valued neutrosophic set (SVNS) T in E is classified by a truth χ_T, indeterminacy ψ_T and a falsity membership τ_T. $T = \{\langle e, (\chi_N(e), \psi_T(e), \tau_T(e))\rangle, e \in E, \chi_T, \psi_T, \tau_T \in [0, 1]\}$.

2.3. Definition [16]. $S = (D, I, T)$ is called single valued neutrosophic automaton (SV NA for short); where D and I are non-empty set of states and input symbols respectively, and $T = \{\langle e, (\chi_T(e), \psi_T(e), \tau_T(e))\rangle\}$ is an SV NS in $D \times I \times D$. The set of all words of I is denoted by I^*. The empty word is denoted by λ, and the length of each $e \in I^*$ is denoted by $|e|$.

2.4 Definition [16]. $S = (D, I, T)$ be an SVNA. Define an SV NS $T^* = \{\langle e, (\chi_T(e), \psi_T(e), \tau_T(e))\rangle\}$ in $D \times I^* \times D$ by

$$\chi_{T^*}(d_i, \lambda, d_j) = \begin{cases} 1 & \text{if } d_i = d_j \\ 0 & \text{if } d_i \neq d_j \end{cases}$$

$$\psi_{T^*}(d_i, \lambda, d_j) = \begin{cases} 0 & \text{if } d_i = d_j \\ 1 & \text{if } d_i \neq d_j \end{cases}$$

$$\tau_{T^*}(d_i, \lambda, d_j) = \begin{cases} 0 & \text{if } d_i = d_j \\ 1 & \text{if } d_i \neq d_j \end{cases}$$

$$\chi_{T^*}(d_i, ee', d_j) = \vee_{q_r \in D}[\chi_{T^*}(d_i, e, q_r) \wedge \chi_{T^*}(q_r, e', d_j)],$$

$$\psi_{T^*}(d_i, ee', d_j) = \wedge_{q_r \in D}[\psi_{T^*}(d_i, e, q_r) \vee \psi_{T^*}(q_r, e', d_j)],$$

$$\tau_{T^*}(d_i, ee', d_j) = \wedge_{q_r \in D}[\tau_{T^*}(d_i, e, q_r) \vee \tau_{T^*}(q_r, e', d_j)], \forall d_i, d_j \in D, e \in I^*$$

and $e' \in I$.

3. Local Necks of Single Valued Neutrosophic Automata

3.1 Definition. Let $S = (D, I, T)$ be an SVNA and $d_i \in D$. If d_i is called local neck of S. If it is neck of some directable sub automaton of S. The collection of all local necks of S is denoted by $LN(S)$.

3.2 Definition. Let $S = (D, I, T)$ be SVNA. If S is called monogenically directable if every monogenic sub automaton of S is directable.

3.3 Definition. Let $S = (D, I, T)$ be SVNA. If S is called monogenically strongly direct able then all monogenic sub automaton directable if every monogenic sub automaton of S is strongly direct able.

3.4 Definition. Let $S = (D, I, T)$ be SVNA. If S is called monogenically trap-direct able if every monogenic sub automaton of S has a single neck.

3.5 Definition. Let $S = (D, I, T)$ be SVNA. If $z \in I^*$ is said to be common directing word of S then z is a directing word of every monogenic sub automaton of S. The collection of all common directing words of S will be denoted by $CDW(S)$. In other words, $CDW(M) = \bigcap_{q_i \in D} DW(\langle d_i \rangle)$.

3.6 Definition. Let $S = (D, I, T)$ be SVNA. S is called uniformly monogenically (strongly) directable SVNA if every monogenic sub automaton of S is (strongly) directable and have at least one common directing word.

3.7 Definition. Let $S = (D, I, T)$ be SVNA. S is called uniformly monogenically trap- direct able SVNA if every monogenic sub automaton of S is has a single neck and have at least one common directing word.

3.8 Definition. Let $S = (D, I, T)$ be SVNA. A subset K of semi group J is called an ideal if $JKJ \subseteq I$.

3.9 Locally Direct able. Let $S = (D, I, T)$ be SVNA. S is called locally directable SVNA if each finitely generated sub automaton of S is directable SVNA.

3.10 ω-Relation. We define a relation ω on states set of an arbitrary SVNA $S = (D, I, T)$ as follows:

Let $d_i, d_j \in D$. $d_i \;\omega\; d_j \Leftrightarrow N(\langle d_i \rangle) = N(\langle d_j \rangle)$.

This relation is clearly an equivalence relation.

4. Properties of Local Necks of Single Valued Neutrosophic Automata

Theorem 4.1. *Let* $S = (D, I, T)$ *be SVNA and* $d_i \in D$. *Then the following conditions are equivalent.*

(i) d_i *is local neck of SVNA.*

(ii) $\langle d_i \rangle$ *is a strongly directable SVNA.*

(iii) $\forall z \in I^*, \exists z' \in I^*$ *such that* $\chi_{T^*}(d_i, zz', d_i) > 0, \psi_{T^*}(d_i, zz', d_i) < 1, \tau_{T^*}(d_i, zz', d_i) < 1$.

