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NANO REGULAR *b*-CONTINUOUS FUNCTIONS

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Abstract

This paper is to introduce a new nano continuous functions in nano topological spaces namely nano regular *b*-continuous functions (briefly Nrb-continuous functions) and to compare this with some other already existing nano continuous functions such that nano regular continuous functions, nano generalized continuous functions, etc. in nano topological spaces and to analyze some of the properties. Then the concept of nano topological spaces was applied in some real life situation like data analysis. By using the basis of a nano topological space we found the key factors or key attributes for the particular data. And also the concept of topological reduction of attributes was used in a complete information system to get the corresponding key factors. In this paper the essential key factors for a healthy life are discovered.

1. Introduction

In 1970, Levine [15] introduced the concept of generalized closed sets in topological space. Later N. Palaniappan [19] studied the concept of regular generalized closed sets in a topological space in 1990. In 2011, Sharmistha Bhattacharya [25] has introduced the notion of generalized regular closed sets in topological space. Andrijevic [2] introduced a new class of generalized open sets namely, *b*-open sets in 1996. Later in 2009 A. Al-omari and M. S. M. Noorani [1] introduced the class of generalized closed sets. A. Narmadha and N. Nagaveni [16] introduced regular *b*-closed sets in 2012. In 2013, A. Narmadha, N. Nagaveni and T. Noiri [17] introduced regular *b*-open sets. In 2013 Lellis Thivagar M., Carmel Richard [12] introduced the concept of nano

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topological spaces and also he introduced certain weak forms of nano open sets such as nano α-open sets, nano semi-open sets and nano pre open sets and also he established nano continuity [13] in 2013. Nano b-open sets [21] was introduced in 2016 by A. Parimala, G. Indirani and S. Jafari. In 2016 M. Dhanapackiam and M. Trinita Pricilla introduced the class of nano generalized b-continuous functions [10]. In 2021 P. Srividhya and T. Indira introduced the Nrb-open sets and Nrb-closed sets in nano topological spaces. In this paper we introduced the Nrb-continuous functions in nano topological spaces [27]. Pawlak Z. discovered some application in rough set theory in 2002 [23]. Later Lellis Thivagar applied the concepts of nano topological basis to find the deciding factors in data analysis [14]. In this paper we introduced the application to find the essential food habits for a healthy life.

2. Nano Regular *b*-Closed

Definition 1. A subset A of a nano topological spaces $(U, \tau_R(X))$ is said to be Nrb-closed if $Nrcl(A) \subset G$ whenever $A \subset G$ and G is nano b-open in U. If its complement is a nano regular b-closed, then a subset A of a nano topological space $(U, \tau_R(X))$ is said to be nano regular b-open (Nrb-open).

3. Nano Regular *b*-Continuous Functions (Nrb-Continuous Functions)

Definition 1. Let $(U, \tau_R(X))$ and $(V, \tau_{R^l}(Y))$ be two nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ is nano regular b-continuous function (briefly Nrb-continuous function) on U if the inverse image of every nano open set in V is nano regular b-open (Nrb-open) in U.

Example 2. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$, Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Let $V = \{1, 2, 3, 4\}$, $Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}$. $\tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$, Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b), = 3$, $f(c) = 2$, $f(d) = 1$ and $f^{-1}(V) = U$, $f^{-1}(\varphi) = (\varphi)$, $f^{-1}(\{4\}) = \{a\}$, $f^{-1}(\{1, 3\}) = \{b, d\}$, $f^{-1}(\{1, 3, 4\}) = \{d, b, a\}$. (i.e.)

the inverse image of every nano open set in V is Nrb-open in U. Since the mapping $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ is Nrb-continuous on U.

Theorem 3. *Every nano regular continuous function is Nrb-continuous but not conversely.*

Proof of Theorem 3. Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ be nano regular continuous on $(U, \tau_R(X))$. Since f is nano regular continuous on $(U, \tau_R(X))$, the inverse image of every nano open in $(V, \tau_{R^l}(Y))$ is nano regular open in $(U, \tau_R(X))$. But, we know that by Theorem 3.6 in [27] "Every nano regular open is Nrb-open". Hence the inverse image of every nano regular open in $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. Since f is Nrb-continuous.

Example for the converse part. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nano regular open sets $= \{U, \varphi, \{a\}, \{b, d\}\}$, Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Let $V = \{1, 2, 3, 4\}$, $Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}$, $\tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nano regular open sets $= \{V, \varphi, \{4\}, \{1, 3\}\}$, Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b) = 3$, $f(c) = 2$, $f(d) = 1$. Then the inverse image of the Nrb open set $\{1, 3, 4\} = \{a, b, d\}$ in $(V, \tau_{R^l}(Y))$ is not in nano regular open of $(U, \tau_R(X))$.

