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CONNECTIVITY IN INTUITIONISTIC L-FUZZY GRAPHS

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Abstract

Connectivity is the major aspect of a dynamic network. It has been broadly studied and applied in several perspectives. In this paper, we define and study path, strength of a path and strong arc of an ILFG. Naturally questions regarding connectedness arise. So we also define and examine path connected ILFG, MN-connected ILFG and strongly connected ILFG. We prove several sufficient conditions for an arc of an ILFG to be a strong arc. Further we discuss different connectivity concepts which are preserved under isomorphism and weak isomorphism. These concepts offer a solid background for further work.

1. Introduction

Zadeh [19] developed the concept of fuzzy relation in 1965 to describe the situations in which the data are imprecise or vague. In 1975, Rosenfeld [12] introduced the concept of fuzzy graphs. Researchers in this field developed and studied numerous generalizations of fuzzy graph theory. Intuitionistic fuzzy graphs is one of such generalization and it was introduced by Atanassov [2] in 1999. Parvathy and Karunambigai [10] gave a new definition of

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intuitionistic fuzzy graphs and studied properties of its components. Bhutani and Rosenfeld [3, 4] initiated the study of arcs in fuzzy graphs. Sunitha and Vijayakumar [14] discussed fuzzy bridges and fuzzy cutnodes in detail. Mathew and Sunitha [6] analyzed various types of arcs in fuzzy graphs. They also introduced different types of connectivity parameters in [7]. Sandeep Narayan and Sunitha [13] discussed different types of arcs in fuzzy graph and its complement. Sunitha and Mathew [15] made an excellent survey of the research developments in fuzzy graph theory. Parvathy, Karunambigai and Buvaneswari [5] generalized some connectivity parameters to IFG. Nivethana and Parvathi [9] also carried out an arc analysis of intuitionistic fuzzy graphs. Nagoor Gani and Shajitha Begum [8] studied different properties of strong arcs in intuitionistic fuzzy graphs. Akram and Alshehri [1] discussed different types of intuitionistic fuzzy bridges and cutnodes of intuitionistic fuzzy graphs. Pramada and Thomas [11] introduced L-fuzzy graphs as another generalization of fuzzy graphs. Tintumol, Pramada and Magie [16] introduced intuitionistic L-fuzzy graphs and studied some properties. Further in [17], isomorphism properties of an intuitionistic L-fuzzy graph were studied.

The area of ILFGs present several possibilities. Also the concept of connectivity plays a prime role in ILFGs. This is the first body of work to study paths and connectivity in ILFGs.

In this paper, we introduce several connectivity concepts particularly, path of an ILFG, strength of a path and strength of connectedness between two nodes of an ILFG. Depending on the strength of connectedness between two nodes, we define strong arcs and strong paths of an ILFG and some results. We also define different classes of ILFGs, viz., path connected ILFGs, MN-connected ILFGs and strongly connected ILFGs and prove that these are preserved under isomorphism.

2. Preliminaries

In this section, we review some of the necessary definitions for the understanding of subsequent results.

Definition 2.1 [18]. A lattice is a partially ordered set $(L, \leqslant)$ in which every pair of elements $a, b \in L$ has a greatest lower bound $a \wedge b$ and a least upper bound $a \vee b$.

Definition 2.2 [18]. A lattice is called a complete lattice if each of its nonempty subsets has a least upper bound and a greatest lower bound.

Definition 2.3 [16]. Let L be a complete lattice with an involutive order reversing operation $c : L \to L$. An intuitionistic L-fuzzy graph (ILFG) G^L with underlying set V is defined to be $G = (V, \sigma, \mu)$ where $\sigma = (M_\sigma, N_\sigma)$ and $\mu = (M_\mu, N_\mu)$ such that

(i) The functions $M_\sigma : V \to L$ and $N_\sigma : V \to L$ should satisfy $\mathrm{M}_\sigma(x) \leqslant c(N_\sigma(x)), \forall x \in V$. Here $M_\sigma(x)$ and $N_\sigma(x)$ denote the degree of membership and degree of nonmembership of the vertex $x \in V$ respectively.

