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NOTE ON $Nmg^{\#}$- CLOSED SETS IN NANO MINIMAL STRUCTURE SPACES

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Abstract

In this paper, we introduce the notions of $Nmg^{\#}$- closed sets are obtain the unified characterizations for certain families of subsets between nano closed sets and $Nmg^{\#}$- closed sets. Also the relations of nano minimal structure spaces introduce and studied.

1. Introduction

M. K. R. S. Veera Kumar [11] introduced a new class of sets, namely $g^{\#}$- closed sets in topological spaces. Lellis Thivagar et al. [4] introduced a nano topological space with respect to a subset X of a universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space.

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Bhuvaneswari et al. [3] introduced and investigated nano g-closed sets in nano topological spaces. Rajendran et al. [9] introduced the notion of nano g^*-closed sets and further properties of nano g^*-closed sets are investigated. Pandi et al. [5] introduced the notions of nano minimal g^*-closed sets in nano minimal structure spaces. In this paper, we introduce the notions of $Nmg^{\#}$-closed sets are obtain the unified characterizations for certain families of subsets between nano closed sets and $Nmg^{\#}$-closed sets. Also the relations of nano minimal structure spaces introduce and studied.

2. Preliminaries

Throughout this paper $(U, \tau_R(x))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(U, \tau_R(X))$, $Ncl(A)$ and $N\operatorname{int}(A)$ denote the nano closure of A and the nano interior of A respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1 [6]**.** Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$

(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 [4]**.** If (U, R) is an approximation space and $X, Y \subseteq U$; then

(1) $L_R(X) \subseteq X \subseteq U_R(X)$;

(2) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;

(3) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(4) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

(5) $L_R(X \cup Y) \supset L_R(X) \cup L_R(Y)$;

(6) $L_R(X \cap Y) \subseteq L_R(X) \cup L_R(Y)$;

(7) $L_R(X) \subseteq L_R(Y)$; and $U_R(X) \subseteq U_R(Y)$ whenever $x \subseteq Y$;

(8) $U_R(X^c) = [L_R(X)]^c$ and $U_R(X^c) = [U_R(X)]^c$;

(9) $U_R U_R(X) = U_R L_R(X) = L_R(X)$.

(10) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3 [4]**.** Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$,

(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4 [4]. If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5 [4]. If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $A \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of A and it is denoted by $N\,\text{int}(A)$.

That is, $N\,\text{int}(A)$ is the largest nano open subset of A.

The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $Ncl(A)$.

That is, $Ncl(A)$ is the smallest nano closed set containing A.

Definition 2.6 [7]. A subfamily m_X of the power set $\wp(X)$ of a nonempty set X is called a minimal structure (briefly m-structure) on X if $\phi \in m_x$ and $U \in m_X$.

By $(X \in m_X)$ we denote a nonempty set X with a minimal structure m_X on X and call it an m-space. Each member of m_X is said to be m_X-open (briefly m-open) and the complement of an m_X-open set is said to be m_X-closed (briefly m-closed).

Definition 2.7. A subset A of a nano topological space $(U, \tau_R(X))$ is called

(1) nano α-open [4] if $A \subseteq N\,\text{int}\,(N\,\text{int}\,(A)))$.

(2) nano regular-open [4] if $A = N\,\text{int}\,(NCl(A))$.

(3) nano π-open [1] if the finite union of nano regular-open sets.

The family of nano α-open (resp. nano regular-open, nano π-open, nano open) sets is denoted by $N_\alpha O(U, \tau_R(X))$ (resp. $NRO(U, \tau_R(X))$, $N_\pi O(U, \tau_R(X)))$.

The complements of the above mentioned sets are called their respective closed sets.

The nano α-closure of a subset A of U is, denoted by $\alpha cl(A)$, defined to be the intersection of all nano α-closed sets containing A.

Definition 2.8. A subset A of a nano topological space $(U, \tau_R(X))$ is called;

(1) nano g-closed [2] if $Ncl(A) \subseteq G$, whenever $A \subseteq G$ and G is nano open.

(2) nano g^*-closed [9] if $Ncl(A) \subseteq G$, whenever $A \subseteq G$ and G is nano g-open.

(3) nano rg-closed set [10] if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano regular-open.

(4) nano πg-closed [8] if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano π-open.