Theorem 4. *Every Nrb-continuous function is Ng-continuous but not conversely.*

Proof of Theorem 4. Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ be Nrb-continuous on $(U, \tau_R(X))$. Since f is Nrb-continuous on $(U, \tau_R(X))$, the inverse image of every nano open in $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. But, we know that by Theorem 3.8 in [27] "Every Nrb-open is Ng-open." Hence the inverse image of every Nrb open in $(V, \tau_{R^l}(Y))$ is Ng-open in $(U, \tau_R(X))$. Since f is Ng-continuous.

Example for the converse part. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$,

$U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$, Ng-open sets $= \{U, \varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$. Let $V = \{1, 2, 3, 4\}$, $Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}$, $\tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$, Ng-open sets $= \{V, \varphi, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b) = 3$, $f(c) = 2$, $f(d) = 1$. Then the inverse image of the Ng open sets $= \{\{1\}, \{3\}, \{1, 4\}, \{3, 4\}\}$ in $(V, \tau_{R^l}(Y))$ are not in Nrb-open of $(U, \tau_R(X))$.

Theorem 5. *Every Nrb-continuous is Ngs-continuous but not conversely*.

Proof of Theorem 5. Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ be Nrb-continuous on $(U, \tau_R(X))$. Since f is Nrb-continuous on $(U, \tau_R(X))$, the inverse image of every nano open in $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. But, we know that by Theorem 3.10 in [27] "Every Nrb-open is Ngs-open." Hence the inverse image of every Nrb open in $(V, \tau_{R^l}(Y))$ is Ngs-open in $(U, \tau_R(X))$. Since f is Ngs-continuous.

Example for the converse part. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Ngs open sets $= \{U, \varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $V = \{1, 2, 3, 4\}$, $Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}$, $\tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Ngs-open sets $= \{V, \varphi, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b) = 3$, $f(c) = 2$, $f(d) = 1$. Then the inverse image of the Ngs open sets $= \{\{1\}, \{3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\}$ in $(V, \tau_{R^l}(Y))$ are not in Nrb open of $(U, \tau_R(X))$.

Theorem 6. *Every Nrb-continuous function is Nsg-continuous but not conversely*.

Proof of theorem 6. Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ be Nrb-continuous on $(U, \tau_R(X))$. Since f is Nrb-continuous on $(U, \tau_R(X))$, the inverse image of every nano open in $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. But, we know that by Theorem 3.12 in [27] "Every Nrb-open is Nsg-open". Hence the inverse image of every Nrb-open in $(V, \tau_{R^l}(Y))$ is Nsg-open in $(U, \tau_R(X))$. Since f is Nsg-continuous.

Example for the converse part. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nsg-open sets $= \{U, \varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $V = \{1, 2, 3, 4\}$, $Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}, \tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nsg-open sets $= \{V, \varphi, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b) = 3$, $f(c) = 2$, $f(d) = 1$. Then the inverse image of the Nsg open sets $= \{\{1\}, \{3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\}$ in $(V, \tau_{R^l}(Y))$ are not in Nrb open of $(U, \tau_R(X))$.

Theorem 7. *Every Nrb-continuous function is Ngp-continuous but not conversely.*

Proof of theorem 7. Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ be Nrb-continuous on $(U, \tau_R(X))$. Since f is Nrb-continuous on $(U, \tau_R(X))$, the inverse image of every nano open in $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. But, we know that by Theorem 3.14 in [27] "Every Nrb-open is Ngp-open." Hence the inverse image of every Nrb-open in $(V, \tau_{R^l}(Y))$ is Ngp-open in $(U, \tau_R(X))$. Since f is Ngp-continuous.

Example for the converse part. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Ngp-open sets $= \{U, \varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $V = \{1, 2, 3, 4\}$,

$Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}, \tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Ngp-open sets $= \{V, \varphi, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b) = 3$, $f(c) = 2$, $f(d) = 1$. Then the inverse image of the Ngp open sets $= \{\{1\}, \{3\}, \{1, 4\}, \{3, 4\}, \{1, 2, 4\}, \{2, 3, 4\}\}$ in $(V, \tau_{R^l}(Y))$ are not in Nrb open of $(U, \tau_R(X))$.

Theorem 8. *Every Nrb-continuous function is Npg-continuous but not conversely.*

Proof of theorem 8. Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ be Nrb-continuous on $(U, \tau_R(X))$. Since f is Nrb-continuous on $(U, \tau_R(X))$, the inverse image of every nano open $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. But, we know that by Theorem 3.16 in [27] "Every Nrb-open is Npg-open." Hence the inverse image of every Nrb open in $(V, \tau_{R^l}(Y))$ is Npg-open in $(U, \tau_R(X))$. Since f is Npg-continuous.

Example for the converse part. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Npg-open sets $= \{U, \varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$. Let $V = \{1, 2, 3, 4\}$, $Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}, \tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Npg-open sets $= \{V, \varphi, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b) = 3$, $f(c) = 2$, $f(d) = 1$. Then the inverse image of the Npg-open sets $= \{\{1\}, \{3\}, \{1, 4\}, \{3, 4\}, \{1, 2, 4\}, \{2, 3, 4\}\}$ in $(V, \tau_{R^l}(Y))$ are not in Nrb-open of $(U, \tau_R(X))$.

