Advances and Applications in Mathematical Sciences
Volume 21, Issue 8, June 2022, Pages 4671-4679

$M/M/1/K$ INTERDEPENDENT QUEUEING MODEL WITH VACATION AND CONTROLLABLE ARRIVAL RATES

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Abstract

In this paper an $M/M/1/K$ interdependent queueing model with vacation and controllable arrival rates is considered. The steady state solutions of the model are derived. Numerical examples and graphical analysis are given for better understanding.

1. Introduction

In this paper we consider a queueing model where server takes vacation and the arrival rate is controlled. Earlier, both A. Srinivasan and M. Thiagarajan [5] having studied about $M/M/1/K$ interdependent queueing model with controllable arrival rates.

In some situation, an idle server will start some other uninterruptible tasks which is referred to as a vacation period'. For a comprehensive and complete review on vacation queueing systems, we refer the readers to Doshi (1986) [1], Ke et al. (2010) [2] and Shweta Upadhyaya [3]. Further B. Deepa and K. Kalidass [4] have analysed an $M/M/1/N$ queue with working breakdowns and vacations. Many other similar models also have appeared.

2020 Mathematics Subject Classification: 60G20.
Keywords: Markovian queueing system, Vacation, finite capacity, Interdependent arrival and service process, varying arrival rates, bivariate Poisson distribution.
Received March 24, 2022; Accepted April 5, 2022

These models are useful in computer communication system.

2. Model Description

The arrival process and the service process are $\{X_1(t)\}, \{X_2(t)\}$ respectively are correlated and follow a bivariate Poisson process given by

$$P[X_1(t) = x_1,\ X_2(t) = x_2]$$

$$= e^{-(\lambda_i+\mu-\varepsilon)t} \sum_{j=0}^{\min(x_1,\ x_2)} (\varepsilon t)^j [(\lambda_i - \varepsilon)t]^{x_1-j} [(\mu - \varepsilon)]^{x_2-j} \frac{1}{j!\,(x_1 - j)!\,(x_2 - j)!} \quad (1)$$

where $x_1, x_2 = 0, 1, 2, \ldots; \lambda_i > 0,\ i = 0, 1; \mu > 0,\ 0 \leq \varepsilon < \min(\lambda_i, \mu),\ i = 0, 1.$

(1) Here, we consider a single server queueing system with parameter

λ_0 -Mean faster rate of arrivals

λ_1 -Mean slower rate of arrivals

μ-Mean service rate

ε-Mean dependence rate

υ-Vacation rate

(2) When the system size increases to R from below the arrival rate which was λ_0 until $R-1$, decreases to λ_1 and remains same for subsequent upward movement of the system size.

(3) When the system size decreases to r from above, the arrival rate which was λ_1 until $r+1$, increases to λ_0 and remains same for subsequent downward movement to 0 and upward movement up to $R-1$. This process is repeated.

(4) The states for the model are as follows:

(a) $(0, i)$ is the state in which there are i customers in the queue and the server is in vacation, $i \geq 0$. Its probability is $P_{0,i}$.

(b) $(1, i)$ is the state in which there are i customers in the system during active service, $i \geq 1$. Its probability is $P_{1,i}$.

3. Steady State Equations

We observe that only $P_{0,i}(0)$ and $P_{1,i}(0)$ exists when $n = 0, 1, 2, \ldots, r-1, r$; $P_{0,i}(0), P_{1,i}(0), P_{0,i}(1), P_{1,i}(1)$ exists when $n = r+1, r+2, \ldots, R-1$ and $P_{0,i}(0)$ and $P_{1,i}(0)$ exists only when $n = R, R+1, \ldots, K$.

Further $P_{0,i}(0) = P_{1,i}(0) = P_{0,i}(1) = P_{1,i}(1) = 0$ if $n > K$.

$$(\lambda_0 - \varepsilon)P_{0,0}(0) = (\mu - \varepsilon)P_{1,1}(0) \tag{2}$$

$$(\lambda_0 + v - \varepsilon)P_{0,i}(0) = (\lambda_0 - \varepsilon)P_{1,i-1}(0); (i = 1, 2, \ldots, R-1) \tag{3}$$

$$(\lambda_1 + v - \varepsilon)P_{0,i}(1) = (\lambda_1 - \varepsilon)P_{1,i-1}(1); (i = r+1, \ldots, K) \tag{4}$$

$$(\lambda_0 + \mu - 2\varepsilon)P_{1,1}(0) = (\mu - \varepsilon)P_{1,2}(0) + vP_{0,1}(0) \tag{5}$$