(ii) The functions $M_\mu : E \to L$ and $N_\mu : E \to L$ where should satisfy

$M_\mu(x, y) \leqslant M_\sigma(x) \wedge M_\sigma(y)$

$N_\mu(x, y) \geqslant N_\sigma(x) \vee N_\sigma(y)$

$M_\mu(x, y) \leqslant c(N_\mu(x, y)), \forall (x, y) \in E$. Here $M_\sigma(x, y)$ and $N_\sigma(x, y)$ denote the degree of membership and degree of non-membership of the edge $(x, y) \in E$ respectively.

Definition 2.4 [16]. An ILFG $G^L = (V, \sigma, \mu)$ is said to be a complete ILFG, if $M_\mu(x, y) = M_\sigma(x) \wedge M_\sigma(y)$ and $N_\mu(x, y) = N_\sigma(x) \vee N_\sigma(y)$, $\forall (x, y) \in V \times V$.

Definition 2.5 [16]. An ILFG $H^L = (V, v, \tau)$ is said to be partial intuitionistic L-fuzzy subgraph of the ILFG $G^L = (V, \sigma, \mu)$ if $M_\nu(x) \leqslant M_\sigma(x)$, and $N_\nu(x) \geqslant N_\sigma(x), \forall x \in V$ and $M_\tau(x, y) \leqslant M_\mu(x, y)$ and $N_\tau(x, y) \geqslant N_\mu(x, y)$, $\forall (x, y) \in E$.

Definition 2.6 [17]. Let $G_1^L = (V_1, \sigma_1, \mu_1)$ and $G_2^L = (V_2, \sigma_2, \mu_2)$ be any two ILFGs. Then an isomorphism from G_1^L to G_2^L is a bijective mapping $f : V_1 \to V_2$ such that

(i) $M_{\sigma_1}(x) = M_{\sigma_2}(f(x))$

$N_{\sigma_1}(x) = N_{\sigma_2}(f(x)), \forall x \in V_1$

(ii) $M_{\mu_1}(x, y) = M_{\mu_2}(f(x), f(y))$

$N_{\mu_1}(x, y) = N_{\mu_2}(f(x), f(y)), \forall (x, y) \in V_1 \times V_1$

Definition 2.7 [17]. Let $G_1^L = (V_1, \sigma_1, \mu_1)$ and $G_2^L = (V_2, \sigma_2, \mu_2)$ be any two ILFGs. Then a weak isomorphism from G_1^L to G_2^L is a bijective mapping $f : V_1 \to V_2$ such that

(i) $M_{\sigma_1}(x) = M_{\sigma_2}(f(x))$

$N_{\sigma_1}(x) = N_{\sigma_2}(f(x)), \forall x \in V_1$

(ii) $M_{\mu_1}(x, y) \leqslant M_{\mu_2}(f(x), f(y))$

$N_{\mu_1}(x, y) \geqslant N_{\mu_2}(f(x), f(y)), \forall (x, y) \in V_1 \times V_1$

3. Paths in intuitionistic L-fuzzy graphs

In this section, we introduce path in an ILFG, strength of a path, strength of the connectedness between two nodes, strong arc of an ILFG and discuss its properties.

Definition 3.1. A path P in an ILFG $G^L = (V, \sigma, \mu)$ is a sequence of distinct vertices $v_0, v_1, v_2, \ldots, v_n$ in V such that one of the following conditions is satisfied:

(i) $M_\mu(v_{i-1}, v_i) > 0$ and $N_\mu(v_{i-1}, v_i) > 0$,

(ii) $M_\mu(v_{i-1}, v_i) > 0$ and $N_\mu(v_{i-1}, v_i) = 0$,

(iii) $M_\mu(v_{i-1}, v_i) > 0$ and $N_\mu(v_{i-1}, v_i) > 0$, for $i = 1, 2, \ldots, n$.

Here 0 is the least value of a lattice.

The length of such a path is n.

Consider the ILFG G^L in the figure 1. Then $v_1v_2v_3v_4$, $v_2v_3v_4$, $v_2v_4v_1$ and $v_2v_3v_4v_1$ are some paths of G^L.

Definition 3.2. An ILFG G^L is said to be path connected or simply connected if there exists a path between every pair of nodes of G^L.

