INVENTORY CONTROL POLICY WITH TWO-WAREHOUSE, VARIOUS DEMAND, SHORTAGES, TRADE CREDIT AND FUZZY ENVIRONMENT REVISITED

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Abstract

The analysis of the literature is a critical feature of the research effort which connects the past work with the current work. This paper presents recent review engaged with inventory control of decaying items. The last considerable review on this concern dates back to 1913 (Ford Harris EOQ Model). Articles are emphasized by discussing fundamental structure, modeling method, capacity constraints, trade credit, fuzzy parameters, with shortages and various demand. We look over many papers of different journals belonging to decaying items with distinct parameters and then take the review. We introduce about EGQ model of inventory.

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Keywords: inventory control, deteriorating items, two-warehouses, various demand, trade credit, shortages and fuzzy parameters.
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1. Introduction

Operations research is the systematic application of quantitative method, procedure and implement for exploration of problems including the operations of systems. It is used in many fields like agriculture marketing industry etc. There are many methods of operations research which are given as simplex method, inventory control method, Queuing theory, replacement method, networking method etc. The main motive is to find out the best result to search a problem by using mathematical tools. The main phases of operations research to find out the good solution is Inspection, Define the situation, Model construction, Clarification, Implementation and Improve the solution (if needed). Inventory is one of the good visible features of business. In inventory, goods are stocked in the types of spare parts, ready to sell, partly finished items and raw materials. It is necessary for doing business. It is branch of operations research. The main objective is to provide the protection from stocking and under stocking and maintain the balance between supply and demand. Inventory control is useful in manufacture or purchasing by minimize the price of provided the sufficient level of purchaser service. In inventory control, the mathematical modeling began on deterioration with the EOQ model of Harris was first introduced by Whit in 1953. The aim of this study is to give an overall literature survey of inventory control with deteriorating stocks that have been issued. Economic order quantity model is developed by Ford W. Harris in 1913. EPQ model is developed by E. W. Taft in 1918. In this paper, we also introduce economic growing quantity model.

2. Types of Inventory Models

(i) Economic order quantity model (ii) Economic production quantity model (iii) Economic growing quantity model

Economic order quantity model: It is the size of order which minimize total inventory cost. By taking assumption with uniform demand, infinite rate with no shortages.

\[ Q = \sqrt{\frac{2DC_0}{C_c}} \]

Q-Order Quantity, \(D\)-Annual demand per year, \(C_0\)-Ordering Cost, \(C_c\)-Total Carrying Cost.

Economic production quantity model: It controls the amount that a firm
should order to diminish the total inventory cost by stabilizing carrying and average fixed ordering cost. When production rate is uniform and finite.

\[ Q = \sqrt{\frac{2C_c(rk)}{C_0(k - r)}} \]

\( r \)-number of goods required per unit time, \( k \)-number of goods manufactured per unit time \( q \)-number of goods manufactured per production run, \( C_c \)-Holding Cost, \( C_0 \)-setup cost or ordering cost.

Economic growing quantity model: Economic growing quantity model is designed by focusing on growing items of agriculture and industries. The total cost for EGQ consists of the purchasing cost, disposal cost of Dead items, holding cost, feeding cost and the setup cost for both the live and dead items. This model considers the probability density functions of survival and more mortality of a growing item. The optimal decision variables were derived by minimizing the total inventory cost. If the final weight of all live items should be equal to demand and without shortages. Then Quantity is

\[ Q = \frac{2D(b - a)}{2d_0T^* + d_1T^{*2}}. \]

\( D \)-Demand rate per growth cycle, \( T^* \)-optimum cycle length, \( d_0 \)-intercept, \( a, b \)-the parameters of uniform probability function.

3. Method

3.1. Initial Phase. The aim of this evaluation is to observe papers on perish items related to inventory control that have been published between 1965 to till 2019. We first began with a search of keywords in a choice of papers (listed in table 1) that issue on this title. Five non-identical keywords were entered later. These keywords are (i) Two-warehouses (ii) Various demand (iii) Fuzzy parameters (iv) Shortage (v) Trade credit Articles were criticized on the relevancy by examine or study the title.
### Table 1.

