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FELICITOUS LABELINGS OF GRAPHS RELATED WITH STAR MERGED WITH SHELL GRAPH

S. SRIRAM[1] and S. KAVITHANANDHI[2]

[1,2]Department of Mathematics
National College, Tiruchirappalli
(Affiliated with Bharathidasan University)
E-mail: sriram.priya02@yahoo.com
kavitha.anandhi@yahoo.com

Abstract

Selvam, Thirusangu and Ulaganathan, [2011] investigated and they have proved that. The class of extended duplicate graph of a Twig $T_m, m \geq 1$ is felicitous Labeling graphs. Princy [2012] explained that every harmonious is felicitous, and every (p, q)-graph G can be embedded in a connected felicitous graph L with (2^{P+1}) edges and $(2^{P+1} - q)$ vertices. Lakshmi Alias Gomathi, Nagarajan and Nellai Murugan [2013] have proved that the P_r^{2m-1} for all values of m and r as odd number; P_r^{2m-1} for all values of m and $r \equiv 0, 3 (\text{mod} 4)$ and the graph $C_n \times P_m$ for $m \geq 1$ and $n \equiv 1 (\text{mod} 2)$ are felicitous labeling. In this paper, felicitous labelings are obtained for few graphs $C(5, 2) \Delta S_n$, $C(5, 2) \oplus S_n$, $C(6, 3) \Delta S_n$, $C(6, 3) \oplus S_n$ and $K_4 \oplus S_n$.

1. Introduction

Alias Gomathi, Nagarajan and Nellai Murugan [2012], have proved that the combination of cycles and complete graphs $C_n * K_{1,m}$ for any $m \geq 1$, $n \geq 3$, $C_n * K_1$, $n \equiv 1 (\text{mod} 2)$ for any $n \geq 3$, and $(C_n \blacksquare C_n)_{2n}$ for any $n \geq 1$ are all felicitous labeling.

Jiajuan Zhang, Bing Yao, Zhiqian Wang, Hongyu Wang, Chao Yang, Sihua Yang [2013] have Constructed the larger felicitous Graph from

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smaller graphs having special felicitous labeling and some network models are shown to be felicitous.

Selvaraju, Balaganesan, Renuka, [2013] have proved that the graph $D_2(P_n)$ is an even sequential harmonious and graceful graphs; $D_2(K_{1,n})$ is an even sequential harmonious, graceful graph; odd graceful graph and felicitous graph, $spl(P_n)$ is an even sequential harmonious, and $spl(K_{1,n})$ is even sequential harmonious and felicitous graph.

Arockiaraj, Mahalakshmi and Namasivayam [2014] have proved that the subdivision of the triangular snake, quadrilateral snake, slanting ladder, $C_p \odot K_1$, $H \odot K_1$, $C_m @ C_n$ grid graph $P_m \times P_n$, duplication of a vertex of a path and duplication of a vertex of a cycle.

2. Felicitous Labeling of a Graph

Definition 2.1. A felicitous labeling of a graph G, with q edges is an injection $f : V(G) \to \{0, 1, \ldots, q\}$ so that the induced edge labels $f^*(xy) = (f(x) + f(y)) (\bmod q)$ are distinct. Clearly, a harmonious graph is felicitous.

Definition 2.2. A shell graph $C_{(n, n-3)}$ is a simple graph having a cycle with n vertices such that a vertex on the cycle is attached with $(n-3)$ edges at another vertex on the cycle.

Theorem 2.3. *The connected simple graph $C(5, 2) \Delta S_n$ is a felicitous graph for any positive integer n.*

Proof of Theorem 2.3. Let G be simple connected graph with one vertex of S_n attaching with the root of the cycle $3C_3$ with $(n+6)$ vertices and $(n+8)$ edges, whose labels are shown in figure (1).