Definition 3.3. *M*-strength of a path P, $S_M(P)$ is defined as the infimum of the membership values of all arcs in the path *P*. *N*-strength of a path P, $S_N(P)$ is defined as the supremum of the non-membership values of all arcs in the path *P*. Then the strength of a path P, S_P is $(S_M(P), S_N(P))$

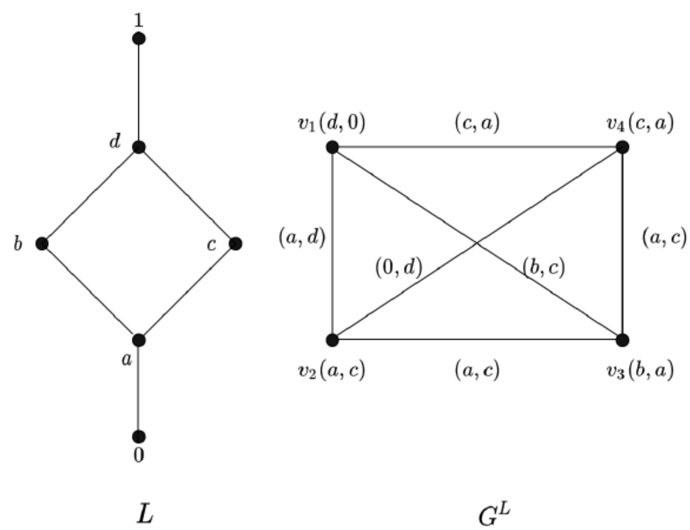

Figure 1. Lattice L and Intuitionistic L-fuzzy graph G^L.

Definition 3.4. If the nodes u and v in V are connected by means of paths of length k, then

$$M_\mu^k(u, v) = \vee\{(M_\mu(u, v_1) \wedge M_\mu(v_1, v_2) \wedge \ldots \wedge M_\mu(v_{k-1}, v))/v_1, v_2, \ldots, u_{k-1} \in V\}$$

$$N_\mu^k(u, v) = \wedge\{(N_\mu(u, v_1) \vee N_\mu(v_1, v_2) \vee \ldots \vee N_\mu(v_{k-1}, v))/v_1, v_2, \ldots, u_{k-1} \in V\}.$$

Definition 3.5. The *M*-strength of the connectedness between two nodes

u and v in G^L is denoted by $\mathcal{CONN}_{M(G^L)}(u, v)$ and is defined as the supremum of M-strength of all paths between u and v. i.e., $\mathcal{CONN}_{M(G^L)}(u, v) = \vee \{M_\mu^k(u, v)/k = 1, 2, \ldots n\}$.

The N-strength of the connectedness between two nodes u and v in G^L is denoted by $\mathcal{CONN}_{N(G^L)}(u, v)$ and is defined as the infimum of N-strength of all paths between u and v. i.e., $\mathcal{CONN}_{N(G^L)}(u, v) = \vee\{N_\mu^k(u, v)/k = 1, 2, \ldots n\}$

The strength of the connectedness between two nodes u and v in G^L is

$$(\mathcal{CONN}_{M(G^L)}(u, v), \mathcal{CONN}_{N(G^L)}(u, v)).$$

Definition 3.6. An ILFG G^L is MN-connected if one of the following conditions is satisfied:

(i) $\mathcal{CONN}_{M(G^L)}(u, v) > 0$ and $\mathcal{CONN}_{N(G^L)}(u, v) > 0$

(ii) $\mathcal{CONN}_{M(G^L)}(u, v) > 0$ and $\mathcal{CONN}_{N(G^L)}(u, v) = 0$

(iii) $\mathcal{CONN}_{M(G^L)}(u, v) = 0$ and $\mathcal{CONN}_{N(G^L)}(u, v) > 0, \forall u, v \in V$.

Here 0 is the least element of the lattice L.

Theorem 3.7. *If for every arc e in* G^L, $M_\mu(e) = l_1$ *and* $N_\mu(e) = l_2$ *where* $l_1, l_2 \in L$, *then the strength of connectedness between every pair of vertices in* G^L *is* (l_1, l_2).

Proof. Let u and v be any two nodes in G^L and P be an arbitrary path joining the nodes u and v of length k.