Theorem 9. *Every Nrb-continuous function is Ngr-continuous but not conversely.*

Proof of theorem 9. Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ be Nrb-continuous on $(U, \tau_R(X))$. Since f is Nrb-continuous on $(U, \tau_R(X))$, the inverse image of every nano open in $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. But, we know that by Theorem 3.24 in [27] "Every Nrb-open is Ngr-open." Hence the inverse image of every Nrb open in $(V, \tau_{R^l}(Y))$ is Ngr-open in $(U, \tau_R(X))$. Since f is Ngr-continuous.

Example for the converse part. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Ngr-open sets $= \{U, \varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}\}$. Let $V = \{1, 2, 3, 4\}$, $Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}$, $\tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Ngr-open sets $= \{V, \varphi, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b) = 3$, $f(c) = 2$, $f(d) = 1$. Then the inverse image of the Ngr open sets $= \{\{1\}, \{3\}, \{1, 4\}, \{3, 4\}\}$ in $(V, \tau_{R^l}(Y))$ are not in Nrb open of $(U, \tau_R(X))$.

Theorem 10. *Every Nrb-continuous function is Nrg-continuous but not conversely.*

Proof of theorem 10. Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ be Nrb-continuous on $(U, \tau_R(X))$. Since f is Nrb-continuous on $(U, \tau_R(X))$, the inverse image of every nano open in $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. But, we know that by Theorem 3.26 in [27] "Every Nrb-open is Nrg-open". Hence the inverse image of every Nrb-open in $(V, \tau_{R^l}(Y))$ is Nrg-open in $(U, \tau_R(X))$. Since f is Nrg-continuous.

Example for the converse part. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nrg-open sets $= \{U, \varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}\}$. Let $V = \{1, 2, 3, 4\}$, $Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}$, $\tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\},$

$\{1, 3, 4\}\}$. Nrg-open sets $= \{V, \varphi, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b) = 3$, $f(c) = 2$, $f(d) = 1$. Then the inverse image of the Nrg-open sets $= \{\{1\}, \{3\}, \{1, 4\}, \{3, 4\}\}$ in $(V, \tau_{R^l}(Y))$ are not in Nrb-open of $(U, \tau_R(X))$.

Theorem 11. *Every Nrb-continuous function is Nbg-continuous but not conversely.*

Proof of theorem 11. Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ be Nrb-continuous on $(U, \tau_R(X))$. Since f is Nrb-continuous on $(U, \tau_R(X))$, the inverse image of every nano open in $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. But, we know that by Theorem 3.28 in [27] "Every Nrb-open is Nbg-open". Hence the inverse image of every Nrb-open in $(V, \tau_{R^l}(Y))$ is Nbg-open in $(U, \tau_R(X))$. Since f is Nbg-continuous.

Example for the converse part. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nbg-open sets $= \{U, \varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $V = \{1, 2, 3, 4\}$, $Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}$, $\tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nbg-open sets $= \{V, \varphi, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b) = 3$, $f(c) = 2$, $f(d) = 1$. Then the inverse image of the Nbg-open sets $= \{\{1\}, \{3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\}$ in $(V, \tau_{R^l}(Y))$ are not in Nrb open of $(U, \tau_R(X))$.

Theorem 12. *Every Nrb-continuous function is Ngb-continuous but not conversely.*

Proof of theorem 12. Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ be Nrb-continuous on $(U, \tau_R(X))$. Since f is Nrb-continuous on $(U, \tau_R(X))$, the inverse image of every nano open in $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. But, we know that by Theorem 3.30 in [27] "Every Nrb-open is Ngb-open".

Hence the inverse image of every Nrb-open in $(V, \tau_{R^l}(Y))$ is Ngb-open in $(U, \tau_R(X))$. Since f is Ngb-continuous.

Example for the converse part. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Nrb-open sets $= \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Ngb-open sets $= \{U, \varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $V = \{1, 2, 3, 4\}$, $Y = \{3, 4\}$, $V/R^l = \{\{2\}, \{4\}, \{1, 3\}\}$, $\tau_{R^l}(Y) = \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Nrb-open sets $= \{V, \varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Ngb-open sets $= \{V, \varphi, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$. Define $f : U \to V$ as $f(a) = 4$, $f(b) = 3$, $f(c) = 2$, $f(d) = 1$. Then the inverse image of the Ngb-open sets $= \{\{1\}, \{3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\}$ in $(V, \tau_{R^l}(Y))$ are not in Nrb-open of $(U, \tau_R(X))$.