$$(\lambda_0 + \mu - 2\varepsilon)P_{1,i}(0) = (\lambda_0 - \varepsilon)P_{1,i-1}(1) + (\mu - \varepsilon)P_{1,i+1}(0) + vP_{0,i}(1);$$
$$(i = 2, 3, \ldots, r-1) \tag{6}$$

$$(\lambda_0 + \mu - 2\varepsilon)P_{1,r}(0) = (\lambda_0 - \varepsilon)P_{1,r-1}(1) + (\mu - \varepsilon)P_{1,r+1}(0) + (\mu - \varepsilon)P_{1,r+1}(1)$$
$$+ vP_{0,i}(1); \tag{7}$$

$$(\lambda_0 + \mu - 2\varepsilon)P_{1,i}(0) = (\lambda_0 - \varepsilon)P_{1,i-1}(0) + (\mu - \varepsilon)P_{1,i+1}(0) + vP_{0,i}(0);$$
$$(i = r+1, \ldots, R-2) \tag{8}$$

$$(\lambda_0 + \mu - 2\varepsilon)P_{1,R-1}(1) = (\lambda_0 - \varepsilon)P_{1,R-2}(0) + vP_{0,R-1}(0); \tag{9}$$

$$(\lambda_1 + \mu - 2\varepsilon)P_{1,r+1}(1) = (\mu - \varepsilon)P_{1,r+2}(1) + vP_{0,r+1}(1); \tag{10}$$

$$(\lambda_1 + \mu - 2\varepsilon)P_{1,i}(1) = (\mu - \varepsilon)P_{1,i+1}(1) + (\lambda_1 - \varepsilon)P_{1,i-1}(1) + vP_{0,i}(1);$$
$$(i = r+2, \ldots, R-1) \tag{11}$$

$$(\lambda_1 + \mu - 2\varepsilon)P_{1,R}(1) = (\mu - \varepsilon)P_{1,R+1}(1) + (\lambda_1 - \varepsilon)P_{1,R-1}(1) + (\lambda_0 - \varepsilon)P_{1,R-1}(0) +$$
$$vP_{0,R}(1); \tag{12}$$

$$(\lambda_1 + \mu - 2\varepsilon)P_{1,i}(1) = (\mu - \varepsilon)P_{1,i+1}(1) + (\lambda_1 - \varepsilon)P_{1,i-1}(1) + vP_{0,i}(1);$$

$$(i = R+1, \ldots, K-1) \tag{13}$$

$$(\mu - \varepsilon)P_{1,K}(1) = (\lambda_1 - \varepsilon)P_{1,K-1}(1) + vP_{0,K}(1); \tag{14}$$

Let

$$A = \frac{\lambda_0 - \varepsilon}{\mu - \varepsilon},\ B = \frac{\lambda_1 - \varepsilon}{\mu - \varepsilon},\ C = \frac{v}{\mu - \varepsilon},\ D = \frac{A}{A + C},\ E = \frac{B}{B + C}$$

From equation (2) we derive

$$P_{1,1}(0) = AP_{0,0}(0) \tag{15}$$

And from equation (3) we recursively get

$$\sum_{n=1}^{R-1} P_{0,n}(0) = \sum_{n=1}^{R-1} D^n P_{0,0}(0) \tag{16}$$

Using (5) in equation (6) we recursively get,

$$\sum\nolimits_{n=1}^{r} P_{1,n}(0) = \{\sum\nolimits_{n=1}^{r}[A + A^2 + \ldots + A^n] - \sum\nolimits_{n=2}^{r}[1 + A + \ldots + A^{n-2}]CD$$

$$- \sum\nolimits_{n=3}^{r}[CD^2 + CD^3 + \ldots + CD^{n-1}]\}P_{0,0}(0) \tag{18}$$

Using (7) in (8) we recursively get,

$$\sum\nolimits_{n=r+1}^{R-1} P_{1,n}(0) = \sum\nolimits_{n=r+1}^{R-1}\{([A^n + A^{n-1} - 1] - [1 + \ldots 2A^{n-r}][CD^2 + \ldots CD^{r-2}]$$

$$- \ldots CD^{n-1})P_{0,0}(0) - (1 + \ldots A^{n-r-1})P_{1,r+1}(1)\} \tag{19}$$

From (9) we derive

$$P_{1,r+1}(1) = FP_{0,0}(0) \tag{20}$$

Where

$$F = \frac{\{A^R + A^{R-1} - 1 - (1 + 2A + \ldots 2A^{R-r})(CD^2 + \ldots + CD^{r-2}) - \ldots CD^{R-2}\}}{1 + A + \ldots A^{R-r-2}}$$