Then

$$\begin{aligned} M_\mu^k(u, v) &= \vee\{(M_\mu(u, v_1) \wedge M_\mu(v_1, v_2) \wedge \ldots \wedge M_\mu(v_{k-1}, v))/v_1, v_2, \ldots, u_{k-1} \in V\} \\ &= \vee\{(l_1 \wedge l_1 \wedge \ldots \wedge l_1)/v_1, v_2, \ldots, u_{k-1} \in V\} \\ &= l_1 \end{aligned}$$

Hence

$$\mathcal{CONN}_{M(G^L)}(u, v) = \vee\{M_\mu^k(u, v)/\ k = 1, 2, \ldots n\}$$

$$= l_1 \vee l_1 \vee \ldots \vee l_1$$

$$= l_1$$

Similarly $N_\mu^k(u, v) = l_2$ and hence $\mathcal{CONN}_{M(G^L)}(u, v) = l_2$.

i.e., Strength of the connectedness between u and v in G^L is (l_1, l_2). □

Note 3.8. The strength of the connectedness between two nodes u and v in G^L, obtained by deleting the arc (u, v) is denoted by $(\mathcal{CONN}_{M(G^L)-(u, v)}(u, v), \mathcal{CONN}_{N(G^L)-(u, v)}(u, v))$.

Theorem 3.9. *Let $G^L = (V, \sigma, \mu)$ be an ILFG and $H^L = (V, \nu, \tau)$ be the partial intuitionistic L-fuzzy subgraph of G^L. Then*

$$\mathcal{CONN}_{M(H^L)}(u, v) \leqslant \mathcal{CONN}_{M(G^L)}(u, v)$$

$$\mathcal{CONN}_{N(H^L)}(u, v) \geqslant \mathcal{CONN}_{M(G^L)}(u, v),\ \forall u, v \in V.$$

Proof. We have

$$M_\mu^k(u, v) = \vee\{(M_\mu(u, v_1) \wedge M_\mu(v_1, v_2) \wedge \ldots \wedge M_\mu(v_{k-1}, v))/v_1, v_2, \ldots, u_{k-1} \in V\}$$

$$\geqslant \vee\ \{(M_\tau(u, v_1) \wedge M_\tau(v_1, v_2) \wedge \ldots \wedge M_\tau(v_{k-1}, v))/v_1, v_2, \ldots, u_{k-1} \in V\}$$

$$\text{since } M_\mu(e) \geq M_\tau(e),$$

$$= M_\tau^k(u, v)$$

Now

$$\mathcal{CONN}_{M(G^L)}(u, v) = \vee\{M_\mu^k(u, v)/k = 1, 2, \ldots, n\}$$

$$\geqslant \vee\ \{M_\tau^k(u, v)/k = 1, 2, \ldots n\}$$

$$= \mathcal{CONN}_{M(H^L)}(u, v)$$

Similarly, $N_{\mu}^{k}(u, v) \leqslant N_{\tau}^{k}(u, v)$

Hence $\mathcal{CONN}_{N(H^L)}(u, v) \geqslant \mathcal{CONN}_{N(G^L)}(u, v)$. □

Definition 3.10. A $u-v$ path P is called M-strongest $u-v$ path, if its M-strength equals $\mathcal{CONN}_{M(G^L)}(u, v)$. A $u-v$ path P is called N-strongest $u-v$ path, if its N-strength equals $\mathcal{CONN}_{N(G^L)}(u, v)$. If a $u-v$ path P is both M-strongest and N-strongest, then the path P is known as a strongest path.

From the figure 1, it can be observed that the only strongest $v_1 - v_4$ path is the arc $v_1 v_4$.

Remark 3.11. In an ILFG, the M-strongest path and N-strongest path between two nodes does not always exist, since the supremum and infimum of a set of lattice elements need not be an element in that set.

Remark 3.12. The M-strongest path and N-strongest path between two nodes exist in an ILFG does not imply strongest path between that nodes exists. Because strongest path between two nodes exists iff M-strongest path and N-strongest path should be the same path.

Remark 3.13. In a connected ILFG G^L, if the lattice L is a chain, then M-strongest path and N-strongest path always exist. But there is no assurance of strongest paths.