Theorem 13. *A function $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ is Nrb-continuous if and only if the inverse image of every nano closed set in V is Nrb-closed in U.*

Proof of Theorem 13. Necessary part: Let $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ is Nrb-continuous function and F be nano closed in $(V, \tau_{R^l}(Y))$. (i.e.) $V - F$ is nano open in $(V, \tau_{R^l}(Y))$. Since f is Nrb - continuous, the inverse image of every nano open set in $(V, \tau_{R^l}(Y))$ is Nrb-open in $(U, \tau_R(X))$. Since $f^{-1}(V - F)$ is Nrb-open in $(U, \tau_R(X))$. (i.e.) $f^{-1}(V - F) = f^{-1}(V) - f^{-1}(F) = U - f^{-1}(F)$ is Nrb-open in $(U, \tau_R(X))$. Therefore $f^{-1}(F)$ is Nrb closed in $(U, \tau_R(X))$.

Sufficient part. Let the inverse image of every nano closed set in $(V, \tau_{R^l}(Y))$ is Nrb-closed in $(U, \tau_R(X))$. Let G be nano open in $(V, \tau_{R^l}(Y))$. Then $V - G$ is nano closed in $(V, \tau_{R^l}(Y)) \Rightarrow f^{-1}(V - G)$ is Nrb-closed in $(U, \tau_R(X))$. (i.e.) $f^{-1}(V - G) = f^{-1}(V) - f^{-1}(G) = U - f^{-1}(G)$ is Nrb closed in

$(U, \tau_R(X))$. Since $f^{-1}(G)$ is Nrb open in $(U, \tau_R(X))$. Thus, the inverse image of every nano open set G is $(V, \tau_{R^l}(Y))$ is Nrb open in $(U, \tau_R(X))$. (i.e.) f is Nrb-continuous on $(U, \tau_R(X))$.

Theorem 14. *A function* $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ *is Nrb-continuous then* $f(Nrbcl(A)) \subseteq Nrbcl(f(A))$ *for every subset A of U.*

Proof of Theorem 14. Let f be Nrb-continuous and $A \subseteq U$. Then $f(A) \subseteq V$. Since f is Nrb-continuous and $Nrbcl(f(A))$ is Nrb-closed in V, then $f^{-1}(Nrbcl(f(A)))$ is Nrb closed in U. Since $f(A) \subseteq Nrbcl(f(A))$, $A \subseteq f^{-1}(Nrbcl(f(A)))$. Here $f^{-1}(Nrbcl(f(A)))$ is a Nrb closed set containing A. But, we know that $Nrbcl(A)$ is the smallest Nrb closed set containing A. Since $Nrbcl(A) \subseteq f^{-1}(Nrbcl(f(A)))$. (i.e.) $f(Nrbcl(A)) \subseteq Nrbcl(f(A))$.

Theorem 15. *Let* $(U, \tau_R(X))$ *and* $(V, \tau_{R^l}(Y))$ *be two nano topological spaces where* $X \subseteq U$ *and* $Y \subseteq V$. *Then* $\tau_{R^l}(Y) = \{V, \varphi, L_{R^l}(Y), U_{R^l}(Y), B_{R^l}(Y)\}$ *and its basis is given by* $\mathcal{B}_{R^l}(Y) = \{V, L_{R^l}(Y), B_{R^l}(Y)\}$. *A function* $f : (U, \tau_R(X)) \to (V, \tau_{R^l}(Y))$ *is Nrb-continuous if and only if the inverse image of every member of* $\mathcal{B}_{R^l}(Y)$ *is Nrb-open in U.*

Proof of Theorem 15. Necessary part: Let f be Nrb-continuous in U. Let $B \in \beta_R$. Then B is Nano open in V. (i.e.) $B \in \tau_{R^l}(Y)$. Since f is Nrb-continuous, $f^{-1}(B)$ is Nrb-open. Hence the inverse image of every member of $\beta_{R^l}(Y)$ is Nrb open in U.

Sufficient part. Conversely, assume that the inverse image of every member of $\beta_{R^l}(Y)$ is Nrb-open in U and G be Nano open in V. Then $G = \bigcup\{B : B \in \mathcal{B}_{R^l}(Y)\}$. Then $f^{-1}(G) = f^{-1}(\bigcup \{B : B \in \mathcal{B}_{R^l}(Y)\}) = \bigcup (f^{-1}B : B \in \mathcal{B}_{R^l}(Y))$, where each $f^{-1}(B)$ is Nrb-open in U and hence their union, (i.e.) $f^{-1}(G)$ is Nrb-open in U. Hence f is Nrb-continuous in U.

4. Applications of Nano Topology

Definition 1[16]**.** Let (U, A) be an information system, where A is divided into a set C of condition attributes and a set D of decision attribute. A subset R of C is said to be a core, if $\mathcal{B}_R = \mathcal{B}_C$ and $\mathcal{B}_R \neq \mathcal{B}_{R-\{r\}}$ for all $r \in R$, where $\mathcal{B}_R$ is the basis of nano topology corresponding to $R \subseteq C$. That is, a core is a minimal subset of attributes which is such that none of its elements can be removed without affecting the classification power of attributes.