<table>
<thead>
<tr>
<th>KEYWORDS</th>
<th>PAPERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWO WAREHOUSES</td>
<td>42[8,14,21,33,34,38,43,44,45,47,60,69,70,74,88,90,91,108,113,125,133,137,142,153,165,170,174,179,195,196,197,201,203,211,216,222,234,236,242,247,248]</td>
</tr>
<tr>
<td>SHORTAGES</td>
<td>38[9,12,28,29,48,49,54,56,68,80,82,84,92,93,96,98,99,100,115,121,124,125,134,136,139,144,154,169,177,178,198,200,206,210,230,234,238,240,241,244,245,246,247]</td>
</tr>
<tr>
<td>FUZZY PARAMETERS</td>
<td>30[1,2,6,11,17,18,24,32,47,59,63,66,78,81,90,111,122,129,131,138,140,142,160,183,184,185,191,209,210,217,248]</td>
</tr>
<tr>
<td>TRADE CREDIT</td>
<td>40[7,10,19,20,23,27,37,41,52,54,61,64,66,73,85,87,94,95,114,116,118,119,120,135,141,151,158,159,164,167,173,174,188,204,213,221,233,239,241,248]</td>
</tr>
</tbody>
</table>

#### 3.2. Second Phase. In second phase, 252 papers were surveyed. These features were distributed to the class of decline as reported in section 2. The types of deterioration are fixed lifetime; age-based deterioration; time and stock based deterioration and the sort of various demand such as stock-based demand, time-based demand, price dependent demand, linear demand, increasing demand, exponential demand, decreasing demand, quadratic demand, ramp type demand.
3.3. Final phase. In the last aspect, the references of the 252 research articles were explored. All the papers were related to decaying inventory model. See the Table 2 for a general summary.

Table 2

<table>
<thead>
<tr>
<th>Journals</th>
<th>Papers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied Mathematical Modelling</td>
<td>10</td>
</tr>
<tr>
<td>American Journal of Operational Research</td>
<td>5</td>
</tr>
<tr>
<td>American Journal of Engineering Research</td>
<td>3</td>
</tr>
<tr>
<td>Applied Mathematics and Computation</td>
<td>2</td>
</tr>
<tr>
<td>Applied Mathematics and Computation</td>
<td>2</td>
</tr>
<tr>
<td>Advances in Production Engineering and Management</td>
<td>1</td>
</tr>
<tr>
<td>AIIE Transaction</td>
<td>1</td>
</tr>
<tr>
<td>Applied Mathematical Science</td>
<td>2</td>
</tr>
<tr>
<td>Advances of Operation Research in Commodities and Financial Modeling</td>
<td>1</td>
</tr>
<tr>
<td>Advances in Computational Sciences and Technology</td>
<td>1</td>
</tr>
<tr>
<td>Annals of Operations Research</td>
<td>1</td>
</tr>
<tr>
<td>Computers and Industrial Engineering</td>
<td>4</td>
</tr>
<tr>
<td>Computers and Operations Research</td>
<td>2</td>
</tr>
<tr>
<td>Cogent Business and Management</td>
<td>2</td>
</tr>
<tr>
<td>Communications in Applied Analysis</td>
<td>1</td>
</tr>
<tr>
<td>Decision Science Letters</td>
<td>1</td>
</tr>
<tr>
<td>European Journal of Operational Research</td>
<td>6</td>
</tr>
<tr>
<td>Expert Systems with Application</td>
<td>1</td>
</tr>
<tr>
<td>Economic Modelling</td>
<td>1</td>
</tr>
<tr>
<td>Engineering Optimization</td>
<td>1</td>
</tr>
<tr>
<td>Fuzzy Sets and System</td>
<td>1</td>
</tr>
<tr>
<td>Global Journal of Pure and Applied Mathematics</td>
<td>4</td>
</tr>
<tr>
<td>International Journal of Engineering Research, Management and Technology</td>
<td>4</td>
</tr>
<tr>
<td>International Journal of Operational Research</td>
<td>7</td>
</tr>
<tr>
<td>International Journal of Production Economics</td>
<td>6</td>
</tr>
<tr>
<td>International Journal of Supply and Management</td>
<td>6</td>
</tr>
<tr>
<td>IOSR Journal of Mathematics</td>
<td>10</td>
</tr>
<tr>
<td>International Journal of Science Engineering and Technology</td>
<td>2</td>
</tr>
</tbody>
</table>
The choice procedure reported in the foregoing field listed 252 pertinent papers that have been published from 1965 to till 2019. In table 2, we display the articles issued by journals. In table 3, articles are categorized according to the modeling of deterioration and various types of rate. We will describe a number of key modeling parameters indifferent subsections. There are the number of warehouses in section 4.1, with various demand in section 4.2,
deteriorating inventory under fuzzy parameters in section 4.3, inventory modeling on decaying item with shortages in section 4.4 and a permissible delay in payments.