Definition 3.14. An arc (u, v) in G^L is said to be a strong arc if

$$M_{\mu}(u, v) \geqslant \mathcal{CONN}_{M(G^L)-(u, v)}(u, v) \text{ and } N_{\mu}(u, v) \leqslant \mathcal{CONN}_{N(G^L)-(u, v)}(u, v).$$

Definition 3.15. If all arcs in a path P of G^L are strong, then the path P is said to be a strong path.

The arcs v_1v_4, v_1v_3, v_2v_3 and v_3v_4 are the strong arcs of the graph G^L in the figure 1. Hence the paths $v_1v_4v_3v_2$, $v_2v_3v_1v_4$, $v_2v_3v_1$, $v_2v_3v_4$, $v_3v_1v_4$, $v_3v_4v_1$ and $v_1v_3v_4$ are strong paths.

Remark 3.16. If for an arc (u, v) in G^L, the membership value is maximum and nonmembership value is minimum among all the arcs in G^L, then certainly the arc (u, v) is a strong arc in G^L. But the converse need not be true.

Definition 3.17. A connected ILFG G^L is said to be strongly connected if there exists a strong path between every pair of nodes in G^L.

Theorem 3.18. *An arc* (u, v) *in* G^L *is strong if and only if* $M_\mu(u, v) = \mathcal{CONN}_{M(G^L)}(u, v)$ *and* $N_\mu(u, v) = \mathcal{CONN}_{N(G^L)}(u, v)$.

Proof. Let the arc (u, v) be a strong arc in G^L. Then

$M_\mu(u, v) \geqslant \mathcal{CONN}_{M(G^L)-(u, v)}(u, v)$. and $N_\mu(u, v) \leqslant \mathcal{CONN}_{N(G^L)-(u, v)}(u, v)$.

But we have

$$\mathcal{CONN}_{M(G^L)}(u, v) = \mathcal{CONN}_{M(G^L)-(u, v)}(u, v) \vee M_\mu(u, v) \text{ and}$$

$$\mathcal{CONN}_{N(G^L)}(u, v) = \mathcal{CONN}_{N(G^L)-(u, v)}(u, v) \wedge N_\mu(u, v)$$

Hence

$$\mathcal{CONN}_{M(G^L)-(u, v)}(u, v) \vee M_\mu(u, v) = M_\mu(u, v) \text{ and}$$

$$\mathcal{CONN}_{N(G^L)-(u, v)}(u, v) \wedge N_\mu(u, v) = N_\mu(u, v)$$

i.e.,

$$\mathcal{CONN}_{M(G^L)}(u, v) = M_\mu(u, v) \text{ and } \mathcal{CONN}_{N(G^L)}(u, v) = N_\mu(u, v)$$

Conversely, assume $\mathcal{CONN}_{M(G^L)}(u, v) = M_\mu(u, v)$ and $\mathcal{CONN}_{N(G^L)}(u, v) = N_\mu(u, v)$

Since

$$\mathcal{CONN}_{M(G^L)}(u, v) \geqslant \mathcal{CONN}_{M(G^L)-(u, v)}(u, v) \text{ and}$$

$$\mathcal{CONN}_{N(G^L)}(u, v) \leqslant \mathcal{CONN}_{N(G^L)-(u, v)}(u, v)$$

We have

$$M_\mu(u, v) \geqslant \mathcal{CONN}_{M(G^L)-(u, v)}(u, v) \text{ and } N_\mu(u, v) \leqslant \mathcal{CONN}_{N(G^L)-(u, v)}(u, v)$$

i.e., The arc (u, v) is a strong arc in G^L. □

Theorem 3.19. *If* $M_\mu(u, v) = M_\sigma(u) \wedge M_\sigma(v)$ *and* $N_\mu(u, v) = N_\sigma(u) \vee N_\sigma(v)$ *in* G^L, *then* (u, v) *is a strong arc in* G^L.