Proof.

$(i) \Rightarrow (ii)$

Let d_i be a local neck of S. Then $\exists$ a directable sub automaton S' of S such that $d_i \in N(S')$. Thus $N(S')$ is a strongly direct able SVNA. Also, $\langle d_i \rangle \subseteq N(S')$ and $N(S')$ is strongly connected, then $\langle d_i \rangle = N(S')$. Therefore, $\langle 2d_i \rangle$ is a strongly directable SVNA.

$(ii) \Rightarrow (ii)$

Let $\langle d_i \rangle$ be a strongly directable SVNA. Then $\langle d_i \rangle$ is a z-neck of $\langle d_i \rangle$ for some $z \in I^*$. Since $\langle d_i \rangle$ is strongly directable SVNA $\forall z' \in I^*$ there exists some $d_l \in \langle d_i \rangle$ such that $\chi_{T^*}(d_i, z', d_l) > 0, \psi_{T^*}(d_i, z', d_l) < 1$, $\tau_{T^*}(d_i, z', d_l) < 1$.

$\chi_{T^*}(d_i, z'z, d_i) = \wedge_{d_l \in D}\{\chi_{T^*}(d_i, z', d_l), \chi_{T^*}(d_l, z, d_i) > 0\}$,

$\psi_{T^*}(d_i, z'z, d_i) = \vee_{d_l \in D}\{\psi_{T^*}(d_i, z', d_l), \psi_{T^*}(d_l, z, d_i) < 1\}$,

$\tau_{T^*}(d_i, z', d_i) = \vee_{d_l \in D}\{\tau_{T^*}(d_i, z', d_l), \tau_{T^*}(d_l, z, d_i) < 1\}$,

$(ii) \Rightarrow (i)$

It clearly shows that d_i is a z-neck of $\langle q_i \rangle$, thus it is a local neck of S.

Theorem 4.2. *Let $S = (D, I, T)$ be SVNA. $LN(S) \neq \phi$ then $LN(S)$ is sub automaton of S.*

Proof. Let $d_i \in LN(S)$ and $y \in I$. Then the monogenic sub automaton $\langle d_i \rangle$ of S is strongly directable. $\chi_{T^*}(d_i, y, d_l) > 0, \psi_{T^*}(d_i, y, d_l) < 1$, $\tau_{T^*}(d_i, y, d_l) < 1$ for some $d_i \in \langle d_i \rangle$. Since $\langle d_i \rangle$ is strongly connected, $\langle d_i \rangle = \langle d_l \rangle$. Thus, d_l is also a local neck of S. $d_l \in LN(S)$. Hence, $LN(S)$ is a sub automaton of S.

Theorem 4.3. *Let $S = (D, I, T)$ be SVNA and S' be a sub automaton of S. Then $LN(S') = LN(S) \cap S'$.*

Proof. Let $d_i \in LN(S')$. Then $\exists$ a sub automaton S'' of S' such that $q_i \in N(S'')$. But S'' is also sub automaton of S, $d_i \in LN(S)$. Thus, $LN(S') \subseteq LN(S) \cap S'$.

Conversely, assume $d_i \in LN(S) \cap S'$. Then $d_i \in N(S_1')$, for some directable sub automaton S_1' of S. Let $S'' = S' \cap S_1'$. Then S'' is a sub automaton of S_1' and S'. Also, S'' is directable and $N(S'') = N(S_1')$. It follows that $d_i \in N(S'')$. Thus $d_i \in LN(S')$. Therefore, $LN(S) \cap S' \subseteq LN(S')$. Hence $LN(S') = LN(S) \cap S'$.

Theorem 4.4. *Let $S = (D, I, T)$ be SVNA. Then the following conditions are equivalent*:

(i) *Each state of D in S is a local neck*;

(ii) *S is monogenically strongly directable SVNA*;

(iii) *S is monogenically directable and reversible SVNA*;

(iv) *S is a direct sum of strongly directable SVNA*;

(v) $(\forall\ d_i \in D)\ (\exists\ z \in I^*)\ (\forall\ z' \in I^*)$ *such that* $\chi_{T^*}(d_i, z'z, d_i) > 0$, $\psi_{T^*}(d_i, z'z, d_i) < 1$, $\rho_{T^*}(d_i, z'z, d_i) < 1$.

Proof.

$(i) \Rightarrow (ii)$

If each state $d_i \in D$ is a local neck of S. Then for each $d_i \in D$ the monogenic sub automaton $\langle d_i \rangle$ of D in S is strongly directable.

Hence, S is monogenically strongly directable SVNA.