To Find the Key Factors for a Healthy Life:

In this section we discussed a new application in data analysis by using the basis of nano topology. We found the essential key factors for a healthy lifestyle. Here the concepts of the topological reduction of attributes in a complete information system is used to get the key factors. The general algorithm for identify the key factors is given below.

Algorithm.

Step 1. Let U be the finite universe, A be the finite set of attributes which is divided into two classes such that condition attributes and decision attributes where condition attributes are denoted by C and decision attributes are denoted by D. Then R is the equivalence relation on U corresponding to the condition attributes C and a subset X of U. The given tabular column was represented by the data, in which coloumns are the attributes and rows are the objects. The entries of the table are known as the attribute values.

Step 2. Find the lower approximation $L_C(X)$, upper approximation $U_C(X)$ and the boundary region $B_C(X)$ of X with respect to R.

Step 3. Find the nano topology $\tau_C(X)$ on U and its basis $\mathcal{B}_C(X)$ corresponding to the conditional attribute set C.

Step 4. Remove an attribute x from C and find the lower and upper approximations and the boundary region of X with respect to the equivalence relation on $C - \{x\}$.

Step 5. Find the nano topology $\tau_{C-\{X\}}(X)$ on U and its basis $\mathcal{B}_{C-\{X\}}(X)$.

Step 6. Repeat steps 4 and 5 for all attributes in C.

Step 7. Those attributes in C for which $\mathcal{B}_{C-\{X\}}(X) \neq \mathcal{B}_C(X)$ form the CORE.

Example 2. Food habits are the most important thing for a healthy life. Here we collect the data of food habits for the working people. In their busy morning routine they don't have much time to cook their breakfast. And also they move on to the instant foods due to their less interest in cooking. Sometimes they skip their breakfast or ordered foods in online too. Based on their daily food habits we find out the essential food habits for their healthy life.

Persons	Ordered foods	Fruits and vegetables	Beverages	Instant foods	Properly cooked foods	Healthy / unhealthy
P_1	✓	✓	✓	×	×	HEALTHY
P_2	×	✓	×	✓	✓	HEALTHY
P_3	✓	×	✓	×	✓	HEALTHY
P_4	✓	✓	✓	×	×	UNHEALTHY
P_5	✓	×	✓	×	×	UNHEALTHY
P_6	✓	×	✓	×	✓	UNHEALTHY
P_7	✓	✓	×	×	✓	HEALTHY
P_8	✓	✓	×	✓	×	UNHEALTHY

From the above table we got the different types of food habits. Here $U = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$ be the set of Persons and $A = \{$Ordered Foods, Fruits and Vegetables, Beverages, Instant Foods and Properly Cooked Foods$\}$ are the set of attributes. "Yes" or "No" are the entries in the table which known as attribute Values. The attributes in A are the condition attributes and Healthy/Unhealthy is the decision attribute. Condition attribute is denoted by C such as $C = \{OF, F$ and $V, B, IF, PCF\}$ and decision attribute is denoted by $D = \{H/UH\}$. The family of classes, U/C corresponding to C is given by $U/R(C) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$.

Case 1. (Persons who are healthy)

Let $X = \{P_1, P_2, P_3, P_7\}$, the set of persons who are healthy. Then $L_C(X) = \{P_2, P_7\}$, $U_C(X) = \{P_1, P_2, P_3, P_4, P_6, P_7\}$ and $B_C(X) = U_C(X) - L_C(X) = \{P_1, P_3, P_4, P_6\}$. Then $\tau_C(X) = \{U, \emptyset, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_6, P_7\}, \{P_1, P_3, P_4, P_6\}\}$. The basis of $\tau_C(X)$ is given by $\mathcal{B}_C(X) = \{U, \{P_2, P_7\}, \{P_1, P_3, P_4, P_6\}\}$.

Step 1.

(i) Remove the attribute "Ordered Foods" from C, $U/R(C - OF) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ then the lower and upper approximations of X corresponding to $C - \{OF\}$ are given by $L_{C-\{OF\}}(X) = \{P_2, P_7\}$, $U_{C-\{OF\}}(X) = \{P_1, P_2, P_3, P_4, P_6, P_7\}$ and the boundary region is given by $B_{C-\{OF\}}(X) = \{P_1, P_3, P_4, P_6\}$. Then the nano topology $\tau_{C-\{OF\}}(X) = \{U, \emptyset, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_6, P_7\}, \{P_1, P_3, P_4, P_6\}\}$ and the basis $\mathcal{B}_{C-\{OF\}}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3, P_4, P_6\}\} = \mathcal{B}_C(X)$.

