Advances and Applications in Mathematical Sciences
Volume 19, Issue 12, October 2020, Pages 1337-1366

RELIABILITY AND MEAN TIME TO FAILURE OF UNREPAIRABLE SYSTEMS USING INTUITIONISTIC FUZZY RANDOM LIFETIMES

DEEPAK KUMAR[1], PAWAN KUMAR[2] and ANITA KUMARI[1]

[1]Department of Mathematics
D.S.B. Campus, Kumaun University
Nainital-263002, India
E-mail: anita.shiv2010@gmail.com
deepakdev16@gmail.com

[2]Department of Mathematics,
Statistics and Computer Science
G.B. Pant University of Agriculture and Technology
Pantnagar-263145, Uttarakhand, India
E-mail: cb44330d@gbpuat.ac.in

Abstract

The present study proposes to determine the reliability and mean time to failure (MTTF) of different unrepairable systems using intuitionistic fuzzy random lifetimes. In real life problems, the lifetimes of unrepairable systems contain both randomness and fuzziness. So in this study we assume that the lifetimes of the unrepairable systems to be intuitionistic fuzzy random lifetimes (variables). The reliability and MTTF of different unrepairable systems are evaluated. A numerical example is also presented to demonstrate that how to calculate the reliability and MTTF of the systems with intuitionistic fuzzy random lifetimes.

Introduction

The traditional reliability theory has been successfully used for solving various engineering problems, in which the lifetimes of the systems are assumed to be random variables and the system behaviour can be fully characterized by probability theory. The reliability is defined by the

2010 Mathematics Subject Classification: 03E72.
Keywords: Intuitionistic fuzzy random lifetimes, reliability, Mean time to failure (MTTF), unrepairable systems.
Received October 5, 2019; Accepted November 25, 2019

probability of the random event and mean time to failure (MTTF) is the expected value of random lifetime of the system. But one can face difficulty in reliability evaluation if the lifetime of the system cannot be known precisely due many factors including human errors or any other errors. For example, the lifetime of the machine depends on temperature, humidity and many other factors. In this situation, it is hard to determine the lifetime of the machine precisely. To handle these situations researchers are applying fuzzy set theory in reliability analysis.

The concept of fuzzy set theory introduced by Zadeh (1965) is based on the assumption that the membership degree is equal to one minus non-membership degree. But in reality it may not always be easy to determine membership values, there may exist some hesitation between membership and non membership. Intuitionistic fuzzy sets (IFSs) are appropriate to deal with such problems. IFS, one of the generalization of fuzzy set introduced by Atanassov (1986), is characterized by three functions expressing the membership, non membership and the degree of hesitation. An IFS is one of the generalizations of the fuzzy sets theory. In these sets the nonmembership of an element x of the universe need not be one minus membership degree. Rather, it might be any number lying between 0 and 1. The degree of membership and the degree of non-membership hold the condition $0 \leq \xi(x) + \eta(x) \leq 1$.

In 1975 Kaufmann first used fuzzy theory in reliability engineering. After Kaufmann, Cai et al. (1991, 1993 and 1995) introduced various forms of fuzzy reliability theory, including profust reliability theory, Posbist reliabilit y theory and Posfust reliability theory. Singer (1990) presented a fuzzy approach for fault tree and reliability of both series and parallel system. Chen (1994) discussed a method for fuzzy system reliability analysis using fuzzy number 2 arithmetic operations. Huang (1995) developed the formulas of fuzzy reliability of unrepairable systems. Ying Liu et al. (2007, 2014) evaluated the reliability and MTTF of unrepairable systems with fuzzy random lifetimes and random fuzzy unrepairable systems. Chaube & Singh (2014) analysed the fuzzy reliability of different systems using conflicting bifuzzy set. They assumed that the reliability of all components of a systems follow conflicting bifuzzy set. Needless to say, in above mentioned studies researchers have done good work but they did not give emphasis on the importance of intuitionistic fuzzy random lifetime rate.

In this study, we discuss the reliability and MTTF of different unrepairable systems. There are many reasons for a system to be unrepairable: (1) technical reasons, (2) some of economic reasons. The expression of reliability and MTTF are given for series, parallel, parallel-series and series-parallel unrepairable systems respectively. The lifetimes of the components of unrepairable systems are assumed to intuitionistic fuzzy random variables. The reliability and MTTF of above systems are also evaluated using intuitionistic fuzzy random lifetimes. The concept of the fuzzy random variables introduced by Kwakernaak (1978, 1979). Zainali et al. (2015) give the definition of intuitionistic fuzzy random variable and find the variance using intuitionistic fuzzy random variable. Parvathi et al. (2015) defined intuitionistic fuzzy random variable and give some properties of intuitionistic fuzzy random variables.

