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A STUDY ON FUZZY TRANSPORTATION PROBLEM USING DECAGONAL FUZZY NUMBER

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Abstract

In this paper fuzzy transportation problem in which the values of transportation costs are represented as Decagonal fuzzy number is considered. It has different optional ways to transport the goods. The aim of this study is to find the suitable defuzzification method to find the minimum transportation cost.

1. Introduction

In mathematics and economics, transportation theory is a name given to the study of optimal transportation and allocation of resources. The problem was formalized by the French mathematician Gaspard Monge in 1781. Unfortunately, understanding the process and interpreting the results are not easy tasks. The method is very complex. Transportation problem deals with the problem of how to plan production and transportation in such as industry given several plans at different location and large number of customers of their product. The transportation problem deals with distribution of goods from several points, such as factories often known as sources, to a number of points of demand, such as warehouses often known as destination. Each source is able to supply a fixed number of units of products, usually called the capacity or availability and each destination has a fixed demand usually known as requirements. Because of its major application in solving problems which involving several product sources and several

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destination of products, this type of problem is frequently called "The Transportation Problem". In recent times the demand and supply of any commodity cannot be fixed and due to various reasons it has been changing from time to time. This lead to variation in the transportation cost also. So this study concentrates on Transportation problem where costs are given as fuzzy numbers. Fuzzy set is a mathematical model of vague qualitative or quantitative data, frequently generated by means of the natural language. The model is based on the generalization of the classical concepts of set and its characteristic function.

In particular fuzzy transportation problem in which the values of transportation costs are represented as Decagonal fuzzy number is considered. It has different optional ways to transport the goods. The aim of this study is to find the suitable defuzzification method to find the minimum transportation cost.

Section 2 explains the basic definitions and derivation of different defuzzification methods which are relevant to this study. Decagonal Fuzzy Transportation Problem is solved using various defuzzification techniques in Section 3. Conclusion of this study is given in Section 4.

2. Basic Definitions

2.1. In this section, we give some basic definitions which are relevant to this study.

Definition 2.1. Let X be a non empty set. Then a fuzzy set A in X (i.e., a fuzzy subset A of X) is characterized by a function of the form $\mu_A : X \to [0, 1]$. Such a function μ_A is called the membership function and for each $x \in X$, $\mu_A(x)$ is the degree of membership of x (membership grade of x) in the fuzzy set A.

In other words, A fuzzy set $A = \{(x, \mu_A(x)) | x \in X\}$ where $\mu_A : X \to [0, 1]$.

The characteristic function $\mu_A(x)$ has only values 0 (false) and 1 (true). Such sets are crisp sets.

Definition 2.2. Let M be a fuzzy subset of set of real numbers $\mathbb{R}$. Then M is a fuzzy number if and only if there exists a closed interval $[a, b] \neq \phi$ such that

$$\mu_M(x) = \begin{cases} 1 & \text{for } x \in [a, b] \\ l(x) & \text{for } x \in (-\infty, a) \\ r(x) & \text{for } x \in (b, \infty), \end{cases}$$

where $l(x)$ is a function from $(-\infty, a)$ to [0, 1] that is monotonic increasing, continuous from the right and such that $l(x) = 0$ for $x \in (-\infty, \omega_1)$; $r(x)$ is a function from (b, ∞) to [0, 1] that is monotonic decreasing, continuous from the left and such that $r(x) = 0$ for $x \in (\omega_2, \infty)$.

Definition 2.3 [1]**.** A Decagonal fuzzy number A_D denoted as $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ and the membership function is defined as