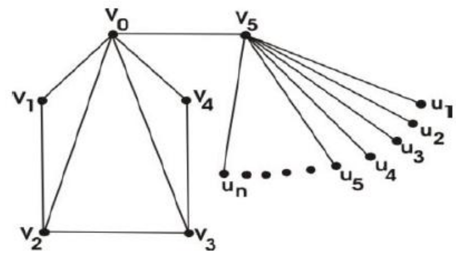

Figure 1. Arbitrary labeling of the connected graph $C(5, 2)\Delta S_n$.

Define $f : V(G) \to \{0, 1, 2, \ldots, q\}$ by

$f(v_0) = 0,\ f(v_1) = 4,\ f(v_2) = 1,\ f(v_3) = 2,\ f(v_4) = 6,\ f(v_5) = q,\ f(u_1) = 7,$

$$f(u_i) = 7 + i \text{ for } i = 2, 3, 4, \ldots, n \quad (1)$$

The induced edge map $f^+ : E(G) \to \{0, 1, 2, 3, \ldots, q\}$ by $f^+(xy) = (f(x) + f(y))(\text{mod } q)$ where this sum run over all edges through v.

(2)

The above labeling from (1) and (2), both *f* and f^+ find the distinct labels for vertices, and also the edge labelings are distinct such that they satisfy the conditions of felicitous graph for the given graph.

Example 2.4. The connected simple graph $C(5, 2)\Delta S_8$ is felicitous.

Solution. The connected graph $C(5, 2)\Delta S_8$ has 15 edges such that 13 verities. Due to the condition (1) and (2) in (2.3), the graph $C(5, 2)\Delta S_8$ is felicitous whose labelings are shown in figure (2).

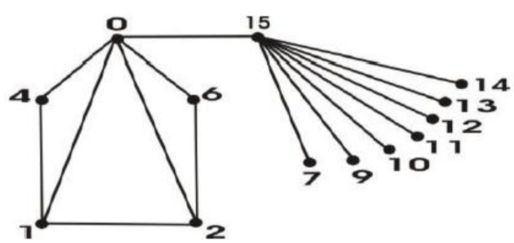

Figure 2. Felicitous graceful labeling of graph $C(5, 2)\Delta S_8$.

Definition 2.5. The graph $C(5, 2) \oplus S_n$ obtained by attaching to the center of S_n (the center of star graph with n verities is merged with the graph $3C_3$).

Theorem 2.6. *The connected simple graph* $C(5, 2) \oplus S_n$ *is a felicitous graph for any a positive integer* n.

Proof of Theorem 2.6. Let G be simple connected graph with the center of S_n is attaching with vertex of the maximum degree vertex in the cycle $C(5, 2)$ with $(n+6)$ vertices and $(n+8)$ edge, whose labels are shown in figure (3).

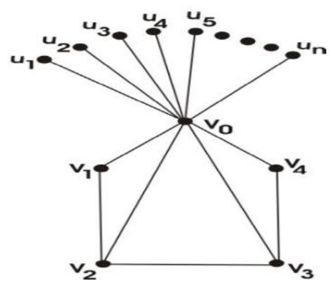

Figure 3. Arbitrary labeling of the connected graph.

Define $f : V(G) \to \{0, 1, 2, \ldots, q\}$ by $f(v_0) = 0$, $f(v_1) = 4$, $f(v_2) = 1$, $f(v_3) = 2$, $f(v_4) = 6$, $f(u_1) = 7$, $f(u_i) = (7 + i)$, for $i = 2, 3, 4, \ldots, n$. (3)

The induced edge map $f^+ : E(G) \to \{0, 1, 2, 3, \ldots, q\}$ by $f^+(xy) = (f(x) + f(y))(\bmod q)$ where this sum run over all edges through v. (4)

The above labeling from (3) and (4), both f and f^+ find the distinct labels for vertices, and also the edge labelings are distinct such that they satisfy the conditions of felicitous graph for the given graph.

Example 2.7. The connected simple graph $C(5, 2) \oplus S_9$ is felicitous.