The family of all nano g-open sets of U is denoted by $N_gO(U, \tau_R(X))$.

The complements of the above mentioned sets are called their respective closed sets.

The nano g-closure of a subset A of U is, denoted by $Ngcl(A)$, defined to be the intersection of all nano g-closed sets containing A.

Definition 2.9 [5]**.** A nano subfamily Nm_U of the power set $\wp(U)$ of a nonempty set U is called a nano minimal structure (briefly Nm-structure) on U if $\phi \in Nm_U$ and $U \in Nm_U$.

By (U, Nm_U), we denote a nonempty set U with a nano minimal structure Nm_U on U and call it a nano m-space (briefly Nm-space). Each member of Nm_U is said to be nano m_U-open (briefly Nm-open) and the complement of a nano Nm-open set is said to be nano m_U-closed (briefly Nm-closed).

Definition 2.10 [5]**.** A nano topological space $(U, \tau_R(X))$ with a nano minimal structure Nm_U on U is called a nano minimal structure space $(U, \tau_R(X), Nm_U)$.

Remark 2.11 [5]. Let $(U, \tau_R(X))$ be a nano topological space. Then the families $\tau_R(X)$, $O(, ( ))$, $(, ( ))$, $(, ( ))$, $(, ( ))$ and $(, ( ))$ are all nano minimal structure space $(, ( ))$.

Definition 2.12 [5]. Let $(,)$ be an Nm-space. For a subset A of U, the Nm_u-closure of A and the Nm_u-interior of A are defined in as follows:

(1) $Nm_U - cl(A) = \bigcap\{F : A \subseteq F, F^c \in Nm_U\}$.

(2) $Nm_U - \text{int}(A) = \bigcap\{V : V \subseteq A, V \in Nm_U\}$.

Remark 2.13 [5]. Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space and A a subset of U. If $Nm_U = \tau_R(X)$ (resp. $N\alpha O(U, \tau_R(X))$, $NgO(U, \tau_R(X))$), then we have $Nm_U - cl(A) = Ncl(A)$ (resp. $N\alpha cl(A)$, $Ngcl(A)$).

Definition 2.14 [5]. Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space. A subset A of U is said to be

(1) nano minimal generalized closed (briefly Nmg-closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nm_U-open.

(2) nano minimal generalized open (briefly Nmg-open) if its complement is called Nmg-closed. The family of all Nmg-open sets in U is an Nm-structure on U and denoted by $NmgO(U, \tau_R(X), Nm_U)$.

Definition 2.15 [5]. Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space. A subset A of U is said to be

(1) nano minimal generalized closed (briefly Nmg-closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nmg-open.

(2) nano minimal generalized open (briefly Nmg-open) if its complement is Nmg-closed. The family of all Nmg-open sets in U is an Nm-structure on U and denoted by $NmgO(U, \tau_R(X), Nm_U)$.

3. Nano Minimal $g^{\#}$- Closed Sets

We obtain several basic properties of nano minimal $g^{\#}$- closed sets.

Definition 3.1. Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space. A subset A of U is said to be

(1) nano minimal $g^{\#}$- closed (briefly $Nmg^{\#}$- closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is $Nmg^{\#}$- open.

(2) nano minimal $g^{\#}$- open(briefly $Nmg^{\#}$- open) if its complement is $Nmg^{\#}$- closed.

Definition 3.2. Let $(U, Nmg^{\#}O(U, \tau_R(X), Nm_U))$ be a nano minimal structure space. For a subset A of U, the Nmg^{*}- closure of A and the Nmg^{*}- interior of A are defined as follows:

(1) $Nmg^{*} - cl(A) = \bigcap\{F : A \subseteq F, U - F \in Nmg^{*}O(U, \tau_R(X), Nm_U)\}$,

(2) $Nmg^{*} - \text{int}(A) = \bigcap\{V : V \subseteq A, V \in Nmg^{*}O(U, \tau_R(X), Nm_U)\}$.

Definition 3.3. Let $(U, Nmg^{*}O(U, \tau_R(X), Nm_U))$ be a nano minimal structure space and A be a subset of U. Then Nmg^{*}- Frontier of A, $Nmg^{*} - Fr(A)$, is defined as follows: $Nmg^{*} - Fr(A) = Nmg^{*} - cl(A) \bigcap Nmg^{*} - cl(U - A)$.