In this study, section 2 presents the definition of fuzzy set, intuitionistic fuzzy set and intuitionistic fuzzy random variable. Section 3 describes the expressions of reliability and MTTF for series, parallel, parallel-series and series-parallel system respectively. A numerical example is presented in this paper to illustrate how to calculate the reliability and MTTF of considered system.

2. Definitions

Definition 2.1. Fuzzy set: If a set X be fixed, then a fuzzy set $\widetilde{A}$ is given by

$$\widetilde{A} = \{\langle x, \mu_{\widetilde{z}}(x)\rangle : x \in X\}$$

where $\mu_{\widetilde{A}}(x) \in [0, 1]$ is the membership degree of the element $x \in X$.

Definition 2.2. Intuitionistic fuzzy set: If a set X is fixed then a intuitionistic fuzzy set (IFS) $\widetilde{A}$ of X is an object having the following form

$$\widetilde{A} = \{\langle x, \mu_{\widetilde{A}}(x), \upsilon_{\widetilde{A}}(x) : x \in X\rangle\}$$

where $\mu_{\widetilde{A}} X \to [0, 1]$ is the degree of positive $x, (x \in X)$ with respect to $\widetilde{A}$

and $\upsilon_{\widetilde{A}} : X \to [0, 1]$ is the degree of negative $x, (x \in X)$ with respect to $\widetilde{A}$ and $0 \le \mu_{\widetilde{A}} + \upsilon_{\widetilde{A}} \le 1$.

Definition 2.3. (Zainali (2015)) Intuitionistic fuzzy random variable: Suppose that the probability space (Ω, A, p) describes a random variable, where Ω is a set of all possible outcomes, A is an σ-algebra of subsets of Ω and P is a probability measure on the measurable space (Ω, A) then an intuitionistic fuzzy random variable is a Borel measurable function $\widetilde{X} : \Omega \to IF(R)$, such that

$$\{(\omega, x) : \omega \in \Omega, x \in \widetilde{X}_{\alpha}^{\mu}(\omega) \cap \widetilde{X}_{\beta}^{1-\upsilon}(\omega)\} \in F \times B, \forall 0 \le \alpha \le \beta \le 1$$

where B is a σ-algebra of open sets and $0 \le \mu_{\widetilde{X}(\omega)}(x) + \upsilon_{\widetilde{X}(\omega)}(x) \le 1$.

Definition 2.4 (Ying Liu (2007))**.** Let X be a random lifetime of an unrepairable system then the reliability of an unrepairable system is defined as

$$R(t) = ch\{X \ge t\}. \tag{1}$$

Definition 2.5 (Ying Liu (2007))**.** Let X be a fuzzy random variable then the mean chance, denoted by Ch of fuzzy event is defined as,

$$Ch\{X \le 0\} = \frac{1}{2}\int_0^1 (\Pr\{X_\alpha^L \le 0\} + \Pr\{X_\alpha^U \le 0\})d\alpha, \tag{2}$$

where X_α^L and X_α^U are infimum and supremum of α-cut of fuzzy random variable X respectively. If the fuzzy random variable ξ_i follows exponential distribution, then the density function of exponential distribution is defined as:

$$f(\lambda, t) = \begin{cases} \lambda \exp(-\lambda t), & \text{if } t \ge 0, \\ 0, & if\ t < 0. \end{cases} \tag{3}$$

3. Reliability and MTTF of Unrepairable Systems Using Intuitionistic Fuzzy Random Lifetimes:

3.1. Reliability of Unrepairable Series System

Consider a series system consisting n components as shown in Figure 1.

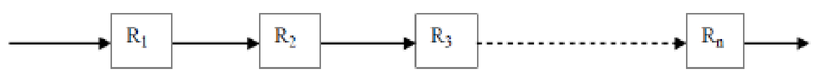

Figure 1. Series system.