4.1. Capacity Constraints for Decaying Inventory. Many writers model the type in which two-warehouses are introduced for the storehouse facilities of decaying goods in the listed papers [8, 14, 33, 34, 43, 71, 108, 133, 137, 195, 201, 234, 247]. In these models, one storehouse owned and the other is rented. Items in the rented house are exhausted first before the owned warehouse items are used. Both warehouses have different cost, deterioration rates, demands and other parameter and rented warehouse has higher cost. Model [21, 88] has multivariate demand with two warehouses. In [69, 38] authors presented two warehouses model with FIFO policy and in [197] authors developed the model with different dispatching policies. In [74, 165, 142, 248] authors described two-warehouses model in fuzzy environment and in [44, 47, 60, 113, 196] two-warehouses model with trade credit. In [45, 70, 88, 90, 125, 153, 179, 211, 216, 222, 242] authors developed two-warehouses model with the effect of inflation. In [174, 91] author introduced model with two storage facilities with trade credit and inflation. In [170, 203, 236] author took effect of preservation technology with warehouse model.

4.2. Decaying Inventory Model with Fuzzy Environment. To formulate fuzzy inventory model, we need the successive definitions:

(i) Let \( Y \) be space of points and \( \mu : Y \to [0, 1] \) be such that for every \( y \in Y \), \( \mu(y) \) is a real number in the interval \([0, 1]\). We define a fuzzy set \( A \) in \( Y \) as the order pair \( A = \{(y, \mu_A(y) : y \in Y)\} \), where \( y \) is called a generic element and \( \mu_A(y) \) is a membership function.

(ii) A fuzzy set \( A = \{(y, \mu_A(y))\} \subseteq Y \), is called a convex fuzzy set, if all \( A_x \) are convex sets for every \( y \in Y \).

(iii) Let \( \alpha, \beta \in R \) such that \( \alpha < \beta \). Then, for \( 0 \leq x \leq 1 \), the fuzzy set \([\alpha, \beta]_x\) is called a fuzzy interval, if its membership functions is \( \mu_{[\alpha, \beta]_x} = \begin{cases} x, & \alpha \leq x \leq \beta \\ 0, & \text{otherwise} \end{cases} \).
(iv) Let $\alpha, \beta, \gamma \in R$ such that $\alpha < \beta < \gamma$. Then the fuzzy number $A = (\alpha, \beta, \gamma)$, is called a triangular fuzzy number if its membership is

$$
\mu_A(y) = \begin{cases} 
\frac{y - \alpha}{\beta - \alpha}, & \alpha \leq y \leq \beta \\
\frac{\gamma - y}{\gamma - \beta}, & \beta \leq y \leq \gamma \\
0, & \text{otherwise}
\end{cases}
$$

(v) If $A = (\alpha, \beta, \gamma)$ is triangular fuzzy number, then the signed distance of $A$ is defined as $S_A = \frac{1}{4} (\alpha + 2\beta + \gamma)$.

(vi) The Centroid method on the triangular fuzzy number $A = (\alpha, \beta, \gamma)$ is defined as $C_A = \frac{\alpha + \beta + \gamma}{3}$.

A number of authors model the instance in which inventory model are formulated with fuzzy environment. In [1, 2, 6, 18, 32, 63, 111, 131, 140, 142, 160, 183, 185, 191, 217] authors investigated the model with deteriorating inventory under fuzzy parameters. Here deterioration rates are fuzzy. In 11, 138 model inventory cost is fuzzy. In paper [17] author takes both the parameter fuzzy and crisp. Ln [66, 78, 184, 210, 248] author established the fuzzy model with shortages. In some model demand is fuzzy and under the permissible delay in payment. In paper [109, 129] author offered a fuzzy model under the effect of inflation. In fuzzy inventory model, author used signed distance method, graded mean integration method, triangular fuzzy numbers and defuzzification. In paper [24, 131, 209] author proposed the fuzzy production quantity model

### 4.3. Deteriorating inventory model with Various Demand

<table>
<thead>
<tr>
<th>Various Demand</th>
<th>Related Equation</th>
</tr>
</thead>
</table>
| Stock dependent demand  | $D(t) = \begin{cases} \{a + bI(t), 0 \leq t \leq T_1\} \\
|                         | \{a, T_1 \leq t \leq T\} \end{cases}$                                           |
| Quadratic Demand        | $D(t) = a + bt + ct^2$                                                           |
| Multivariate Demand     | A Unique combination of time and on hand inventory. $D(t) = a + bt + c[I(t)], a > b$ |
Exponential Demand

Increasing function of Time: \( D(t) = a e^{bt} \)

Decreasing function of Time: \( D(t) = a e^{-bt} \)

Linear Demand

\( D(t) = a + bt \)

Selling price demand

\( D(t) = \frac{\gamma}{\rho^t} \), \( \rho \) = Selling price, \( \gamma, \delta \) = demand parameters

Cubic demand

\( D(t) = a + bt + c t^2 + dt^3 \)

Ramp type demand

\( D(t) = \begin{cases} ae^{bt}, & t < \mu, \ a > b, \ 0 \leq b \leq 1 \\ ae^{b\mu}, & t \geq \mu \end{cases} \), \( \mu \) is position of time.