Solution. The connected graph $C(5, 2) \oplus S_9$ has 16 edges such that 14 verities. Due to the condition (3) and (4) in (2.6), the graph $C(5, 2) \oplus S_9$ is felicitous whose labelings are shown in figure (4).

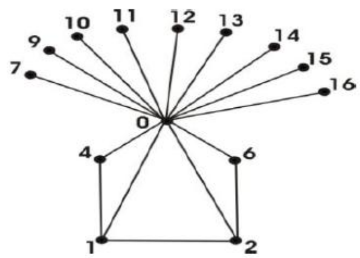

Figure 4. Felicitous graceful labeling of graph $C(5, 2) \oplus S_9$.

Definition 2.8. The graph $C(6, 3) \oplus S_n$ obtained by attaching to the center of S_n (the center of star graph with n verities is merged with a vertex of maximum degree in the cycle $C(6, 3)$).

Theorem 2.9. *The connected simple graph* $C(6, 3) \oplus S_n$ *is felicitous graph for any n positive integer* n.

Proof of Theorem 2.9. Let $G = C(6, 3) \oplus S_n$ be simple connected graph which has $(n + 6)$ vertices and $(n + 9)$ edges, whose are shown in figure (5).

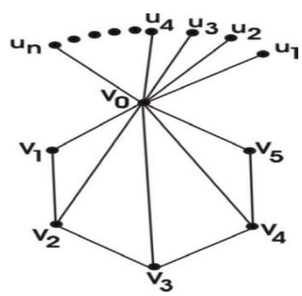

Figure 5. Arbitrary labeling of the connected graph.

Define $f : V(G) \to \{0, 1, 2, \ldots, q\}$ by $f(v_0) = 0,\ f(v_1) = 8,\ f(v_2) = 2,\ f(v_3) = 1,\ f(v_4) = 5,\ f(v_5) = 4,\ f(u_1) = 7,\ f(u_i) = 9 + i,$ for $i = 2, 3, 4, \ldots, n.$ (5)

The induced edge map $f^+ : E(G) \to \{0, 1, 2, 3, \ldots, q\}$ by $f^+(xy) = (f(x) + f(y)) (\bmod q)$ where this sum run over all edges through v. (6)

The above labeling from (5) and (6), both f and f^+ find the distinct labels for vertices, and also the edge labelings are distinct such that they satisfy the conditions of felicitous graph for the given graph.

Example 2.10. The connected simple graph $C(6, 3) \oplus S_7$ is a felicitous graph.

Solution. The connected graph $C(6, 3) \oplus S_7$ has 16 edges such that 13 verities. Due to the condition (5) and (6) in (2.9), the graph $C(6, 3) \oplus S_7$ is felicitous whose labels are shown in figure (6).

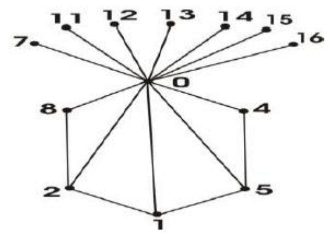

Figure 6. Felicitous graceful labeling of graph $C(6, 3) \oplus S_7$.

Definition 2.11. The graph $C(6, 3)\Delta S_n$ obtained by attaching one vertex of S_n (a non-center of star graph with n verities is merged to a vertex of maximum degree in the cycle $C(6, 3)$).

Theorem 2.12. *The connected simple graph* $C(6, 3)\Delta S_n$ *is a felicitous graph for any* n.

Proof of Theorem 2.12. Let $G = C(6, 3)\Delta S_n$ be simple connected graph which has $(n+7)$ vertices and $(n+10)$ edges whose labels are shown in figure (7).

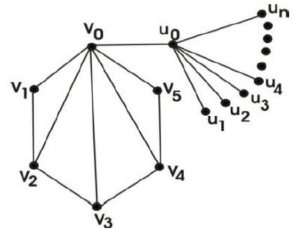

Figure 7. Arbitrary labeling of the connected graph.

