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CERTAIN FAMILIES OF ACYCLIC GRAPHS WITH EVEN-ODD AVERAGE HARMONIOUS LABELING

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Abstract

Even odd average harmonious labeling is an innovative harmonious labeling that we have presented in this article. If a function that can be bijective then $f : V \rightarrow \{1, 3, 5, \ldots, (2n-1)\}$ as the induced mapping $f^{*} : E \rightarrow \{0, 1, 2, \ldots, m-1\}$ described as $f^{*}(uv) = \frac{f(u)+f(v)}{2} (\bmod m)$, G is said to be an even odd average harmonic graph. A graph which admit an Even-Odd Average harmonious graph. In this paper we proved that the graph 1-regular lobster, The coconut tree graph $CT_{p,q}$, The caterpillar graph $cat_{q}^{+t}(1, r)$, The spider tree graph, The star graph $S_{q,3}$ are Certain Families of Acyclic Graphs with Even-odd harmonious labeling.

1. Introduction

Harmonious labeling were defined by Graham and Sloane [3]. As part of their study of additive basis and are applicable to error-correcting codes. Since their introduction in 1980, harmonious labeling [1] has piqued the interest of many researchers, and they are now among the most widely used

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labeling in the literature. Indeed, if we were to pick between two popular labeling, we would select the latter. We would undoubtedly prefer labels that are graceful and harmonious. Liang, Bai [10] introduced odd harmonious labeling and Sarasija and Binthiya [6, 7] introduced an even harmonious labeling. Even-odd harmonious labeling was pioneered by M. Kalaimathi, B. J. Balamurugan [4, 5] and Adalin Beatress and Sarasija [8, 9] also introduced the average harmonious graph in 2015.

2. Preliminaries

Definition 2.1. If there is no coherently directed cycle in an orientation of G, it is acyclic. Any acyclic orientation on a graph G produces a posset in a natural way.

Definition 2.2. A graph depicting caterpillars when all of the leaves are removed from G, it produces a path.

Definition 2.3. A graph depicts a regular lobster formed from a path of n vertices p_n by connecting each vertex of p_n with a path of length 2.

Definition 2.4. A spider tree is a tree T with n legs and l length that has exactly one vertex of degree greater than or equal to three.

3. Main Results

Definition. G is an even odd average harmonious graph if there exists a bijective function $f : V \rightarrow \{1, 3, 5, \ldots, (2n-1)\}$ as the induced mapping $f^* : E \rightarrow \{0, 1, 2, \ldots, m-1\}$ as well as being $f^*(uv) = \frac{f(u)+f(v)}{2} (\bmod m)$, is a bijection. f is defined as an Even-odd harmonious labeling of graph G. A graph which admit an Even-Odd Average harmonious graph.

Theorem 3.1. *The even odd average harmonic graph is possible with the 1-regular lobster graph.*

Proof. Let $V = \{u_a : 1 \leq a \leq r, u_{ab} : 1 \leq a \leq r$ and $b = 1, 2\}$ denote the vertex set of $cat_q^{+t}(1, r)$ where the vertices of the path p_n are denoted by u_a and u_{ab} are the path u_a vertices that are adjacent to each other for, $1 \leq a \leq r$ and $b = 1, 2$.

Let $E = \{u_a u_{a+1} : 1 \le a \le r-1,\ u_a u_{ab},\ 1 \le a \le r,\ b = 1, 2\}$ be the 1-regular lobster edge set.

There are $n = 3r$ vertices and $m = 3r - 1$ edges in the graph.

Define a injective function $f : V \to \{1, 3, 5, \ldots, 2(3r) - 1\}$ such that

Case (i) $r \equiv 0 (\bmod 2)$

$$f(u_a) = \begin{cases} 3(a-1)+1, & \text{if } a \text{ is odd} \\ 3r + 4 + 3(a-2) + 1, & \text{if } a \text{ is even} \end{cases}$$

$$f(u_{a1}) = \begin{cases} 3(a-1) + 3n + 1, & \text{if } a \text{ is odd} \\ 3(a-2) + 5, & \text{if } a \text{ is even} \end{cases}$$

$$f(u_{a2}) = \begin{cases} 3(a-1) + 3, & \text{if } a \text{ is odd} \\ 3(a-2) - 3, & \text{if } a \text{ is even} \end{cases}$$