Theorem 3.4. *Let $(U, Nmg^{*}O(U, \tau_R(X), Nm_U))$ be a nano minimal structure space and A be a subset of U. Then $x \in Nmg^{*} - cl(A)$ if and only if $V \bigcap A \neq \phi$, for every Nmg^{*}- open set V containing x.*

Proof. Suppose that there exists Nmg^{*}- open set V containing x such that $V \bigcap A = \phi$. Then $A \subseteq U - V$ and $U - (U - V) = V \in Nmg^{*}O(U, \tau_R(X), Nm_U)$. Then by definition 3.12, $Nmg^{*} - cl(A) \subseteq U - V$. Since $x \in V$, we have $x \notin Nmg^{*} - cl(A)$.

Conversely, suppose that $x \notin Nmg^* - cl(A)$. There exists a subset F of U such that $U - F \in Nmg^*O(U, \tau_R(X), Nm_U)$, $A \subseteq F$ and $x \notin F$. Then there exists Nmg^*-open set $U - F$ containing x such that $(U - F) \cap A = \phi$.

Definition 3.5. A Nm-structure $Nmg^*O(X)$ on a nonempty set U is said to have property $\mathcal{R}_{\mathcal{N}}$ if the union of any family of subsets belonging to $Nmg^*O(U, \tau_R(X), Nm_U)$ belongs to $Nmg^*O(U, \tau_R(X), Nm_U)$.

Example 3.6. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c, d\}\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{\phi, U, \{a\}, \{b, c, d\}\}$ and $Nm_U = \{\emptyset, U, \{a, b\}, \{a, c\}\}$. Then Nmg^*-open sets are $\emptyset, U, \{a\}, \{b\}, \{c\}, \{d\}$, $\{a, b\}, \{a, c\}, \{b, d\}$ and $\{b, c, d\}$. It is shown that $Nmg^*O(U, \tau_R(X), Nm_U)$ does not have property $\mathcal{R}_{\mathcal{N}}$.

Remark 3.7. Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space. Then the families $\tau_R(X), N\alpha O(U, \tau_R(X))$ and $NgO(U, \tau_R(X))$ are all Nm-structure with property $\mathcal{R}_{\mathcal{N}}$.

Lemma 3.8. *Let U be a nonempty set and $Nmg^*O(U, \tau_R(X), Nm_U)$ be a Nm-structure on U satisfying property $\mathcal{R}_{\mathcal{N}}$. For a subset A of U, the following properties hold*:

(1) $A \in Nmg^*O(U, \tau_R(X))$ *if and only if* $Nmg^* - \text{int}(A) = A$.

(2) A is Nmg^*-*closed if and only if* $Nmg^* - cl(A) = A$.

(3) $Nmg^* - \text{int}(A) \in Nmg^*O(U, \tau_R(X))$ *and* $Nmg^* - cl(A)$ *is* Nmg^*-*closed*.

Remark 3.9. Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space and A be a subset of U. If $Nmg^*O(U, \tau_R(X), Nm_U) = NgO(U, \tau_R(X))$ (resp. $\tau_{r^*}(X)$, $N\pi O(U, \tau_R(X))$, $NRO(U, \tau_R(X))$) and A is nano g^*-closed, then A is $Nmg^{\#}$-closed (resp. nano g-closed, nano πg-closed, nano rg-closed).

Proposition 3.10. *Let* $Nmg^{*}O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$. *Then the following implications hold*:

$$\text{nano close} \rightarrow \text{nano } g^{*}\text{-closed} \rightarrow Nmg^{\#}\text{-closed.}$$

Proof. It is obvious that every nano closed set is nano g^{*}- closed. Suppose that A is a nano g^{*}- closed set. Let $A \subseteq V$ and $V \in Nmg^{*}O(U, \tau_R(X), Nm_U)$. Since $Nmg^{*}O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$, $Ncl(A) \subseteq V$ and hence A is $Nmg^{\#}$- closed.