Let ξ_i be the lifetime of the $i^{th}(i = 1, 2, 3, \cdots, n)$ component, which is a positive intuitionistic fuzzy random variable on the probability space. The lifetime of the series system is $\xi = \min\{\xi_1, \xi_2, \cdots, \xi_n\}$. Here ξ is an intuitionistic fuzzy random variable on the probability space $(\Omega, A, \Pr)$, where $\Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_n$, $A = A_1 \times A_2 \times \cdots \times A_n$ and $\Pr = \Pr_1 \times \Pr_2 \times \cdots \times \Pr_n$. Let $\alpha, \beta \in [0, 1]$, and $\xi_{i,\alpha}^{L}$ and $\xi_{i,\alpha}^{U}$ are infimum and supremum values of α-cut respectively and $\xi_{i,\beta}^{L}$ and $\xi_{i,\beta}^{U}$ are infimum and supremum values of β-cut of intuitionistic fuzzy random variable ξ_i respectively.

Proposition 1. *Let ξ_i be the lifetime of the $i^{th}(i = 1, 2, 3, \cdots, n)$ component and $\omega \in \Omega$ varies all over Ω and α is fixed, then the reliability of system is given by,*

$$R(t) = \frac{1}{2}\int_0^1 \left(\prod_{i=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{i,\alpha}^{L}(\omega) \geq t\} + \prod_{i=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{i,\alpha}^{U}(\omega) \geq t\} \right) d\alpha.$$

Proof. For any given $\alpha \in [0, 1]$, ξ_α^L and ξ_α^U are the infimum and supremum values of α-cut of ξ, which is intuitionistic fuzzy random variable of the series system then the probability of system is

$$\Pr\{\omega \in \Omega \mid \xi_\alpha^L(\omega) \geq t\} = \prod_{i=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{i,\alpha}^{L}(\omega) \geq t\},$$

$$\Pr\{\omega \in \Omega \mid \xi_\alpha^U(\omega) \geq t\} = \prod_{i=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{i,\alpha}^{U}(\omega) \geq t\}. \quad (4)$$

The reliability of series system is defined as:

$R(t) = ch\{\xi \geq t\}$

$$= \frac{1}{2}\int_0^1 (\Pr\{\omega \in \Omega \mid \xi_\alpha^L(\omega) \geq t\} \Pr\{\omega \in \Omega \mid \xi_\alpha^U(\omega) \geq t\}) d\alpha. \quad (5)$$

By equation (4)-(5), we have

$$R(t) = \frac{1}{2}\int_0^1 \left(\prod_{i=1}^n \Pr\{\omega \in \Omega \mid \xi_{i,\alpha}^L(\omega) \geq t\} + \prod_{i=1}^n \Pr\{\omega \in \Omega \mid \xi_{i,\alpha}^U(\omega) \geq t\} \right) d\alpha. \quad (6)$$

Corollary 1. *If the fuzzy random variable* ξ_i *follows exponential distribution then the probability of series system using* (3) *is given by*,

$$\Pr\{\lambda \in \Omega \mid \xi_\alpha^L(\lambda) \geq t\} = \prod_{i=1}^n e^{-\lambda_{i,\alpha}^L \cdot t} = e^{-\sum_{i=1}^n \lambda_{i,\alpha}^L \cdot t}$$

$$\Pr\{\lambda \in \Omega \mid \xi_\alpha^U(\lambda) \geq t\} = \prod_{i=1}^n e^{-\lambda_{i,\alpha}^U . t} = e^{-\sum_{i=1}^n \lambda_{i,\alpha}^U . t} \quad (7)$$

and the reliability of series system using (6)-(7) is obtained as:

$$R(t) = \frac{1}{2}\int_0^1 (\Pr\{\lambda \in \Omega \mid \xi_\alpha^L(\lambda) > t\} + \Pr\{\lambda \in \Omega \mid \xi_\alpha^U(\lambda) > t\}) d\alpha$$

$$= \frac{1}{2}\int_0^1 (e^{-\sum_{i=1}^n \lambda_{i,\alpha}^L . t} = e^{-\sum_{i=1}^n \lambda_{i,\alpha}^U . t}) d\alpha \quad (8)$$

Proposition 2. *Let* ξ_i *be the lifetime of the* $i^{th}(i = 1, 2, 3, \cdots, n)$ *component and* $\omega \in \Omega$ *varies all over* Ω *and* β *is fixed, then the reliability function of system using* β*-cut is*,

$$R(t) = \frac{1}{2}\int_0^1 \left(\prod_{i=1}^n \Pr\{\omega \in \Omega \mid \xi_{i,\beta}^L(\omega) \geq t\} + \prod_{i=1}^n \Pr\{\omega \in \Omega \mid \xi_{i,\beta}^U(\omega) \geq t\} \right) d\beta$$