Proof. Let P be any arbitrary path from u to v other than the arc (u, v). Then P contains the arcs (u, v_1) and (v_1, v) for some $u \neq v_1$ and $v_1 \neq v$ in V. Now

$$\begin{aligned} S_M(P) &\leqslant M_\mu(u, v_1) \wedge M_\mu(v_1, v) \\ &\leqslant (M_\sigma(u) \wedge M_\sigma(v_1)) \wedge (M_\sigma(v_1) \wedge M_\sigma(v)) \\ &\leqslant M_\sigma(u) \wedge M_\sigma(v) \end{aligned}$$

But $M_\mu(u, v) = M_\sigma(u) \wedge M_\sigma(v)$. Hence $S_M(P) \leq M_\mu(u, v)$.

Since P is an arbitrary path, for every path P from u to v other than the arc (u, v) $S_M(P) \leqslant M_\mu(u, v)$. Therefore $\vee_P S_M(p) \leqslant M_\mu(u, v)$

But $\mathcal{CONN}_{M(G^L)-(u, v)}(u, v) = \vee_P S_M(P)$

Hence $\mathcal{CONN}_{M(G^L)-(u, v)}(u, v) \leqslant M_\mu(u, v)$

Similarly $\mathcal{CONN}_{N(G^L)-(u, v)}(u, v) \geqslant N_\mu(u, v)$

i.e., (u, v) is a strong arc in G^L. □

Theorem 3.20. *Every path in a complete intuitionistic L-fuzzy graph is a strong path.*

Proof. Let $G^L = (V, \sigma, \mu)$ be a complete ILFG. Then $M_\mu(u, v)$

$= M_\sigma(u) \wedge M_\sigma(v)$ and $N_\mu(u, v) = N_\sigma(u) \vee N_\sigma(v)\ \forall u, v \in V$. Hence by theorem 3.19, for every u, v in $V, (u, v)$ is a strong arc in G^L. i.e., Every arc in G^L is a strong arc. Hence every path in a complete intuitionistic L-fuzzy graph G^L is strong. □

Remark 3.21. Every arc in a complete ILFG $G^L = (V, \sigma, \mu)$ is a strong arc and hence for every, $(u, v) \in \mu* = E$, $M_\mu(u, v) = \mathcal{CONN}_{M(G^L)}(u, v)$ and $N_\mu(u, v) = \mathcal{CONN}_{N(G^L)}(u, v)$.

4. Isomorphism and Connectivity of an ILFG

In this section, we study connectedness properties of isomorphic, weak isomorphic intuitionistic L-fuzzy graphs.

Theorem 4.1. *Let $G_1^L = (V_1, \sigma_1, \mu_1)$ be isomorphic to $G_2^L = (V_2, \sigma_2, \mu_2)$. Then G_1^L is path connected iff G_2^L is path connected.*

Proof. Firstly assume that G_1^L is path connected and let u, v be any two arbitrary nodes in G_1^L. Since G_1^L is path connected, there is a path say $uv_1v_2 \ldots v_{k-1}v$ connecting u and v.

Then $M_\mu(v_{i-1}, v_i) > 0$ and $N_\mu(v_{i-1}, v_i) > 0$ or $M_\mu(v_{i-1}, v_i) > 0$ and $N_\mu(v_{i-1}, v_i) = 0$ or $M_\mu(v_{i-1}, v_i) = 0$ and $N_\mu(v_{i-1}, v_i) > 0$ for $i = 1, 2, \ldots, k$, where $v_0 = u$ and $v_k = v$.

But since G_1^L is isomorphic to G_2^L, $M_{\mu_1}(u, v) = M_{\mu_2}(f(u), f(v))$ and $N_{\mu_1}(u, v) = N_{\mu_2}(f(u), f(v))$. Hence $f(u)$ and $f(v)$ are connected by the path $f(u)f(v_1)f(v_2)\ldots f(v_{k-1})f(v)$. Since u and v are two arbitrary nodes in G_1^L and f is a bijection from V_1 to V_2, we have every two nodes in G_2^L are connected by a path. Hence G_2^L is path connected.