Define $f : V(G) \to \{0, 1, 2, \ldots, q\}$ by $f(v_0) = 0$, $f(v_1) = 8$, $f(v_2) = 2$, $f(v_3) = 1$, $f(v_4) = 5$, $f(v_5) = 4$, $f(u_1) = q$, $f(u_i) = q - i$, for $i = 1$ to $(n-1)$, $f(u_n) = 7$ (7)

The induced edge map $f^{+} : E(G) \to \{0, 1, 2, 3, \ldots, q\}$ by $f^{+}(xy)$ $= (f(x) + f(y))(\bmod q)$ where this sum run over all edges through v. (8)

The above labeling from (7) and (8), both f and f^{+} find the distinct labels for vertices, and also the edge labelings are distinct such that they satisfy the conditions of felicitous graph for the given graph.

Example 2.13. The connected simple graph $C(6, 3)\Delta S_8$ is a felicitous graph.

Solution. The connected graph $C(6, 3)\Delta S_8$ has 17 edges such that 14 verities. Due to the condition (7) and (8) in (2.9), the given graph is felicitous whose labels are shown in figure (8).

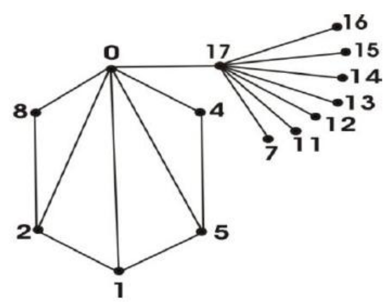

Figure 8. Felicitous graceful labeling of graph $C(6, 3)\Delta S_8$.

Definition 2.14. The graph $K_4 \oplus S_n$ obtained by attaching to the center of S_n (star graph with n verities) to a vertex of the complete graph K_4.

Theorem 2.15. *The connected simple graph* $K_4 \oplus S_n$ *is a felicitous graph for any n is positive integer*.

Proof of Theorem 2.15. Let $G = K_4 \oplus S_n$ be simple connected graph S_n whose labels are shown in figure (9).

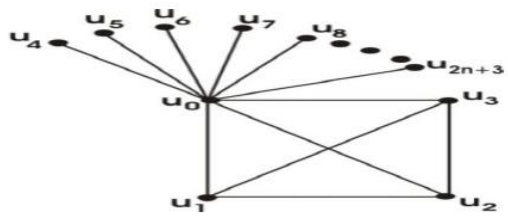

Figure 9. Arbitrary labeling of the connected graph.

Define $f : V(G) \to \{0, 1, 2, \ldots, q\}$ by $f(v_0) = 0$, $f(u_1) = 2$, $f(u_2) = 4$, $f(u_3) = 1$, $f(u_i) = i + 3$, for $i = 4$ to $(n + 3)$. (9)

The induced edge map $f^+ : E(G) \to \{0, 1, 2, 3, \ldots, q\}$ by $f^+(xy) = (f(x) + f(y))(\text{mod}\, q)$ where this sum run over all edges through v. (10)

The above labeling from (9) and (10), both f and f^+ find the distinct labels for vertices, and also the edge labelings are distinct such that they satisfy the conditions of felicitous graph for the given graph.

Example 2.16. The connected simple graph $K_4 \oplus S_{10}$ is felicitous graph.

Solution. The connected graph $G = K_4 \oplus S_{10}$ has 16 edges such that 14 verities. Due to the condition (9) and (10) in (2.15), $K_4 \oplus S_{10}$ is felicitous whose labels are shown in figure (10).

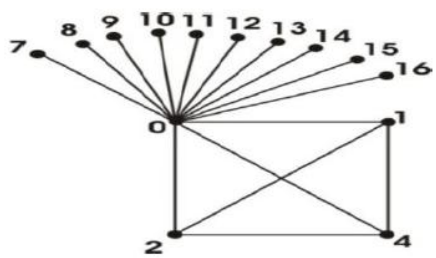

Figure 10. Felicitous graceful labeling of graph $K_4 \oplus S_8$.

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