Using (10) in (11) we get

$$\sum_{n=r+1}^{R} P_{1,n}(1) = \sum_{n=r+1}^{R} (1 + B + \dots + B^{n-r-1}) F P_{0,0}(0) \tag{21}$$

Using (12) in (13) and (15) we get

$$\sum_{n=R+1}^{K} P_{1,n}(1) = \sum_{n=R+1}^{K} \{F[B + \dots B^{n-r} + (B^{n-R-1} + \dots B + 1)$$

$$(A + \dots A^{R-r-1})] - (B^{n-R-1} + \dots B + 1)(A^{R} + \dots A^{R-1} - A) + \dots ACD^{R-2}\} P_{0,0}(0) \tag{22}$$

4. Characteristics of the Model

$$P(0) = \sum_{n=0}^{K} P_{1,n}(0)$$

$P(0)$ exists only when $n = 1, 2, \dots r-1, r, r+1, \dots R-1,$ we get

$$P(0) = \sum_{n=1}^{r} P_{1,n}(0) + \sum_{n=r+1}^{R-1} P_{1,n}(0) \tag{23}$$

From (18), (19), (20) and (23), we get

$$P(0) = \{\sum_{n=1}^{r} [A + \dots + A^{n}] - \sum_{n=2}^{r} [1 + \dots A^{n-2}] CD$$

$$- \sum_{n=3}^{r} [CD^{2} + \dots + CD^{n-1}] + \sum_{n=r+1}^{R-1} \{([A^{n} + A^{n-1} - 1] - \dots CD^{n-1})$$

$$P_{0,0}(0) - F(1 + \dots A^{n-r-1})\}\} P_{0,0}(0) \tag{24}$$

Now,

$$P(1) = \sum_{n=0}^{\infty} P_{1,n}(1)$$

$P(1)$ exists only when $n = r+1, r+2 \dots K$

$$P(1) = \sum_{n=r+1}^{R} P_{1,n}(1) + \sum_{n=R+1}^{K} P_{1,n}(1). \tag{25}$$

From (21), (22) and (23) we get

$$P(1) = \{\sum_{n=r+1}^{R}(1 + \ldots . B^{n-r-1})F + \sum_{n=R+1}^{K}\{F[B + \ldots . B^{n-r}$$

$$+ (B^{n-R-1} + \ldots . B + 1)(A + \ldots . A^{R-r-1})] - \ldots . ACD^{n-2}\}\}P_{0,0}(0) \tag{26}$$

The system is empty can be calculated from the normalizing condition

$$P(0) + P(1) = 1$$

$$[G_1 + G_2]P_{0,0}(0) = 1$$

$$P_{0,0}(0) = [G_1 + G_1]^{-1} \tag{27}$$

we have

$$L_s = L_{s_0} + L_{s_1} \tag{28}$$

Where

$$L_{s_0} = \sum_{n=1}^{r} nP_{1,n}(0) + \sum_{n=r+1}^{R-1} nP_{1,n}(0) \tag{29}$$

and

$$L_{s_1} = \sum_{n=r+1}^{R} P_{1,n}(1) + \sum_{n=R+1}^{K} P_{1,n}(1) \tag{30}$$

Now by using Little's formula,

$$W_s = L_s \bar{\lambda} \tag{32}$$

Where $\bar{\lambda} = \lambda_1 P(0) + \lambda_1 P(1)$.

5. Numerical Illustrations

For various values of $\lambda_0, \lambda_1, \mu, \varepsilon, \upsilon$ the values of $P_{0,0}(0), P(0), P(1), L_s, W_s$ are computed

Let $r = 4$, $R = 7$, $K = 8$, $\upsilon = 20$.

Table

λ_0	λ_1	μ	ε	$P_{0,0}(0)$	$P(0)$	$P(1)$	L_s	W_s
6	6	5	1	0.1259	0.5170	0.4098	11.4961	2.0672
6	6	5	0.5	0.1458	0.5472	0.3695	11.8891	2.1615
6	6	5	0	0.1640	0.5130	0.3343	12.2230	2.2452
6	6	4	1	0.0164	0.2313	0.7452	8.0279	1.3702
6	6	4	0.5	0.0253	0.2798	0.6883	8.6146	1.4832
5	5	5	1	0.5604	0.8191	0.0425	14.6064	3.3905
5	5	5	0	0.5506	0.8284	0.0165	14.8318	3.5107
7	6	6	0	0.2612	0.8224	0.0317	15.2184	2.5592
7	6	6	0.5	02569	0.8477	0.0081	15.5025	2.5913
6	7	4	0	0.0203	0.1852	0.7915	7.7787	1.1694
8	7	5	0	0.0427	0.5212	0.4095	11.0623	1.5723
8	7	5	0.5	0.0349	0.5025	0.4351	10.6700	1.5101