$$\mu_{A_D}(x) = \begin{cases} \frac{1}{4}\frac{(x - a_1)}{(a_2 - a_1)}, & a_1 \le x \le a_2 \\ \frac{1}{4} + \frac{1}{4}\frac{(x - a_2)}{(a_3 - a_2)}, & a_2 \le x \le a_3 \\ \frac{1}{2} + \frac{1}{4}\frac{(x - a_3)}{(a_4 - a_3)}, & a_3 \le x \le a_4 \\ \frac{3}{4} + \frac{1}{4}\frac{(x - a_4)}{(a_5 - a_4)}, & a_4 \le x \le a_5 \\ 1, & a_5 \le x \le a_6 \\ 1 - \frac{1}{4}\frac{(x - a_6)}{(a_7 - a_6)}, & a_6 \le x \le a_7 \\ \frac{3}{4} - \frac{1}{4}\frac{(x - a_7)}{(a_8 - a_7)}, & a_7 \le x \le a_8 \\ \frac{1}{2} - \frac{1}{4}\frac{(x - a_8)}{(a_9 - a_8)}, & a_8 \le x \le a_9 \\ \frac{1}{4}\frac{(x - a_9)}{(a_{10} - a_9)}, & a_9 \le x \le a_{10} \\ 0 & \text{otherwise}. \end{cases}$$

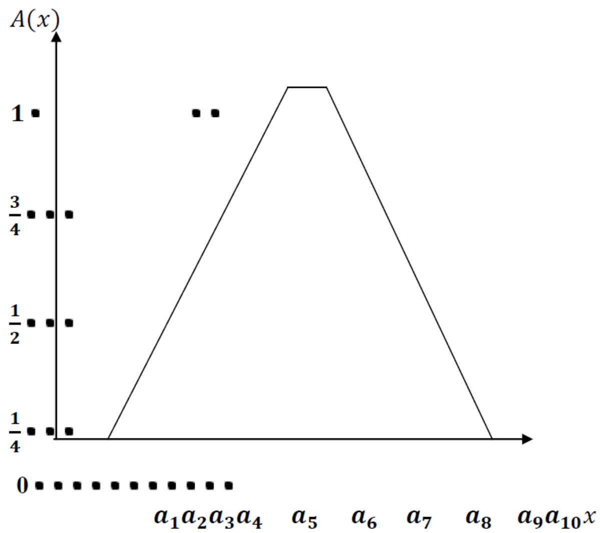


Figure 2.3. Graphical Representation of normal Decagonal fuzzy number for $x \in [0, 1]$.

2.2. Mathematical Formulation of Fuzzy Transformation Problem.

The fuzzy transportation problems, in which a decision maker is uncertain about precise value of transportation cost, availability and demand, can be formulated as follows

$$\text{Minimize } \widetilde{Z} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c}_{ij}\, \widetilde{x}_{ij}.$$

$$\sum_{j=1}^{n} \widetilde{X}_{ij} \approx \widetilde{a}_i,\ i = 1, 2, 3, \ldots, m$$

subject to

$$\sum_{i=1}^{m} \widetilde{X}_{ij} \approx \widetilde{b}_j,\ j = 1, 2, 3, \ldots, n$$

$$\sum_{i=1}^{m} \widetilde{a}_i \approx \sum_{j=1}^{n} \widetilde{b}_j,\ i = 1, 2, 3, \ldots, m,$$

$j = 1, 2, 3, \ldots, n$ and $\tilde{x}_{ij} \geq 0$

$\sum_{j=1}^{m} \tilde{b}_j$ = total fuzzy availability of the product

$\sum_{j=1}^{n} \tilde{b}_j$ = total fuzzy demand of the product

$\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$ = total fuzzy transportation cost.

If $\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.

2.3. Algorithm for Vogel Approximation method

Step 1. Convert the given fuzzy parameters in to crisp values by using proposed ranking method.

Step 2. If it is unbalanced convert the given fuzzy transportation problem to balanced transportation problem.

Step 3. Determine the penalty cost for each row and column by subtracting the lowest cell cost in the row or column from the next cell cost in the same row or column.

Step 4. Select the row or column with the highest penalty cost (breaking tiles arbitrarily or choosing lowest cost cell).

Step 5. Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.

Step 6. Repeat 3 and 4 until all requirements have been meet.