Example 3.11. (1) Let $U = \{a, b, c\}$ with $U/R = \{\{b\}, \{a, c\}, \{a, c\}\}$ and $X = \{c\}$. Then the nano topology $\tau_R(X) = \{\emptyset, U, \{a, c\}\}$ and $Nm_U = \{\emptyset, U\}$. Then nano g-closed are $\emptyset, U, \{b\}, \{a, b\}, \{b, c\}$; and nano g^{*}- closed sets are $\emptyset$, U, $\{b\}$, $\{a, b\}$, $\{b, c\}$. It is clear that $\{a, b\}$ is nano g^{*}- closed set but it is not nano closed.

(2) In Example 3.6, $Nmg^{\#}$- closed sets are $\emptyset, U, \{a\}, \{a, d\}, \{b, c\}, \{c, d\}$, $\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ and nano g^{*}- closed sets are $\emptyset, U, \{a\}$, $\{b, c, d\}$. It is clear that $\{a, d\}$ is $Nmg^{\#}$- closed set but it is not nano g^{*}- closed.

Proposition 3.12. *If* A *and* B *are* $Nmg^{\#}$- *closed sets, then* $A \cup B$ *is* $Nmg^{\#}$- *closed*.

Proof. Let $A \cup B \subseteq V$ and $V \in Nmg^{*}O(U, \tau_R(X), Nm_U)$. Then $A \subseteq V$ and $B \subseteq V$. Since A and B are $Nmg^{\#}$- closed, we have $Ncl(A \cup B) = Ncl(A) \cup Ncl(B) \subseteq V$. Therefore, $A \cup B$ is $Nmg^{\#}$- closed.

Proposition 3.13. *If* A *is* $Nmg^{\#}$- *closed and* $Nmg^{\#}$- *open, then* A *is nano closed*.

Proof. Since A is $Nmg^{\#}$-closed and Nmg^{*}-open, then $Ncl(A) \subseteq A$ but $A \subseteq Ncl(A)$. Therefore $Ncl(A) = A$. Hence, A is nano closed.

Proposition 3.14. If A is $Nmg^{\#}$-closed and $A \subseteq B \subseteq Ncl(A)$, then B is $Nmg^{\#}$-closed.

Proof. Let $B \subseteq V$ and $AV \in Nmg^{*}O(U, \tau_R(X), Nm_U)$. Then $A \subseteq V$ and A is $Nmg^{\#}$-closed. Hence $Ncl(B) \subseteq Ncl(A) \subseteq A$ and B is $Nmg^{\#}$-closed.

Proposition 3.15. *If A is $Nmg^{\#}$-closed and $A \subseteq V \in Nmg^{\#}\mathrm{O}(U, \tau_R(X), Nm_U)$, then $Nmg^{*} - Fr(V) \subseteq N\,\mathrm{int}\,(U - A)$.*

Proof. Let A be $Nmg^{\#}$-closed and $A \subseteq V \in Nmg^{*}\mathrm{O}(U, \tau_R(X), Nm_U)$. Then $Ncl(A) \subseteq A$. Suppose that $x \in Nmg^{*} - Fr(V)$. Since $V \in Nmg^{*}O(U, \tau_R(X), Nm_U)$, $Nmg^{*} - Fr(V) = Nmg^{*} - cl(V) \cap Nmg^{*} - cl(U - V)$ $= Nmg^{*} - cl(V) \cap (U - V) = Nmg^{*} - cl(V) - V$. Therefore, $x \notin V$ and $x \notin Ncl(A)$. This shows that $x \in N\,\mathrm{int}\,(U - A)$ and hence $Nmg^{\#} - Fr(V) \subseteq N\,\mathrm{int}\,(U - A)$.

Proposition 3.16. *A subset A of U is $Nmg^{\#}$-open if and only if $F \subseteq N\,\mathrm{int}\,(U - A)$. whenever $F \subseteq A$ and A is Nmg^{*}-closed.*

Proof. Suppose that A is $Nmg^{\#}$-open. Let $F \subseteq A$ and F be Nmg^{*}-closed. Then $U - A \subseteq U - F \in Nmg^{*}O(U, \tau_R(X), Nm_U)$ and $U - A$ is $Nmg^{\#}$-closed. Therefore, we have $U - N\,\mathrm{int}\,(A) = Ncl(U - A) \subseteq U - F$ and hence $F \subseteq N\,\mathrm{int}\,(A)$.