Case (ii) $r \equiv 1 (\bmod 2)$

$$f(u_a) = \begin{cases} 3(a-1)+1, & \text{if } a \text{ is odd} \\ 3(r+a), & \text{if } a \text{ is even} \end{cases}$$

$$f(u_{a1}) = \begin{cases} 3(r+a) - 1, & \text{if } a \text{ is odd} \\ 3(a-2) + 5, & \text{if } a \text{ is even} \end{cases}$$

$$f(u_{a2}) = \begin{cases} 3(a-1) + 3, & \text{if } a \text{ is odd} \\ 3(r + a - 1) + 1, & \text{if } a \text{ is even} \end{cases}$$

and a bijective function $f^* : E \to \{0, 1, 2, \ldots, (3r-1) - 1\}$ by

Case (i) $r \equiv 0 (\bmod 2)$

$$f^*(u_a u_{a+1}) = \left(\frac{3r}{2} + 2 + 3(a-1) + 1\right)(\bmod m),\ 1 \le a \le r-1$$

$$f^*(u_a u_{a1}) = \begin{cases} \left(\frac{3r}{2} + 2 + 3(a-1) + 1\right)(\bmod m), & \text{if } a \text{ is odd} \\ \left(\frac{3r}{2} + 3a - 1\right)(\bmod m), & \text{if } a \text{ is even} \end{cases}$$

$$f^*(u_a u_{a2}) = \begin{cases} \left(\frac{3r}{2} + 2 + 3(a-1) + 2\right)(\bmod m), & \text{if } a \text{ is odd} \\ 3\left(\frac{r}{2} + a - 1\right)(\bmod m), & \text{if } a \text{ is even} \end{cases}$$

Case (ii) $r \equiv 1 (\bmod 2)$

$$f^*(u_a u_{a+1}) = 3\left(\frac{r}{2} + a\right) + 1 (\bmod m),\ 1 \leq a \leq r-1$$

$$f^*(u_a u_{a1}) = \begin{cases} \left(\frac{3r}{2} + 3(a-1) + 1\right)(\bmod m), & \text{if } a \text{ is odd} \\ 3\left(\frac{r}{2} + a\right) - 1 (\bmod m), & \text{if } a \text{ is even} \end{cases}$$

$$f^*(u_a u_{a2}) = \begin{cases} \left(3\left(\frac{r}{2}\right) + 3(a-1) + 3\right)(\bmod m), & \text{if } a \text{ is odd} \\ 3\left(\frac{r}{2} + a - 1\right) + 2 (\bmod m), & \text{if } a \text{ is even} \end{cases}$$

As a result, the even odd average hormonious graph is possible with the 1-regular lobster graph.

Example 1. The even odd average hormonious graph is possible with the 1-regular lobster graph as figure 1.

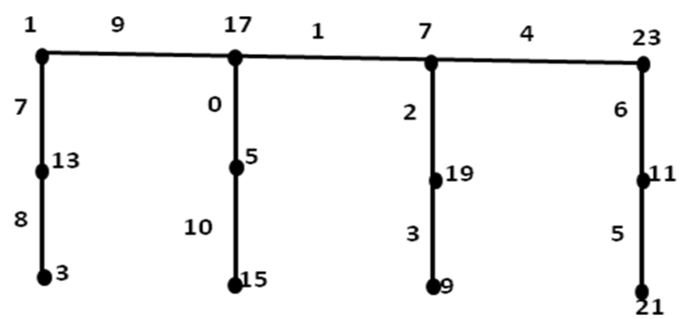

Figure 1. The 1-regular lobster graph is an Even-Odd Average harmonious graph.

Theorem 3.2. *The coconut tree graph $CT_{p,q}$, admits an Even-Odd Average harmonious graph when* $p \geq 2$, $q \geq 2$.

Proof. $V = \{u_a : 1 \leq a \leq p, u_b : 1 \leq b \leq q\}$, where the vertices of the path p_n are denoted by u_a and the m additional vertices that are pendent at the path's p_n end vertex are denoted by the symbol u_b.

Let $E = \{u_a u_{a+1} : 1 \le a \le p-1,\ u_a u_b,\ 1 \le b \le q,\ a = p\}$.