2.4. Defuzzification Methods. There are different types of defuzzification methods. They are

- Total Integral Value Method (TI)
- Graded Mean Integration Representation (GMI)
- Total Area Method (TA)

- Ranking technique based on Centroid of Centroid Method (CC)

2.4.1. Total Integral Value of a Decagonal Fuzzy Number

We define

$$f_{A_D}(x) = \begin{Bmatrix} f_A^{L_1(x)} \\ f_A^{L_2(x)} \\ f_A^{L_3(x)} \\ f_A^{L_4(x)} \\ 1 \\ f_A^{R_1(x)} \\ f_A^{R_2(x)} \\ f_A^{R_3(x)} \\ f_A^{R_4(x)} \end{Bmatrix},$$

where

$$f_A^{L_1(x)} = \frac{1}{4}\frac{(x-a_1)}{(a_2-a_3)},\ f_A^{L_2(x)} = \frac{1}{4}+\frac{1}{4}\frac{(x-a_2)}{(a_3-a_2)}$$

$$f_A^{L_3(x)} = \frac{1}{2}+\frac{1}{4}\frac{(x-a_3)}{(a_4-a_3)},\ f_A^{L_4(x)} = \frac{3}{4}+\frac{1}{4}\frac{(x-a_4)}{(a_5-a_4)}$$

$$f_A^{R_1(x)} = 1-\frac{1}{4}\frac{(x-a_6)}{(a_7-a_6)},\ f_A^{R_2(x)} = \frac{3}{4}+\frac{1}{4}\frac{(x-a_7)}{(a_8-a_7)}$$

$$f_A^{R_3(x)} = \frac{1}{2}-\frac{1}{4}\frac{(x-a_8)}{(a_9-a_8)},\ f_A^{R_2(x)} = \frac{1}{4}\frac{(x-a_9)}{(a_{10}-a_9)}.$$

The inverse function of $f_A^{L_1(x)}$, $f_A^{L_2(x)}$, $f_A^{L_3(x)}$, $f_A^{L_4(x)}$, $f_A^{R_1(x)}$, $f_A^{R_2(x)}$, $f_A^{R_3(x)}$ and $f_A^{R_4(x)}$ and $f_A^{R_4(x)}$ are $g_A^{L_1(x)}$, $g_A^{L_2(x)}$, $g_A^{L_3(x)}$, $g_A^{L_4(x)}$, $g_A^{R_1(x)}$, $g_A^{R_2(x)}$, $g_A^{R_3(x)}$ and $g_A^{R_4(x)}$.

Where $g_A^{L_1(x)} = 4h(a_2-a_1)+a_1$

$g_A^{L_2(x)} = (4h-1)(a_3-a_2)+a_2$

$$g_A^{L_3(x)} = (4h-1)(a_4 - a_3) + a_3$$

$$g_A^{L_4(x)} = (4h-1)(a_5 - a_4) + a_4$$

$$g_A^{R_1(x)} = -(4h-1)(a_7 - a_6) + a_6$$

$$g_A^{R_2(x)} = -(4h-1)(a_8 - a_7) + a_7$$

$$g_A^{R_3(x)} = -(4h-1)(a_9 - a_8) + a_8$$

$$g_A^{R_4(x)} = -(4h-1)(a_{10} - a_9) + a_9.$$

The inverse function of decagonal fuzzy number is

$$R(TI_D) = \int_0^1 \begin{matrix} (g_A^{L_1(x)} + g_A^{L_2(x)} + +g_A^{L_3(x)} + g_A^{L_4(x)} \\ g_A^{R_1(x)} + g_A^{R_2(x)} + g_A^{R_3(x)} + g_A^{R_4(x)})dt. \end{matrix}$$

Solving the above integration, using MATLAB Program we get the ranking of Decagonal fuzzy number using Total integral value method is

$$R(TI_D) = 2(a_2 + a_3 + a_4 + a_7 + a_8 + a_{10}) - (a_1 + a_2 + a_6 + a_9). \tag{I}$$

2.4.2 Graded Mean Integration Ranking Method [5]. Graded Mean Integration Representation method is used to defuzzify Decagonal fuzzy numbers.