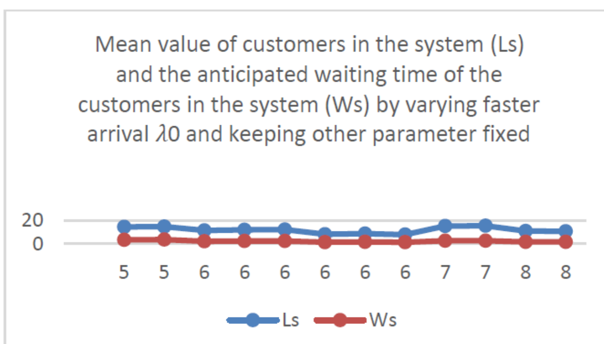

Figure 1.

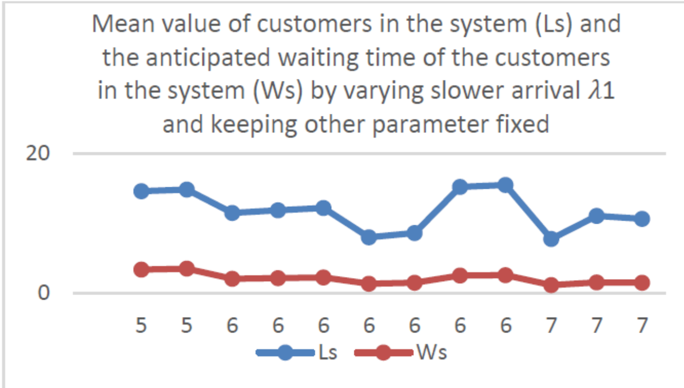

Figure 2.

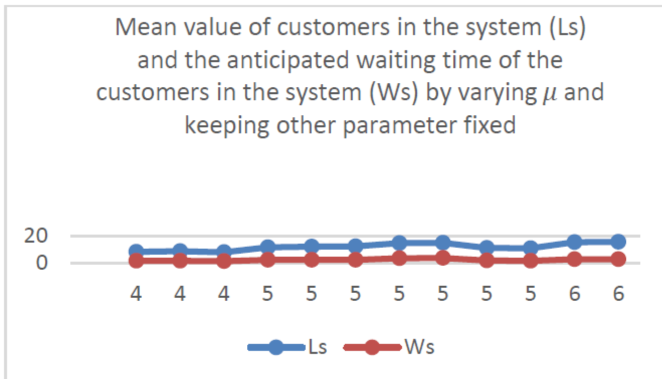

Figure 3.

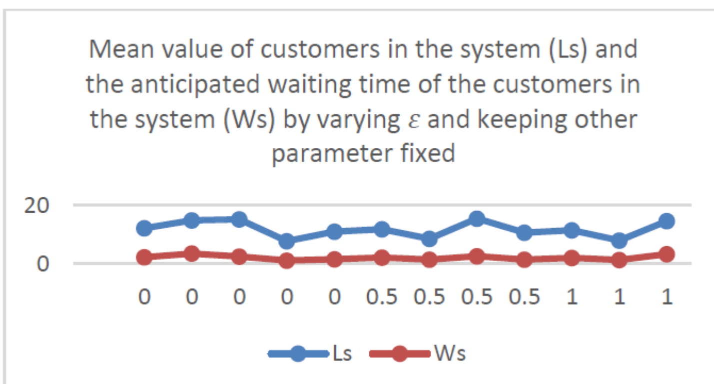

Figure 4.

6. Conclusion

It is observed from the tables I and II that when the mean dependence rate increases and the other parameters are kept fixed, both L_s and W_s decreases. When the service rate increases and the other parameter are kept fixed, both L_s and W_s increases. When the arrival rate increases and the other parameter are kept fixed, both L_s and W_s decreases. The model includes the earlier models as particular cases. For example, when $\upsilon = 0$, this model reduces to the $M/M/1/K$ interdependent queueing model with controllable arrival rates [5]. When λ_0 tends to λ_1 and $\varepsilon = 0$, this model reduces to the $M/M/1/K$ queueing model with vacation [4]. When λ_0 tends to λ_1, $\varepsilon = 0$ and $\upsilon = 0$, this model reduces to the conventional $M/M/1/K$ queueing model.

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