There are $n = p + q$ vertices and $m = p + q - 1$ edges in the graph.

Define a injective function $f : V \to \{1, 3, 5, \dots, 2(p+q)-1\}$

Case (i) $p \equiv 1 (\mathrm{mod}\ 2)$

$$f(u_a) = \begin{cases} a, & \text{if } a \text{ is odd} \\ p + a, & \text{if } a \text{ is even} \end{cases}$$

$$f(u_b) = 2p + 2b - 1,\ 1 \le b \le q$$

Case (ii) $p \equiv 0 (\mathrm{mod}\ 2)$

$$f(u_{2a-1}) = p + a,\ 1 \le a \le \frac{p}{2}$$

$$f(u_{2a}) = 2a - 1,\ 1 \le a \le \frac{p}{2}$$

$$f(u_b) = 2a + 2b - 1,\ 1 \le b \le q$$

and a bijective function $f^* : E \to \{0, 1, 2, \dots, (p+q-1)-1\}$ by

Case (i) $p \equiv 1 (\mathrm{mod}\ 2)$

$$f^*(u_a u_{a+1}) = (p + a - 1)(\mathrm{mod}\ m),\ 1 \le a \le p-1$$

$$f^*(u_a u_b) = (p + q - 1) + b(\mathrm{mod}\ m),\ a = p,\ 1 \le b \le q$$

Case (ii) $p \equiv 0 (\mathrm{mod}\ 2)$

$$f^*(u_a u_{a+1}) = (p + a)(\mathrm{mod}\ m),\ 1 \le a \le p$$

$$f^*(u_a u_b) = (p + b + 1)(\mathrm{mod}\ m),\ a = p,\ 1 \le b \le q$$

As a result, the vertex and edge labels are distinct.

The coconut tree graph $CT_{p,q}$, is an Even-Odd Average harmonious graph.

Example 2. The coconut tree $CT_{3,5}$, is an Even -Odd Average harmonious graph figure 2.

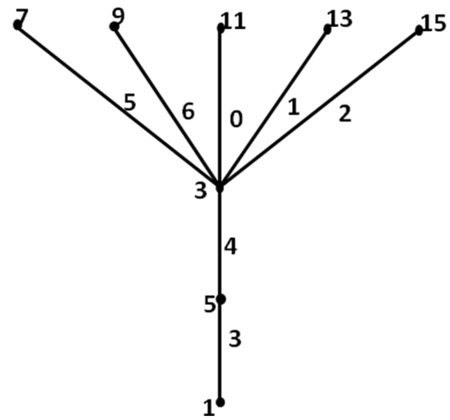

Figure 2. The coconut tree graph $CT_{3,5}$ is an Even -Odd Average harmonious graph.

Theorem 3.3. *The caterpillar graph* $cat_q^{+t}(1, r)$ *admits an Even-Odd Average harmonious graph when* $q \geq 3$, $t \geq 1$.

Proof. $V = \{l_a : 1 \leq a \leq l\} \cup \{r_b : 1 \leq b \leq r\}$ be the vertex set of $cat_q^{+t}(1, r)$ where l_a and are r_b, are the vertices on the left and right, respectively.

Let $E(L_n) = \{e_{ab} = l_a r_b : 1 \leq a \leq l, 1 \leq b \leq r\}$.

There are $n = q + r$ vertices and $m = q + r - 1$ edges.

Define a bijective function $f : V \to \{1, 3, 5, \ldots, 2(q + t) - 1\}$ by

$$f(l_a) = 2i - 1, 1 \leq a \leq l \text{ and}$$

$$f(r_b) = 2l + 2b - 1, 1 \leq b \leq r$$

Then follows, f brings about a bijection $f^* : E \to \{0, 1, 2, \ldots, m - 1\}$ as well as

$$f^*(l_a r_b) = l + a (\bmod m), 1 \leq a \leq l, 1 \leq b \leq r.$$

As a result, the vertex and edge labels are distinct.

Hence the caterpillar graph $cat_q^{+t}(1, r)$ is an Even-Odd Average harmonious graph.

Example 3. The caterpillar graph $cat_q^{+t}(6, 7)$ admits an Even-Odd Average harmonious graph as figure 3.