If $\tilde{a} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_1; w)$ is a generalized fuzzy number, then the defuzzified value $R(GMI_D)$ by the Graded Mean Integration Representation method is given by the following formula:

$$R(GMI_D) = \int_0^w h\left[\frac{L^{-1}(h) + R^{-1}(h)}{2}\right]dh / \int_0^w h dh \text{ where } 0 < h \le w \text{ and } 0 < e \le 1.$$

If $\tilde{a} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_1)$ is a Decagonal number, then the Graded Mean Integration Representation of $R(GMI_D)$ from the above formula is calculated as follows The inverse function of Decagonal fuzzy number are $g_A^{L_1(x)}, g_A^{L_2(x)}, g_A^{L_3(x)}, g_A^{L_4(x)}, g_A^{R_1(x)}, g_A^{R_2(x)}, g_A^{R_3(x)}$ and

$g_A^{R_4(x)}$

$$R(GMI_D) = \int_0^1 \begin{matrix} h[g_A^{L_1(x)} + g_A^{L_2(x)} + g_A^{L_3(x)} + g_A^{L_4(x)} \\ + g_A^{R_1(x)} + g_A^{R_2(x)} + g_A^{R_3(x)} + g_A^{R_4(x)}]dh / \int_0^1 hdh. \end{matrix}$$

Solving the above integration, using MATLAB Program we get the ranking of Decagonal fuzzy number using Graded Mean Integration Representation method is

$$R(GMI_D) = \frac{1}{6}(-5a_1 + 6a_2 + 6a_3 + 6a_4 - a_5 - a_6 + 6a_7 + 6a_8 - 7a_9 + 8a_{10}). \quad \text{(II)}$$

2.4.3 Ranking techniques based on Centroid Method [4]

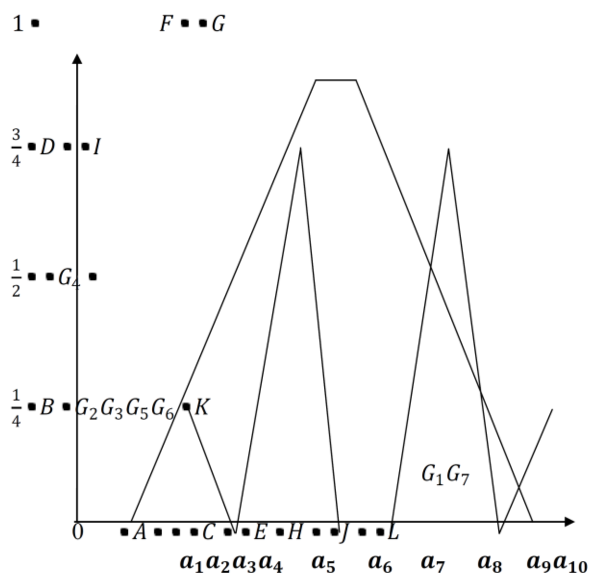


Figure 2.4. Ranking technique based on Centroid Method.

From Figure 2.4 Decagonal is divided into six triangles and one Hexagonal. Here G_1, G_2, G_3, G_5, G_6, G_7 and G_4 are the centroids of ABC, BCD, DCE, HIJ, IJK, JKL and FDEHIG respectively. $A(a_1, 0)$, $B\left(a_2, \frac{1}{4}\right)$ and $C(a_3, 0)$

$$G_1 = \left(\frac{a_1 + a_2 + a_3}{3}, \frac{1}{12}\right) B\left(a_2, \frac{1}{4}\right), C(a_3, 0) \text{ and}$$

$$D\left(a_3, \frac{3}{4}\right) G_2 = \left(\frac{a_2 + a_3 + a_4}{3}, \frac{4}{12}\right) C(a_3, 0), D\left(a_3, \frac{3}{4}\right) \text{ and}$$

$$E(a_5, 0) \quad G_3 = \left(\frac{a_3 + a_4 + a_5}{3}, \frac{3}{12}\right)$$

$D\left(a_5, \frac{3}{4}\right)$, $E(a_5, 0)$, $F(a_5, 1)$, $H(a_6, 0)$ and $I\left(a_7, \frac{3}{4}\right)$

$$G_4 = \left(\frac{a_4 + 2a_5 + 2a_6 + a_7}{6}, \frac{7}{12}\right)$$

$H(a_6, 0)$, $I\left(a_7, \frac{3}{4}\right)$ and $J(a_8, 0)$

$$G_5 = \left(\frac{a_6 + a_7 + a_8}{3}, \frac{3}{12}\right)$$

$I\left(a_7, \frac{3}{4}\right)$, $J(a_6, 0)$ and $K\left(a_9, \frac{1}{4}\right)$