Similarly we can show the converse part. □

Theorem 4.2. *Let $G_1^L = (V_1, \sigma_1, \mu_1)$ be isomorphic to $G_2^L = (V_2, \sigma_2, \mu_2)$.*

Then G_1^L *is MN-connected iff* G_2^L *is MN-connected.*

Proof. Since is G_1^L isomorphic to G_2^L, there exists a bijection f from V_1 to V_2 such that $M_{\mu_1}(u, v) = M_{\mu_2}(f(u), f(v))$, $N_{\mu_1}(u, v) = N_{\mu_2}(f(u), f(v))$, $\forall (u, v) \in V_1 \times V_1$.

Hence

$$\mathcal{CONN}_{M(G_1^L)}(u, v) = \vee_k\{M_{\mu_1}^k(u, v)/k = 1, 2, \ldots n\}$$

$$= \bigvee_{k=1}^{n}\{\bigwedge_{i=1}^{k} M_{\mu_1}(u_{i-1}, u_i)/u = u_0, u_1, \ldots u_k = v \in V_1\}$$

$$= \bigvee_{k=1}^{n}\{\bigwedge_{i=1}^{k} M_{\mu_2}(f(u_{i-1}), f(u_i))/u = u_0, u_1, \ldots u_k = v \in V_1\}$$

$$= \bigvee_{k=1}^{n}\{M_{\mu_2}^k(f(u), f(v))/k = 1, 2, \ldots n\}$$

$$= \mathcal{CONN}_{M(G_2^L)}(f(u), f(v))$$

Similarly we can show that

$$\mathcal{CONN}_{N(G_1^L)}(u, v) = \mathcal{CONN}_{N(G_2^L)}(f(u), f(v))$$

Hence from the definition of MN-connectivity, we can see that G_1^L is MN-connected iff G_2^L is MN-connected. □

Remark 4.3. It is evident that if $G_1^L = (V_1, \sigma_1, \mu_1)$ is weak isomorphic to $G_2^L = (V_2, \sigma_2, \mu_2)$, then the path connectivity (*MN*-connectivity) of one ILFG need not imply the path connectivity (*MN*-connectivity) of the other.

Theorem 4.4. *Let f be a isomorphism from* $G_1^L = (V_1, \sigma_1, \mu_1)$ *to* $G_2^L = (V_2, \sigma_2, \mu_2)$, *then the arc* (u, v) *in* G_1^L *is strong iff the arc* $(f(u), f(v))$ *in* G_2^L *is strong.*

Proof. Let (u, v) be a strong arc in G_1^L. Then by theorem 3.18

$$M_{\mu_1}(u, v) = \mathcal{CONN}_{M(G_1^L)}(u, v) \text{ and } N_{\mu_1}(u, v) = \mathcal{CONN}_{N(G_1^L)}(u, v)$$

Since f is the isomorphism from G_1^L to G_2^L,

$$M_{\mu_1}(u, v) = M_{\mu_2}(f(u), f(v)) \text{ and } N_{\mu_1}(u, v) = N_{\mu_2}(f(u), f(v)) \forall u, v \in V_1$$

Now from the proof of theorem 4.2, we have

$$\mathcal{CONN}_{M(G_1^L)}(u, v) = \mathcal{CONN}_{M(G_2^L)}(f(u), f(v)) \text{ and}$$

$$\mathcal{CONN}_{N(G_1^L)}(u, v) = \mathcal{CONN}_{N(G_2^L)}(f(u), f(v)), \forall u, v \in V_1$$

Hence

$$M_{\mu_2}(f(u), f(v)) = \mathcal{CONN}_{M(G_2^L)}(f(u), f(v)) \text{ and}$$

$$N_{\mu 2}(f(u), f(v)) = \mathcal{ONN}_{N(G_2^L)}(f(u), f(v)), \forall u, v \in V_1$$

So by theorem 3.18, the arc $(f(u), f(v))$ is a strong arc in G_2^L.