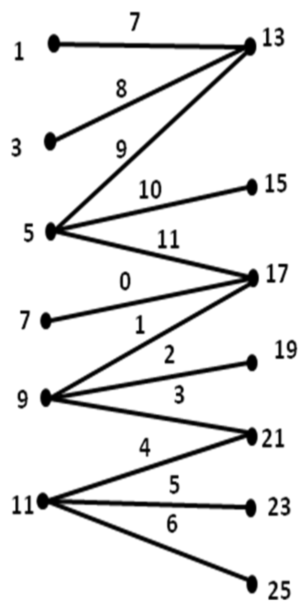

Figure 3. Caterpillar graph $cat_q^{+t}(6, 7)$ is an Even-Odd Average harmonious graph.

Theorem 3.4. *The spider tree graph allows for an Even-Odd Average harmonious graph with r legs and l length when* $r \geq 3, r \equiv 1(\text{mod } 2)$ *and* $l \equiv 0(\text{mod } 2)$.

Proof. Let $V = \{u$ and $u_{ab} : 1 \leq a \leq r, 1 \leq b \leq l-1\}$ be the spider tree vertex set, where u is a center vertex and u_{ab} are the spider tree graph's r legs, l length

Let $E = \{u\, u_{a1} : 1 \leq a \leq r$ and $u_{ab}u_{ab+1} : 1 \leq a \leq r, 1 \leq b \leq l-1\}$ represent the spider tree's edge set.

There are $n = rl + 1$ vertices and $m = rl$ edges in the graph.

Define a injective function $f : V \to \{1, 3, 5, \ldots, 2(rl + 1) - 1\}$ such that

$$f(u) = 1$$

$$f(u_{ab}) = \begin{cases} l(r + a - 1) + b + 2, & \text{if } a \text{ and } b \text{ are add} \\ l(r + a - 1) + b + 1, & \text{if } a \text{ and } b \text{ are even} \end{cases}$$

$$f(u_{ab}) = \begin{cases} l(a - 1) + b + 2, & \text{if } a \text{ is odd and } b \text{ is even} \\ l(a - 1) + b + 1, & \text{if } a \text{ is even and } b \text{ is odd} \end{cases}$$

and a bijective function $f^* : E \to \{0, 1, 2, \ldots, rl - 1\}$ by

$$f^*(u\,u_{a1}) = \begin{cases} \frac{l}{2}(r + a - 1) + 1, & \text{if } a \text{ is odd} \\ \frac{l}{2}(a - 1) + 1, & \text{if } a \text{ is even} \end{cases}$$

$$f^*(u_{ab}u_{ab+1}) = \begin{cases} \frac{l}{2}(r + 3a - 2) + (b), & \text{if } a = 1 \\ \frac{l}{2}(r + 3a - 2) + (b - 2), & \text{if } a = 2 \\ \frac{l}{2}(r + 3a - 2) - (4 - b), & \text{if } a = 3 \\ \frac{l}{2}(r + 3a - 2) - (6 - b), & \text{if } a = 4 \\ \frac{l}{2}(r + 3a - 2) - (8 - b), & \text{if } a = 5 \end{cases}$$

Hence both vertex labels and edge label are distinct.

Hence the spider tree graph is an Even-Odd Average harmonious graph.

Example 4. The spider tree graph with 5 legs and 4 length allows an is Even -Odd Average harmonious labeling as figure 4.

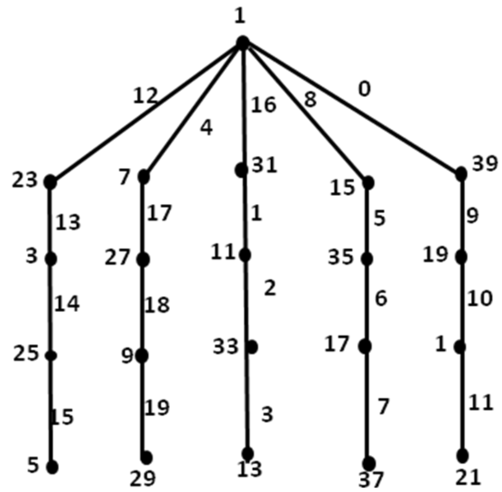

Figure 4. Spider graph is an Even-Odd Average harmonious graph.