$$G_6 = \left(\frac{a_7 + a_8 + a_9}{3}, \frac{4}{12}\right)$$

$J(a_8, 0)$, $K\left(a_9, \frac{1}{4}\right)$ and $L(a_{10}, 0)$

$$G_7 = \left(\frac{a_8 + a_9 + a_{10}}{3}, \frac{1}{12}\right).$$

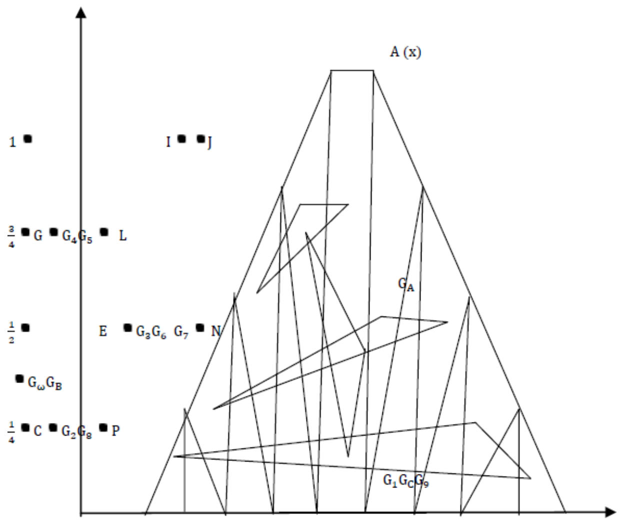
By adding $G_1, G_2, G_3, G_4, G_5, G_6$ and G_7 we get the new ranking as

$$R(CM_D) = \left(\frac{2(a_1 + a_{10}) + 4(a_2 + a_5 + a_6 + a_9) + 5(a_4 + a_7) + 6(a_3 + a_8)}{6}, \frac{23}{12}\right).$$

Therefore the ranking of Decagonal fuzzy number using Total Centroid method is

$$R(CM_D) = \left(\frac{2(a_1 + a_{10}) + 4(a_2 + a_5 + a_6 + a_9) + 5(a_4 + a_7) + 6(a_3 + a_8)}{6}, \frac{23}{12}\right). \quad \text{(III)}$$

2.4.4 Ranking technique based on Centroid of centroid Method [2],[3].



0 ? A ? B ? D ? F ? ? H ? K ? M ? O ? Q ? R

$a_1 a_2 a_3 a_4 \;\; a_5 \; a_6 \; a_7 \; a_8 \; a_9 \; a_{10} x.$

Figure 2.5. Ranking technique based on Centroid of centroid Method.

Let the Decagonal can be divided into eight triangles and one rectangle.

Let $G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8$ and G_9 be the centroids of ABC, CDE, EFG, GHI, IHJK, JKL, LMN, NOP and PQR respectively.