(ii) Remove the attribute "Fruits and Vegetables" from C, U/R(C-F and V) $\{\{P_1, P_4, P_5\}, \{P_2\}, \{P_3, P_6\}, \{P_7\}, \{P_8\}\}$ then $L_{C-\{F\&V\}}(X) = \{P_2, P_7\}$, $U_{C-\{F\&V\}}(X) = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$ and $B_{C-\{F\&V\}}(X) = \{P_1, P_3, P_4, P_5, P_6\}$. Then $\tau_{C-\{F\&V\}}(X) = \{U, \emptyset, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}\}$ and the basis $\mathcal{B}_{C-\{F\&V\}}(X) = \{U, \emptyset, \{P_2, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}\} \neq \mathcal{B}_C(X)$.

(iii) Remove the attribute "Beverages" from C, $U/R(C - B) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ then $L_{C-\{B\}}(X) = \{P_2, P_7\}$, $U_{C-\{B\}}(X) = \{P_1, P_2, P_3, P_4, P_6, P_7\}$, and $B_{C-\{B\}}(X) = \{P_1, P_3, P_4, P_6\}$. Then $\tau_{C-\{B\}}(X) = \{U, \emptyset, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_6, P_7\}, \{P_1, P_3, P_4, P_6\}\}$ and the basis $\mathcal{B}_{C-\{B\}}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3, P_4, P_6\}\} = \mathcal{B}_C(X)$.

(iv) Remove the attribute "Instant Foods" from C, $U/R(C - IF) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ then $L_{C-\{IF\}}(X) = \{P_2, P_7\}$,

$U_{C-\{IF\}}(X) = \{P_1, P_2, P_3, P_4, P_6, P_7\}$, and $B_{C-\{IF\}}(X) = \{P_1, P_3, P_4, P_6\}$. Then the nano topology $\tau_{C-\{IF\}}(X) = \{U, \emptyset, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_6, P_7\}, \{P_1, P_3, P_4, P_6\}\}$ and the basis $\mathcal{B}_{C-\{IF\}}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3, P_4, P_6\}\} = \mathcal{B}_C(X)$.

(v) Remove the attribute "Properly cooked Foods from C, $U/R(C - PCF) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_5, P_6\}, \{P_7\}, \{P_8\}\}$ then $L_{C-\{PCF\}}(X) = \{P_2, P_7\}$, $U_{C-\{PCF\}}(X) = \{P_1, P_2, P_3, P_4, P_6, P_7\}$, and $B_{C-\{PCF\}}(X) = \{P_1, P_3, P_4, P_5, P_6\}$. Then the nano topology $\tau_{C-\{OF\}}(X) = \{U, \emptyset, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_6, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}\}$ and the basis $\mathcal{B}_{C-\{IF\}}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}\} = \mathcal{B}_C(X)$.

Step 2.

If $M = C - \{OF, B, IF\} = \{F \& V, PCF\}$, then $\mathcal{B}_M(X) = \mathcal{B}_C(X)$. From step 1, we get,

(i) $U/R(C - F \& V) = \{\{P_1, P_4, P_5\}, \{P_2\}, \{P_3, P_6\}, \{P_7\}, \{P_8\}\}$. Then $\tau_{C-\{F\&V\}}(X) = \{U, \emptyset, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}\}$ and the basis $B_{C-\{F\&V\}}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3, P_4, P_5, P_6\} \neq B_C(X)$.

(ii) $U/R(C - PCF) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_5, P_6\}, \{P_7\}, \{P_8\}\}$. Then the nano topology $\tau_{C-\{OF\}}(X) = \{U, \emptyset, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}\}$ and the basis $\mathcal{B}_{C-\{OF\}}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}\} \neq \mathcal{B}_C(X)$.

From (i) and (ii) we get, $\mathcal{B}_M(X) = \mathcal{B}_C(X)$ and $\mathcal{B}_{M-\{x\}}(X) \neq \mathcal{B}_M(X)$ for every x in M.

Since CORE(A) = {Fruits and Vegetables, Properly cooked Foods}

Case 2. (Persons who are Unhealthy)

Let $X = \{P_4, P_5, P_6, P_8\}$, the set of persons who are Unhealthy. Then $L_C(X) = \{P_5, P_8\}$, $U_C(X) = \{P_1, P_3, P_4, P_5, P_6, P_8\}$ and $B_C(X) = U_C(X)$

$-L_C(X) = \{P_1, P_3, P_4, P_6\}$. Then $\tau_C(X) = \{U, \emptyset, \{P_5, P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_6\}\}$. The basis of $\tau_C(X)$ is given by $\mathcal{B}_C(X) = \{U, \{P_5, P_8\}, \{P_1, P_3, P_4, P_6\}\}$.

Step 1.

(i) Remove the attribute "Ordered Foods" from C, $U/(C - OF) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ then $L_{C-\{OF\}}(X) = \{P_5, P_8\}$, $U_{C-\{OF\}}(X) = \{P_1, P_3, P_4, P_5, P_6, P_8\}$ and the boundary region is given by $B_{C-\{OF\}}(X) = \{P_1, P_3, P_4, P_6\}$. Then the nano topology $\tau_{C-\{OF\}}(X) = \{U, \emptyset, \{P_2, P_7\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_6\}\}$ and the basis $\mathcal{B}_{C-\{OF\}}(X) = \{U, \{P_5, P_8\}, \{P_1, P_3, P_4, P_6\}\} = \mathcal{B}_C(X)$.