Theorem 3.5. *The star graph* $S_{q,3}$ *allows an even odd average harmonious graph labeling when* $q \equiv 1 (\bmod 2)$.

Proof. Let $U = u$ represent a set of vertex in which u is a point of contact in the center and $V = \{u_i : 1 \le i \le q, v_i : 1 \le i \le q$ and $w_i : 1 \le i \le q\}$ represent the star graph is a point of contact where the vertices of path P_3 are denoted by u_i, v_i, w_i for $1 \le i \le q$.

Let $E = \{u u_i : 1 \le i \le q, u_i v_i : 1 \le i \le q$ and $w_i : 1 \le i \le q\}$ represent the star graph's e the edge set

There are $n = 3q + 1$ vertices and $m = 3q$ edges in the graph.

Define a injective function $f : V \to \{1, 3, 5, \ldots, 2(3q + 1) - 1\}$ such that

$$f(u) = 2q + 1,\ f(u_1) = 6q + 1,\ f(u_{2i}) = 4i - 1,\ 1 \le i \le \frac{m}{2}$$

$$f(u_{2i+1}) = 2q + 4i + 1,\ 1 \le i \le \frac{m}{2}$$

$$f(v_{2i}) = 2q + 4i - 1,\ 1 \le i \le \frac{m}{2}$$

$$f(v_{2i-1}) = 4(i-1),\ 1 \le i \le \frac{m}{2}$$

$$f(w_1) = 4q + 1,$$

$$f(w_{2i}) = 5q + 2 - 2i,\ 1 \le i \le \frac{m}{2}$$

$$f(w_{2i+1}) = 6q - 2i + 1,\ 1 \le i \le \frac{m}{2}$$

and a bijective function $f^* : E \to \{0, 1, 2, \ldots, 3q-1\}$ by

$$f^*(u\,u_1) = (q+1)(\operatorname{mod} m)$$

$$f^*(u\,u_{2i}) = (q + 2i)(\operatorname{mod} m),\ 1 \le i \le \frac{m}{2}$$

$$f^*(u\,u_{2i+1}) = (2q + 2i + 1)(\operatorname{mod} m),\ 1 \le i \le \frac{m}{2}$$

$$f^*(u_1 v_1) = 1$$

$$f^*(u_{i+1}v_{i+1}) = ((q+1) + 2i)(\operatorname{mod} m),\ 1 \le i \le m$$

$$f^*(v_1 w_1) = (2q+1)(\operatorname{mod} m),\ 1 \le i \le \frac{m}{2}$$

$$f^*(v_{2i}w_{2i}) = \frac{1}{2}(q + 2i + 1)(\operatorname{mod} m),\ 1 \le i \le \frac{m}{2}$$

$$f^*(v_{2i+1}w_{2i+1}) = (i+1)(\operatorname{mod} m),\ 1 \le i \le \frac{m}{2}$$

As a result, the vertex labels and edge labels are distinct.

Hence $S_{q,3}$ admits an Even -Odd Average harmonious graph.

Example 5. The star graph $S_{q,3}$ allows an Even-Odd Average harmonious graph as figure 5.

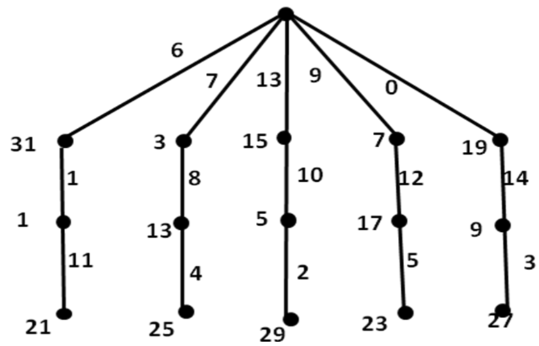

Figure 5. The star graph $S_{q,3}$ allows an Even -Odd Average harmonious graph.

4. Conclusion

In this paper, discusses, the odd average harmonious labeling is discussed. Looked at a variety of graphs of acyclic graphs, such as coconut tree graph $CT_{p,q}$, the caterpillar graph $cat_q^{+t}(l, r)$, the spider tree graph, the star graph $S_{q,3}$ is an Even-Odd Average harmonious graph.

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