$A(a_1, 0)$, $C\left(a_2, \frac{1}{4}\right)$, $B(a_2, 0)$

$$G_1 = \left(\frac{a_1 + 2a_2}{3}, \frac{1}{12}\right)$$

$C\left(a_2, \frac{1}{4}\right)$, $E\left(a_3, \frac{1}{2}\right)$, $D(a_3, 0)$

$$G_2 = \left(\frac{a_2 + 2a_3}{3}, \frac{3}{12}\right)$$

$E\left(a_3, \frac{1}{2}\right)$, $F(a_4, 0)$, $G\left(a_4, \frac{3}{4}\right)$

$$G_3 = \left(\frac{a_3 + 2a_4}{3}, \frac{5}{12}\right)$$

$G\left(a_4, \frac{3}{4}\right)$, $H(a_5, 0)$, $I(a_5, 1)$

$$G_4 = \left(\frac{a_4 + 2a_5}{3}, \frac{7}{12}\right)$$

$H(a_5, 0)$, $I(a_5, 1)$, $J(a_6, 1)$, $K(a_6, 0)$

$$G_5 = \left(\frac{2a_5 + 2a_6}{3}, \frac{2}{4}\right)$$

$J(a_6, 1)$, $K(a_6, 0)$, $L\left(A_7, \frac{3}{4}\right)$

$$G_6 = \left(\frac{2a_6 + a_7}{3}, \frac{7}{12}\right)$$

$L\left(a_7, \frac{3}{4}\right)$, $M(a_7, 0)$, $N\left(a_8, \frac{1}{2}\right)$

$$G_7 = \left(\frac{2a_7 + a_8}{3}, \frac{5}{12}\right)$$

$N\left(a_8, \frac{1}{2}\right)$, $O(a_8, 0)$, $P\left(a_9, \frac{1}{4}\right)$

$$G_8 = \left(\frac{2a_8 + a_9}{3}, \frac{3}{12}\right)$$

$P\left(a_9, \frac{1}{4}\right)$, $Q(a_9, 0)$, $R(a_{10}, 0)$

$$G_9 = \left(\frac{2a_9 + a_{10}}{3}, \frac{1}{12}\right).$$

We have

- G_3, G_4 and G_5 are non-collinear and they form a triangle.

- G_2, G_6 and G_7 are non-collinear and they form a triangle.
- G_1, G_8 and G_9 are non-collinear and they form a triangle.

Now G_A, G_B and G_C are the centroids $G_3 G_4 G_5$, $G_2 G_6 G_7$, $G_1 G_8 G_7$ respectively.

$$G_3\left(\frac{a_3 + 2a_4}{3}, \frac{5}{12}\right), G_4\left(\frac{a_4 + 2a_5}{3}, \frac{7}{12}\right) G_5\left(\frac{2a_5 + 2a_6}{4}, \frac{2}{4}\right)$$

$$G_A = \left(\frac{2a_3 + 6a_4 + 7a_5 + 3a_6}{18}, \frac{18}{12}\right)$$

$$G_2\left(\frac{a_2 + 2a_3}{3}, \frac{3}{12}\right), G_6\left(\frac{2a_6 + a_7}{3}, \frac{7}{12}\right) G_7\left(\frac{2a_7 + a_6}{3}, \frac{5}{12}\right)$$

$$G_B = \left(\frac{2a_2 + 4a_3 + 4a_6 + 6a_7 + 2a_8}{18}, \frac{15}{12}\right)$$

$$G_1\left(\frac{a_1 + 2a_2}{3}, \frac{1}{12}\right), G_6\left(\frac{2a_6 + a_9}{3}, \frac{3}{12}\right) G_9\left(\frac{2a_9 + a_{10}}{3}, \frac{1}{12}\right)$$

$$G_C = \left(\frac{a_1 + 2a_2 + 2a_8 + 3a_3 + a_{10}}{9}, \frac{5}{12}\right).$$

Next we have, G_A, G_B and G_C are non-collinear and they form a triangle. Now G_ω be the centroid of $G_A G_B G_C$

$$R(CC_D) = \left(\frac{2(a_1 + a_{10}) + 6(a_2 + a_3 + a_4 + a_7 + a_8 + a_9) + 7(a_5 + a_6)}{54}, \frac{19}{18}\right). \qquad \text{(IV)}$$

3. Numerical Example

In this Section, Fuzzy Transportation problem is solved using various Defuzzification techniques illustrated in Section 2.

Example 1. Consider the following Fuzzy Transportation problem where supply and demand are given as Decagonal Fuzzy Numbers:

	d_1	d_2	d_3	d_4	supply
O_1	(-1,0,1,2, 3,4,5,6,7,8)	(0,1,2,3,4,5,6,7,8,9)	(8,9,10,11,12, 13,14,15, 16,17)	(4,5,6,7,8,9,10,11,1 2,13)	(1,3,5,6,7,8,10,11, 12,13)
O_2	(-2,1,0, 1,2,3,4, 5,6,7)	(-3,-2,- 1,0,1,2,3,4,5,6)	(2,4,5,6,7,8,9, 11,12,13)	(-3,-1,0,1,2,4, 5,6,7,8)	(-4,2,0,1,2, 3,4,5, 8,11)
O_3	(2,3,4,5,6,7,8,9,10 ,11)	(3,6,7,8,9,10,12,13,1 4,15)	(11,12,14,1516,17,18, 21, 22,23)	(5,6,8,9,10,11, 12,15 16,17)	(5,6,8,10,12,13,15 , 17,18, 19)
demand	(4,5,6,7,8,9,10,11, 12,13)	(1,2,3,5,6,7,8, 10, 11,12)	(0,1,2,3,4,5,6,7,8,9)	(-4,0,1,2,3,4, 5,6, 7,8)	Balanced