(ii) Remove the attribute "Fruits and Vegetables" from C, U/R(C-F and V) $\{\{P_1, P_4, P_5\}, \{P_2\}, \{P_3, P_6\}, \{P_7\}, \{P_8\}\}$ then $L_{C-\{F\&V\}}(X) = \{P_8\}$, $U_{C-\{F\&V\}}(X) = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$ and $B_{C-\{F\&V\}}(X) = \{P_1, P_3, P_4, P_5, P_6\}$. Then $\tau_{C\{F\&V\}}(X) = \{U, \emptyset, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_5, P_6\}\}$ and the basis $\mathcal{B}_{C-\{F\&V\}}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}\} \neq \mathcal{B}_C(X)$.

(iii) Remove the attribute "Beverages" from C, $U/R(C - B) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ then $L_{C-\{B\}}(X) = \{P_2, P_7\}$, $U_{C-\{B\}}(X) = \{P_1, P_3, P_4, P_5, P_6, P_8\}$, and $B_{C-\{B\}}(X) = \{P_1, P_3, P_4, P_6\}$. Then $\tau_{C-\{B\}}(X) = \{U, \emptyset, \{P_5, P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_6\}\}$ and the basis $\mathcal{B}_{C-\{B\}}(X) = \{U, \{P_5, P_8\}, \{P_1, P_3, P_4, P_6\}\} = \mathcal{B}_C(X)$.

(iv) Remove the attribute "Instant Foods" from C, $U/R(C - IF) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_7\}, \{P_8\}\}$ then $L_{C-\{IF\}}(X) = \{P_5, P_8\}$, $U_{C-\{IF\}}(X) = \{P_1, P_3, P_4, P_5, P_6, P_8\}$, and $B_{C-\{IF\}}(X) = \{P_1, P_3, P_4, P_6\}$. Then the nano topology $\tau_{C-\{IF\}}(X) = \{U, \emptyset, \{P_5, P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_6\}\}$ and the basis $\mathcal{B}_{C-\{IF\}}(X) = \{U, \{P_5, P_8\}, \{P_1, P_3, P_4, P_6\}\} = \mathcal{B}_C(X)$.

(v) Remove the attribute “Properly cooked Foods from C, $U/R(C-PCF) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_5, P_6\}, \{P_7\}, \{P_8\}\}$ then $L_{C-\{PCF\}}(X) = \{P_8\}$, $U_{C-\{PCF\}}(X) = \{P_1, P_3, P_4, P_5, P_6, P_7, P_8\}$, and $B_{C-\{PCF\}}(X) = \{P_1, P_3, P_4, P_5, P_6, P_7\}$. Then the nano topology $\tau_{C-\{OF\}}(X) = \{U, \emptyset, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_7\}\}$ and the basis $\mathcal{B}_{C-\{IF\}}(X) = \{U, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_7\}\} \neq \mathcal{B}_C(X)$.

Step 2. If $M = C - \{OF, B, IF\} = \{F \& V, PCF\}$, then $\mathcal{B}_M(X) = \mathcal{B}_C(X)$. From step 1, we get,

(i) $U/R(C - F \& V) = \{\{P_1, P_4, P_5\}, \{P_2\}, \{P_3, P_6\}, \{P_7\}, \{P_8\}\}$. Then $\tau_{C-\{F\&V\}}(X) = \{U, \emptyset, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_5, P_6\}\}$ and the basis $B_{C-\{F\&V\}}(X) = \{U, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6\} \neq B_C(X)$.

(ii) $U/R(C-PCF) = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_5, P_6\}, \{P_7\}, \{P_8\}\}$. Then the nano topology $\tau_{C-\{OF\}}(X) = \{U, \emptyset, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_7\}\}$ and the basis $\mathcal{B}_{C-\{PCF\}}(X) = \{U, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_7\}\} \neq \mathcal{B}_C(X)$.

From (i) and (ii) we get, $\mathcal{B}_M(X) = \mathcal{B}_C(X)$ and $\mathcal{B}_{M-\{x\}}(X) \neq \mathcal{B}_M(X)$ for every x in M.

Since CORE (A) = {Fruits and Vegetables, Properly cooked Foods}.

Result

From the core we conclude that the essential key factors for a healthy life is “Fruits and Vegetables” and “Properly cooked Foods”.

5. Conclusion

In this paper, we introduced a new class of nano continuous functions on nano topological spaces called as Nrb-continuous functions. And also some of their properties are analyzed. Then one real life application was produced by using the concepts of basis in Nano topological spaces. In this paper essential food habits for a healthy life style was identified. In future this work will be extended with some real life applications by using the corresponding set "Nrb-closed" in Nano topological spaces.