Conversely, let $U - A \subseteq G$ and $G \in Nmg^{*}O(U, \tau_R(X), Nm_U)$. Then $U - G \subseteq A$ and $U - G$ is Nmg^{*}-closed. By hypothesis, we have $U - G \subseteq N\,\mathrm{int}\,(A)$ and hence $Ncl(U - A) = U - N\,\mathrm{int}\,(A) \subseteq G$. Therefore, $U - A$ is $Nmg^{\#}$-closed and A is $Nmg^{\#}$-open.

Corollary 3.17. *Let* $Nmg^{*}O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$. *Then the following properties hold*:

(1) *Every nano open set is* $Nmg^{\#}$- *open and every* $Nmg^{\#}$- *open set is nano* g^{*}- *open*,

(2) *If A and B are* $Nmg^{\#}$- *open*, *then* $A \cap B$ *is* $Nmg^{\#}$- *open*,

(3) *If A is* $Nmg^{\#}$- *open and* $Nmg^{\#}$- *closed*, *then A is nano open*,

(4) *If A is* $Nmg^{\#}$- *open and* $N\operatorname{int}(A) \subseteq A$, *then B is* $Nmg^{\#}$- *open*. *This follows from propositions* 3.10, 3.12, 3.13 *and* 3.14.

4. Characterizations of $Nmg^{\#}$- Closed Sets

We obtain some characterizations of $Nmg^{\#}$- closed sets.

Theorem 4.1. *A subset A of U is* $Nmg^{\#}$- *closed if and only if* $Ncl(A) \cap F = \emptyset$ *whenever* $A \cap F = \emptyset$ *and F is* Nmg^{*}- *closed*.

Proof. Suppose that A is $Nmg^{\#}$- closed. Let $A \cap F = \emptyset$ and F be Nmg^{*}- closed. Then $A \subseteq U - F \in Nmg^{*}O(U, \tau_R(X), Nm_U)$ and $Ncl(A) \subseteq U - F$. Therefore, we have $Ncl(A) \cap F = \emptyset$.

Conversely, let $A \subseteq V$ and $V \in Nmg^{*}O(U, \tau_R(X), Nm_U)$. Then $A \cap (U - V) = \emptyset$ and $U - V$ is Nmg^{*}- closed. By the hypothesis, $Ncl(A) \cap (U - V) = \emptyset$ and hence $Ncl(A) \subseteq V$. Therefore, A is $Nmg^{\#}$- closed.

Theorem 4.2. *Let* $Nmg^{*}O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$ *and* $Nmg^{*}O(U, \tau_R(X), Nm_U)$ *have property* $\mathcal{R}_{\mathcal{N}}$. *A subset A of U is* $Nmg^{\#}$- *closed if and only if* $Ncl(A) - A$ *contains no nonempty* Nmg^{*}- *closed*.

Proof. Suppose that A is $Nmg^{\#}$-closed. Let $F \subseteq Ncl(A) - A$ and F be Nmg^{*}-closed. Then $F \subseteq Ncl(A)$ and $F \not\subseteq A$ and so $A \subseteq U - F \in Nmg^{*}O(U, \tau_R(X), Nm_U)$ and hence $Ncl(A) \subseteq U - F$. Therefore, we have $F \subseteq U - Ncl(A)$. Hence $F = \emptyset$.

Conversely, suppose that A is not $Nmg^{\#}$-closed. Then by Theorem 4.1, $\emptyset \neq Ncl(A) - V$ for some $V \in Nmg^{*}O(U, \tau_R(X), Nm_U)$ containing A. Since $\tau_R(X) \subseteq Nmg^{*}O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$ and $Nmg^{*}O(U, \tau_R(X), Nm_U)$ has property $R_N A \subseteq U$, $Ncl(A) - V$ is Nmg^{*}-closed. Moreover, we have $Ncl(A) - V \subseteq Ncl(A) - A$, a contradiction. Hence A is $Nmg^{\#}$-closed.

Theorem 4.3. *Let* $Nmg^{*}O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$ *and* $Nmg^{*}O(U, \tau_R(X), Nm_U)$ *have property* $\mathcal{R}_{\mathcal{N}}$. *A subset* A *of* U *is* $Nmg^{\#}$-*closed if and only if* $Ncl(A) - A$ *is* $Nmg^{\#}$-*open.*

Proof. Suppose that A is $Nmg^{\#}$-closed. Let $F \subseteq Ncl(A) - A$ and F be Nmg^{*}-closed. By Theorem 4.2, we have $F = \emptyset$ and $F \subseteq N\operatorname{int}(Ncl(A) - A)$ it follows from Proposition 3.16, $Ncl(A) - A$ is $Nmg^{\#}$-open.