(i) Total Integral Method: We know

$$R(TI_D) = -a_1 - a_5 - a_6 - a_9 + 2(a_2 + a_3 + a_4 + a_7 + a_8 + a_9 + a_{10}).$$

By defuzzification the given fuzzy transportation table is reduced to Crisp Transportation table as follows. By applying Vogel Approximation Method

	d_1	d_2	d_3	d_4	supply
O_1	31	39	103	71	68
O_2	23	15	67	28	29
O_3	55	86	140	92	102
demand	71	55	39	34	199

Transportation cost $= (55 \times 39) + (13 \times 103) + (29 \times 28) + (71 \times 55)$ $+ (92 \times 5) = 12301$ Units

(ii) Graded Mean Integration Method: We know

$$R(GMI_D) = \frac{1}{12}(-5a_1 + 6a_2 + 6a_3 + 6a_4 - a_5 - a_6 + 6a_7 + 6a_8 - 7a_9 + 8a_{10}).$$

In this method we use VAM (Vogal Approximation Method) directly to

	d_1	d_2	d_3	d_4	supply	RP
O_1	(-1,0,1,2,3,4,5,6,7,8)	(0,1,2,3,4,5,6,7,8,9)	(8,9,10,11,12,13,14,15,16,17)	(4,5,6,7,8,9,10,11,12,13)	(1,3,5,6,7,8,10,11,12,13)	(-8,-6,-4,-2,0,2,4,6,8,10)
O_2	(-2,1,0,1,2,3,4,5,6,7)	(-3,-2,-1,0,1,2,3,4,5,6)	(2,4,5,6,7,8,9,11,12,13)	$\boxed{(-3,-1,0,1,2,4,5,6,7,8)}$	(-4,2,0,1,2,3,4,5,8,11)	(-8,-6,-4,-2,0,2,4,6,8,10)
O_3	(2,3,4,5,6,7,8,9,10,11)	(3,6,7,8,9,10,12,13,14,15)	(11,12,14,15,16,17,18,21,22,23)	(5,6,8,9,10,11,12,15,16,17)	(5,6,8,10,12,13,15,17,18,19)	(-8,-4,-2,0,2,4,7,9,11,13)
demand	(4,5,6,7,8,9,10,11,12,13)	(1,2,3,5,6,7,8,10,11,12)	(0,1,2,3,4,5,6,7,8,9)	(-4,0,1,2,3,4,5,6,7,8)		
CP	(-8,-6,-4,-2,0,2,4 6,8,10)	(-6,-4,-2,0,2,4,6,8,10,12)	(-5,-3,-1,2,4,6,8,10,12,15)	(-4,2,0,2,4,7,9,11,13,16)		

FTP.

$$x_{24}(-4,\ -2,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 8,\ 11)$$

	d_1	d_2	d_3	d_4	supply	RP
O_1	(-1,0,1,2,3,4,5,6,7,8)	$\boxed{(0,1,2,3,4,5,6,7,8,9)}$	(8,9,10,11,12,13,14,15,16,17)	(4,5,6,7,8,9,10,11,12,13)	(1,3,5,6,7,8,10,11,12,13)	(-8,-6,-4,-2,0,2,4,6,8,10)
O_3	(2,3,4,5,6,7,8,9,10,11)	(3,6,7,8,9,10,12,13,14,15)	(11,12,14,15,16,17,18,21,22,23)	(5,6,8,9,10,11,12,15,16,17)	(5,6,8,10,12,13,15,17,18,19)	(-8,-4,-2,0,2,4,7,9,11,13)
demand	(4,5,6,7,8,9,10,11,12,13)	(1,2,3,5,6,7,8,10,11,12)	(0,1,2,3,4,5,6,7,8,9)	(-15,-8,-4,-2,0,2,4,6,9,12)		
CP	(-6,-4,-2,0,2,4,6,8,10,12)	(-6,-2,0,2,4,6,9,11,13,15)	(-6,-4,-1,1,3,5,7,11,13,15)	(-8,-6,-3,-1,1,3,5,9,11,13)		