References

[1] Ahmad Al. Omari and Mohd. Salmi Noorani MD., On generalized b-closed sets, Bulletin of the Malaysian Mathematical Sciences Society 32(1) (2009), 19-30.

[2] D. Andrijevic, On b-open sets, Mat. Vesnik 48 (1996), 59-64.

[3] Bakeir, Nano δ-generalized closed sets in nano topology, 17th International conference on Differential geometry and Application, ICDGA (2015).

[4] K. Bhuvaneshwari and A. Ezhilarasi, Nano generalized semi continuity, International Research Journal of Pure Algebra 6(8) (2016), 361-367.

[5] K. Bhuvaneshwari and A. Ezhilarasi, On nano semi-generalized and nano generalized semi-closed in nano topological spaces, International Journal of Mathematics and computer applications Research 4 (2014), 117-124.

[6] Bhuvaneshwari and K. Mythili Gnanapriya, Nano generalized closed sets in nano topological spaces, International Journal Scientific and Research Publications 4(5) (2014).

[7] Bhuvaneshwari Kandaswamy and K. Mythili Gnanapriya, On nano generalized pre closed sets and Nano pre generalized closed sets in nano topological spaces, International Journal of Innovative Research in science, Engineering and Technology 3 (2014), 16829-16835.

[8] K. Bhuvaneshwari and K. Mythili Gnanapriya, On Nano generalized pre-continuous functions in nano topological spaces, International Journal of Mathematical Archieve 7(7) (2014), 163-167.

[9] K. Bhuvaneshwari and K. Mythili Gnanapriya, On nano generalized continuous function in nano topological space, International Journal of Mathematical Archive 6(6) (2015), 2229-5046.

[10] M. Dhanapackiam and M. Trinita Pricilla, New class of nano generalized b-continuous functions in nano topological spaces, International Journal of Engineering Science and Computing 6(8) (2016), 2264-2269.

[11] A. Dhanis Arul mary and I. Arockiarani, On Nano generalized b-closed sets in nano topological spaces, International Journal of Mathematical Archieve 6(2) (2015), 2229-5046.

[12] Lellis Thivagar and Carmel Richard, On nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention 1(1) (2013), 31-37.

[13] Lellis Thivagar Carmel Richard, On Nano continuity, Mathematical Theory and Modeling 3(7) (2013), 32-37.

[14] M. Lellis Thivagar, Carmel Richard, Mathematical innovations of a Modern topology in Medical Events, International Journal of Information Science 2(4) (2012), 33-36.

[15] Levine, Generalized closed sets in topology, Rendiconti del circolo Matematico di Palermo 19(2) (1970), 89-96.

[16] A. Narmatha and N. Nagaveni On regular *b*-closed sets in topological spaces, Heber International Conference on Applications of Mathematics and Statistics, HICAMS-2012, (2012), 81-87.

[17] A. Narmatha, N. Nagaveni and T. Noiri, On regular *b*-open sets, International Journal of Mathematical Analysis 7(19) (2013), 937-948.

[18] A. A. Nasef, A. I. Aggour and S. M. Darwesh, On some classes of nearly open sets, Journal of the Egyptian Mathematical Society 24(4) (2016), 585-589.

[19] N. Palaniappan and K. Chandrasekhara Rao, Regular generalized closed sets, Kyungpook Mathematics Journal 33 (1993).

[20] C. Pandiammal and K. Deepika, Nano generalized regular continuity in nano topological space, International Journal of Mathematics Trends and Technology (IJMTT) 65(3) (2019), 177-182.

[21] A. Parimala, G. Indirani and S. Jafari, On Nano *b*-open sets, Jordan Journal of Mathematics and Statistics (JJMS) 9(3) (2016), 173-184.

[22] Z. Pawlak, Rough sets, Theoretical aspects of reasoning about data, Kluwer Academic Publishers, London (1991).

[23] Z. Pawlak, Rough set theory and its applications, Journal of telecommunications and Information Technology 3 (2002), 7-10.

[24] P. Sathishmohan, V. Rajendran, A. Devika and R. Vani, On nano semi-continuity and nano pre-continuity in nano topological spaces, International Journal of Applied Research 3(2) (2017), 76-79.

[25] Sharmistha Bhattacharya (Halder), On generalized regular closed sets, International Journal of Mathematical sciences 6 (2011), 145-152.

[26] P. Sulochana Devi and K. Bhuvaneshwari, On Nano regular generalized and Nano generalized regular closed sets in nano topological spaces, International Journal of Engineering Trends and Technology (IJETT) 13(18) (2014), 386-390.

[27] P. Srividhya and T. Indira, On Nano regular b-closed sets and Nano regular *b*-open sets in nano topological spaces, International Journal of Application or Innovation in Engineering and Management (CMA) (communicated).