Conversely, let (u, v) be a strong arc in G_2^L. Then by the bijective and isomorphism property of f, its preimage is a strong arc in G_1^L. □

Theorem 4.5. *Let G_1^L and G_2^L are two ILFGs such that G_1^L is isomorphic to G_2^L. Then G_1^L is strongly connected iff G_2^L is strongly connected.*

Proof. Let f be the isomorphism from G_1^L to G_2^L. Then by the definition of strongly connected ILFG and by theorem 4.4, G_1^L is strongly connected $\Leftrightarrow \exists$ a strong path $P_1 : uu_1u_2 \ldots u_{k-1}v$ between every two nodes u and v in $G_1^L \Leftrightarrow \exists$ a strong path $P_2 : f(u)f(u_1)f(u_2)\ldots f(u_{k-1})f(v)$ between every two nodes $f(u)$ and $f(v)$ in $G_2^L \Leftrightarrow G_2^L$ is strongly connected. □

5. Conclusion

In this paper, we have introduced path of an ILFG, strength of a path and strength of connectedness between two nodes of an ILFG. Depending on the strength of connectedness between two nodes, we have described strong arcs and strong paths of an ILFG and some interesting results. We have also defined different classes of ILFGs, viz., path connected ILFGs, MN-connected ILFGs and strongly connected ILFGs, depending on the connectivity nature of the ILFGs. We have discussed some connectivity properties which are preserved under isomorphism. Connectivity studies are of paramount importance in graph theory. The same importance carries over to the case of ILFGs. Further studies, in this direction can accelerate the use of ILFGs in applications and lead to useful and interesting results.

References

[1] M. Akram and N. O. Alshehri, Intuitionistic fuzzy cycles and intuitionistic fuzzy trees, The Scientific World Journal (2014), doi:10.1155/2014/305836

[2] K. T. Atanassov, Intuitionistic fuzzy sets: Theory and applications, Studies in fuzziness and soft computing, Springer Physica-Verlag, Berlin, 1999.

[3] K. R. Bhutani and A. Rosenfeld, Fuzzy end nodes in fuzzy graphs, Information Sciences 152 (2003), 323-326.

[4] K. R. Bhutani and A. Rosenfeld, Strong arcs in fuzzy graphs, Information Sciences 152 (2003), 319-322.

[5] M. G. Karunambigai, R. Parvathi and R. Buvaneswari, Arcs in intuitionistic fuzzy graphs, Notes on Intuitionistic Fuzzy Sets 18(4) (2012), 48-58.

[6] S. Mathew and M. S. Sunitha, Types of arcs in a fuzzy graph, Information Science 179 (2009), 1760-1768.

[7] S. Mathew and M. S. Sunitha, Node connectivity and arc connectivity in fuzzy graphs, Information Sciences 180 (2010), 519-531.

[8] A. Nagoor Gani and S. Shajitha Begum, Isomorphism properties on strong intuitionistic fuzzy graph, Journal of Altra Scientist of Physical Science 23(3) (2011), 629-638.

[9] V. Nivethana and A. Parvathi, Arc analysis in the intuitionistic fuzzy graph and its applications, Notes on Intuitionistic Fuzzy Sets 22(1) (2016), 53-62.

[10] R. Parvathi, M. G. Karunambigai and K. Atanassov, Operations on Intuitionistic Fuzzy Graphs, Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE) (2009), 1396-1401.

[11] R. Pramada and K. V. Thomas, On isomorphisms of L-fuzzy graphs, Annals of Fuzzy Mathematics and Informatics 11(2) (2016), 301-313.

[12] A. Rosenfeld, Fuzzy graphs, Fuzzy sets and their applications to cognitive and decision processes, Academic Press (1975), 77-95.

[13] K. R. Sandeep Narayan and M. S. Sunitha, Types of arcs in complement of a fuzzy graph, Mapana J. Sci. 10(2) (2011), 75-81.

[14] M. S. Sunitha and A. Vijayakumar, Studies on fuzzy graphs, Ph. D. thesis, Department of Mathematics, 2001.

[15] M. S. Sunitha and S. Mathew, Fuzzy Graph theory: A survey, Annals of Pure and Applied Mathematics 4(1) (2013), 92-110.

[16] S. Tintumol, R. Pramada and J. Magie, On level subsets of intuitionistic L-Fuzzy Graphs, Malaya Journal of Matematik 8(4) (2020), 1942-1946.

[17] S. Tintumol, J. Magie and R. Pramada, Some isomorphism properties of intuitionistic L-fuzzy graphs (submitted).

[18] J. P. Tremblay and R. Manohar, It Discrete mathematical structures with applications to computer science, New Delhi, Tata McGraw-Hill, (1997).

[19] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.