Conversely, let $A \subseteq V$ and $V \in Nmg^{*}O(U, \tau_R(X))$. Then $Ncl(A) \cap (U - V) \subseteq Ncl(A) - A$ and $Ncl(A) - A$ is $Nmg^{\#}$-open. Since $\tau_R(X) \subseteq Nmg^{*}O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$ and $Nmg^{*}O(U, \tau_R(X), Nm_U)$ has property $\mathcal{R}_{\mathcal{N}}$, $Ncl(A) \cap (U - V)$ is $Nmg^{\#}$-closed and by proposition 3.16, $Ncl(A) \cap (U - V) \subseteq N\operatorname{int}(Ncl(A) - A)$. Now $N\operatorname{int}(Ncl(A) - A) = N\operatorname{int}(Ncl(A)) \cap N\operatorname{int}(U - A) \subseteq Ncl(A) \cap N\operatorname{int}(U - A) = Ncl(A) \cap (U - Ncl(A)) = \phi$. Therefore, we have $Ncl(A) \cap (U - V) = \emptyset$ and hence $Ncl(A) \subseteq V$. This shows that A is Nmg^{*}-closed.

Theorem 4.4. *Let* $(U, Nmg^{*}O(U, \tau_R(X), Nm_U))$ *be an* Nm*-structure with property* $\mathcal{R}_{\mathcal{N}}$. *A subset* A *of* U *is* $Nmg^{\#}$*-closed if and only if* $Nmg^{*}-cl(\{x\}) \cap A \neq \emptyset$ *for each* $x \in Ncl(A)$.

Proof. Suppose that A is $Nmg^{\#}$- closed and $Nmg^{*}-cl(\{x\}) \cap A \neq \emptyset$ for some $x \in Ncl(A)$. By lemma 3.8, $Nmg^{*}-cl(\{x\})$ is Nmg^{*}-closed and $A \subseteq U-(Nmg^{*}-cl(\{x\})) \in Nmg^{*}O(U, \tau_R(X))$. Since A is Nmg^{*}-closed, $Ncl(A) \subseteq U-(Nmg^{*}-cl(\{x\})) \subseteq U-\{x\}$, a contradiction, since $x \in Ncl(A)$.

Conversely, suppose that A is not $Nmg^{\#}$- closed. Then by Theorem 4.1, $\emptyset \neq Ncl(A)-V$ for some $V \in Nmg^{*}O(U, \tau_R(X), Nm_U)$ containing A. There exists $x \in Ncl(A)-V$. Since $x \notin V$, by Theorem 3.4, $Nmg^{*}-cl(\{x\}) \cap V = \emptyset$ and hence $Nmg^{*}-cl(\{x\}) \cap A \subseteq Nmg^{*}-cl(\{x\}) \cap V = \emptyset$. This shows that $Nmg^{*}-cl(\{x\}) \cap A = \emptyset$ for some $x \in Ncl(A)$. Hence A is $Nmg^{\#}$- closed.

Corollary 4.5. *Let* $Nmg^{*}O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$ *and* $Nmg^{*}O(U, \tau_R(X), Nm_U)$ *have property* $\mathcal{R}_{\mathcal{N}}$. *For a subset* A *of* U, *the following properties are equivalent*:

(1) *A is $Nmg^{\#}$- open*.

(2) $A-N\operatorname{int}(A)$ *contains no nonempty* Nmg^{*}*-closed set*.

(3) $A-N\operatorname{int}(A)$ *is* $Nmg^{\#}$*- open*.

(4) $Nmg^{*}-cl(\{x\}) \cap (U-A) \neq \emptyset$ *for each* $x \in A-N\operatorname{int}(A)$.

This follows from Theorems 4.2, 4.3 and 4.4.

5. Conclusion

We think that our approach is an important meeting point between nano topology and nano minimal structure spaces. In future, we will discuss more applications of nano topological concepts in nano minimal structure spaces.

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