$(ii) \Rightarrow (iii)$

If S is monogenically strongly detectable SVNA, then it is monogenically directable SVNA. Suppose each monogenic sub automaton of S is strongly connected SVNA, then S is reversible SVNA.

$(iii) \Rightarrow (iv)$

If S is reversible SVNA, then it is a direct sum of strongly connected SVNA $S_\alpha, \alpha \in Y$ and $d_i \in D_\alpha$. Then $\langle d_i \rangle = S_\alpha$. Since S_α is strongly connected SVNA, and by the monogenic directability of S we have $S_\alpha = \langle d_i \rangle$ is directable SVNA. Therefore, S_α is strongly directable SVNA, for any $\alpha \in Y$.

$(iv) \Rightarrow (i)$

Let S be a direct sum of strongly directable SVNA S_α, $\alpha \in Y$. Then for each $d_i \in D$, there exists $\alpha \in Y$ such that $d_i \in D_\alpha$. Thus $d_i \in S_\alpha = N(S_\alpha)$, so d_i is a local neck of S.

$(i) \Rightarrow (v)$

Since, each $d_i \in D$ of S is a local neck. Then for any $d_i \in D, \langle d_i \rangle$ is monogenically strongly directable SVNA. Hence, $\langle d_i \rangle$ is reversible SVNA.

$(v) \Rightarrow (i)$

This statement implies that each $d_i \in D$ of S is a local neck.

Theorem 4.5. *Let S' be an arbitrary* ω*-class of a SVNA S. Then one of the following conditions hold.*

(i) $S' = \{d_i \in D \mid N(\langle d_i \rangle) = \phi\}$

(ii) *S' is a locally direct able sub automaton of S.*

Proof. Suppose that (i) does not hold. Then there exists a strongly directable sub automaton S_1 of S such that $N(\langle d_i \rangle) = S_1, \forall d_i \in S'$.

Consider an arbitrary $d_i \in S'$. Then $N(\langle d_i \rangle) \neq \phi$. It means that $\langle d_i \rangle$ is a directable SVNA. Now, for each $y \in I, \exists d_l \in D$ such that $\chi_{T^*}(d_i, y, d_l) > 0, \psi_{T^*}(d_i, y, d_l) < 1, \tau_{T^*}(d_i, y, d_l) < 1$. It means that $\langle d_i \rangle$ is a directable SVNA and $N(\langle d_i \rangle) = N(\langle d_i \rangle)$ where $\chi_{T^*}(d_i, y, d_l) > 0, \psi_{T^*}(d_i, y, d_l) < 1, \tau_{T^*}(d_i, y, d_l) < 1$. Hence, $d_i \in S'$. Thus, S' is a sub automaton of S.

It remains to prove that S' is a locally directable SVNA.

Let $d_1, d_2, \ldots, d_n \in S'$ and $d_k \in S_1$. For every $i \in [1, n], S_1 = N(\langle d_i \rangle)$, so $\exists z_i \in DW(\langle d_i \rangle)$ such that $\chi_{T^*}(d_i, z_i, d_k) > 0, \psi_{T^*}(d_i, z_i, d_k) < 1, \tau_{T^*}(d_i, z_i, d_k) < 1$ for each $d_i \in \langle d_i \rangle$.

Now, set $z = z_1 z_2 \ldots z_n$ and consider arbitrary $i \in [1, n]$, and $d_i \in \langle d_i \rangle$. Since $DW(\langle d_i \rangle)$ is an ideal of I^* then $z \in DW(\langle d_i \rangle)$.

Now,

$$\chi_{T^*}(d_i, z, d_k) > 0 = \chi_{T^*}(d_i, z_1 z_2 \ldots z_n, d_k) > 0,$$

$$\psi_{T^*}(d_i, z, d_k) < 1 = \psi_{T^*}(d_i, z_1 z_2 \ldots z_n, d_k) < 1,$$

$\tau_{T^*}(d_i, z, d_i) < 1 = \tau_{T^*}(d_i, z_1 z_2 \ldots z_n, d_k) < 1,$

Since $\langle d_1, d_2, \ldots, d_n \rangle = \bigcup_{i=1}^{n} \langle d_i \rangle$, and conclude that $z \in DW(\langle d_1, d_2, \ldots, d_n \rangle)$. Thus, $\langle d_1, d_2, \ldots, d_n \rangle$ is a directable SVNA with $N(\langle d_1, d_2, \ldots, d_n \rangle) = S_1$, so S' is a locally directable SVNA.

5. Conclusion

We study SVNA using necks. We define neck, directable, trap, trap-directable, reverse state of SVNA. Further, we describe the properties of necks and give new structural characterizations of a directable single valued neutrosophic automaton. We prove the set of necks of SVNA is the least sub automaton and it is also a reversible SVNA. Also, we prove a SVNA is strongly directable iff it is strongly connected and directable SVNA. Consequently, we prove a directable SVNA is an extension of a strongly directable SVNA by a trap-directable SVNA.

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