Here \( a, b, c, d > 0, a, b, c, d \) are constraints. \( T \)-cycle of length, \( T_1 \)-time at which shortages start, \( I(t) \)-inventory level at time \( t \).

<table>
<thead>
<tr>
<th>Demand</th>
<th>Deterioration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed life time</td>
<td>Age-based deterioration rate</td>
</tr>
<tr>
<td>Stock -dependent</td>
<td>8(1,1)</td>
</tr>
<tr>
<td>Time -dependent</td>
<td>8(2,1)</td>
</tr>
<tr>
<td>Price -dependent</td>
<td>8(3,1)</td>
</tr>
<tr>
<td>Linear demand</td>
<td>6(4,1)</td>
</tr>
<tr>
<td>Exponential demand</td>
<td>8(5,1)</td>
</tr>
<tr>
<td>Quadratic demand</td>
<td>8(6,1)</td>
</tr>
<tr>
<td>Ramp type demand</td>
<td>3(7,1)</td>
</tr>
<tr>
<td>Constant demand</td>
<td>9(8,1)</td>
</tr>
</tbody>
</table>

(1,1) \[30,35,52,57,79,113,156,199\]
(1,2) \[25,115,118,125,152,166,170,205,231\]
(1,3) \[28,32,36,37,39,109,171,182,184,201,244,248\]
(2,1) \[31,49,69,116,124,148,155,224\]
(2,2) \[23,64,76,106,147,153,181,189,211,218,239\]
(2,3) \[4,10,16,43,54,58,59,63,64,66,67,70,83,108,140,157,164,186,188,207,229\]
(3,1) \[33,73,163,195,198,203,235,240\]
(3,2) \[40,81,103,108,135,215,30,232\]

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4.4. Shortages. Most of inventory models with deteriorating items consider that during the stock out whether all demand is complete backlogged or partial backlogged. In common use, shortage define condition where almost people are not able to get wanted inventory at reasonable cost. If the need of inventory more than stock then shortages occurs. Partial backlogging also called as lost sale case; it described the unfulfilled demand completely lost. Models that take complete backlogging are many times less suitable in real life then model that take partial backlogging. Some inventory model that are formulated with shortages are established with Weibull deterioration and shortages. Model with decaying item under the effect of inflation with shortage are formulated in papers.

4.5. Decaying inventory with Trade Credit. When the economic order quantity inventory model was obtained, it was completely presumed that amount is made to trader immediately. In actuality, however, we frequently find that the traders provide the buyer a period of time of payment for paying the amount, often without paying interest. The main objective is to increase
the buyer to purchase more, to grow demand in market. When the items are decaying, a permissible delay in payment may raise demand rate and prevent the loss of value. When a credit policy may lead much successful marketing strategies. In trade credit a retailer is allowed to pay back dues in a time period without any interest. Many authors formulated the inventory model with different policy of trade credit [7, 10, 20, 23, 37, 41, 52, 54, 61, 64, 67, 85, 87, 94, 114, 116, 118, 120, 141, 159, 167, 204, 213, 221, 233, 241, 248]. In paper [75] inventory modeling with multi-partial prepayments. In papers [19, 27, 73, 119, 151, 158, 164, 173, 174, 188, 239] established deteriorating inventory modeling with trade credit under the effect of inflation. In paper 135, formulated an inventory model with order size-dependent payments.

5. Conclusion

In this paper, we have presented in current review of decaying inventory literature. It is framed that deteriorating inventory model with different parameters are well formulated in up-to-date literature. Analysis of key model features discloses that some of these features are well provided in the literature. A number of authors considered to storage facilities of deteriorating with different type of demand and preservation technology. The impact of inflation and fuzzy environment is also taken in inventory models. The impact of trade credit is also shown in inventory model and shortages are also taken in some of models. Finally, a comment has to be formed in connection with the modeling of substitutability in inventory control of decaying items. The fact that refill decision in relay reliant on the current stock of reserve items should not be ignored any longer. Furthermore, research should extend works towards this very compound, very protected feature of decaying inventory control. The graph of Table 1 and Table 3 are given in the form of bar graph.

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