$$x_{12} = (1,\ 2,\ 3,\ 5,\ 6,\ 7,\ 8,\ 10,\ 11,\ 12)$$

Similarly proceeding,

$$x_{11} = -11, -8, -5, -2, 0, 2, 5, 8, 10, 12$$

$$x_{33} = (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$$

$$x_{34} = (-15, -8, -4, -2, 0, 2, 4, 6, 9, 12)$$

$$x_{31} = (-8, -5, -2, 2, 6, 9, 12, 16, 20, 24).$$

Transportation cost $= (-4, -2, 0, 1, 2, 3, 4, 5, 8, 11)(-3, -1, 0, 1, 2, 4, 5, 6, 7, 8)$ $(1, 2, 3, 5, 6, 7, 8, 10, 11, 12)(0, 1, 2, 3, 4, 5, 6, 7, 8, 9) + (-11, -8, -5, -2, 0, 0, 5, 8, 10, 12)$ $(-1, 0, 1, 2, 3, 4, 5, 6, 7, 8) + (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)(11, 12, 14, 15, 16, 17, 18, 21, 22, 23) +$ $(-15, -8, -4, -, , 2, 4, 6, 9, 12)(5, 6, 8, 9, 10, 11, 12, 15, 16, 17)$ $+(-8, -5, -2, 2, 6, 9, 12, 16, 20, 24)(2, 3, 4, 5, 6, 7, 8, 9, 10, 11).$

Using Graded Mean Integration Method, the above fuzzy values are defuzzified as

$$= (15.83)(15) + (28.16)(20.16) + (11.16)(16.16) + (20.16)(88.66)$$

$$+(20.16)(88.66) + (13.66)(46.33) + (35.83)(28.16) + (35.83)(28.16).$$

Transportation cost = 4414.7 Units

(iii) Ranking technique based on Centroid Method. We know

$$\Re(TA_D) = \left(\frac{2(a_1 + a_{14}) + 4(a_2 + a_5 + a_6 + a_9) + 5(a_4 + a_7) + 6(a_3 + a_8)}{6}, \frac{23}{12}\right).$$

By defuzzification the given fuzzy transportation table is reduced to Crisp Transportation table as follows.

	d_1	d_2	d_3	d_4	supply
O_1	46.9	60.35	167.65	114	104
O_2	33.53	20.11	103.78	39.59	36
O_3	87.17	131.56	227.04	147.85	166
demand	114	87	60	45	306

By applying Vogel Approximation Method, Transportation cost $=(46.9\times 17)+(60.35\times 87)+(39.59\times 36)+(87.17\times 97)+(227.04\times 60)+(147.85\times 9)$

$=30881.53$ Units

(iv) Ranking technique based on Centroid of Centroid Method: We know

$$\Re(CC_D)=\left(\frac{2(a_1+a_{10})+6(a_2+a_3+a_4+a_7+a_8+a_9)+7(a_5+a_6)}{54}\times\frac{19}{18}\right).$$

By defuzzification the given fuzzy transportation table is reduced to Crisp Transportation table as follows.

	d_1	d_2	d_3	d_4	supply
O_1	3.69	4.74	13.18	8.96	8
O_2	2.63	1.58	8.14	3.12	3
O_3	6.85	10.33	17.79	11.46	13
demand	9	7	5	3	24

By applying Vogel Approximation method

Transportation cost $=(4.74\times 7)+(13.18\times 1)+(3.12\times 3)+(6.85\times 9)$

$+(17.79\times 4)=188.53$ Units

Therefore, transportation cost is given as follows:

Table

Method	**Transportation Cost**
Total Integral Method	12301 Units
Graded Mean Integration Method	4414.7 Units
Ranking technique based on Centroid Method	30881.53 Units
Ranking technique based on Centroid of Centroid Method	188.53 Units

4. Conclusion

From the example in section 3, we find that defuzzification using centroid of centroid method is giving the minimum transportation cost. Next suitable method will be Graded Mean Integration Representation Method and Ranking technique based on Centroid Method is not suitable for defuzzification to find the minimum transportation cost for Decagonal Transportation problem.

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