Proof. For any given $\beta \in [0, 1]$, ξ_β^L and ξ_β^U are the infimum and supremum values of β-cut of intuitionistic fuzzy random variable ξ of series system then the probability of system is defined as:

$$\Pr\{\omega \in \Omega \mid \xi_{\alpha}^{L}(\omega) \geq t\} = \prod_{i=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{i,\beta}^{L}(\omega) \geq t\},$$

$$\Pr\{\omega \in \Omega \mid \xi_{\beta}^{L}(\omega) \geq t\} = \prod_{i=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{i,\beta}^{L}(\omega) \geq t\}. \tag{9}$$

The reliability of unrepairable series system is defined as

$$R(t) = ch\{\xi \geq t\}$$

$$= \frac{1}{2}\int_{0}^{1} (\Pr\{\omega \in \Omega \mid \xi_{\beta}^{L}(\omega) \geq t\} + \Pr\{\omega \in \Omega \mid \xi_{\beta}^{U}(\omega) \geq t\}) d\beta. \tag{10}$$

By equation (9)-(10), the reliability is obtained as:

$$R(t) = \frac{1}{2}\int_{0}^{1} \left(\prod_{i=1}^{n} \Pr\{\lambda \in \Omega \mid \xi_{i,\beta}^{L}(\lambda) \geq t\} + \prod_{i=1}^{n} \Pr\{\lambda \in \Omega \mid \xi_{i,\beta}^{U}(\lambda) \geq t\} \right) d\beta. \tag{11}$$

Corollary 2. *Let the intuitionistic fuzzy random variable* ξ_i *follow exponential distribution then the probability of series system using* (3) *is as follows*:

$$\Pr\{\lambda \in \Omega \mid \xi_{\alpha}^{L}(\lambda) \geq t\} = \prod_{i=1}^{n} e^{-\lambda_{i,\alpha}^{L}.t} = e^{-\sum_{i=1}^{n} \lambda_{i,\beta}^{L}.t}$$

$$\Pr\{\lambda \in \Omega \mid \xi_{\beta}^{U}(\lambda) \geq t\} = \prod_{i=1}^{n} e^{-\lambda_{i,\beta}^{U}.t} = e^{-\sum_{i=1}^{n} \lambda_{i,\beta}^{U}.t} \tag{12}$$

and the reliability of series system using (11)-(12) is

$$R(t) = \frac{1}{2}\int_{0}^{1} (\Pr\{\lambda \in \Omega \mid \xi_{\beta}^{L}(\lambda) > t\} + \Pr\{\lambda \in \Omega \mid \xi_{\beta}^{U}(\lambda) > t\}) d\beta$$

$$= \frac{1}{2}\int_0^1 (e^{-\sum_{i=1}^{n}\lambda_{i,\beta}^{L}.t} + e^{-\sum_{i=1}^{n}\lambda_{i,\beta}^{U}.t})d\beta. \tag{13}$$

3.2. MTTF of Unrepairable Series System:

Proposition 3. *Assume that* ξ_i *be the intuitionistic fuzzy random lifetime of component* $i, i = 1, 2, 3, \cdots, n$. *For any given* $\omega \in \Omega$ *and* $\alpha \in [0, 1]$, *then the MTTF is*

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^1 (e^{-\sum_{i=1}^{n}\lambda_{i,\alpha}^{L}.t} + e^{-\sum_{i=1}^{n}\lambda_{i,\alpha}^{U}.t})d\alpha dt.$$

Proof. When ω varies all over Ω and α is fixed than the MTTF of series system is defined as

$$MTTF = \int_0^\infty R(t)dt$$

Using equations (5) and (6), the MTTF is

$$= \frac{1}{2}\int_0^\infty\int_0^1 (\Pr\{\omega \in \Omega \,|\, \xi_\alpha^L(\omega) \geq t\} + \Pr\{\omega \in \Omega \,|\, \xi_\alpha^L(\omega) \geq t\})$$

$$= \frac{1}{2}\int_0^\infty\int_0^1 \left(\prod_{i=1}^{n} \Pr\{\omega \in \Omega \,|\, \xi_{i,\alpha}^L(\omega) \geq t\} + \prod_{i=1}^{n} \Pr\{\omega \in \Omega \,|\, \xi_{i,\alpha}^U(\omega) \geq t\}\right) d\alpha dt$$

and let the intuitionistic fuzzy random variable ξ_i follow exponential distribution then using equation (8), we have

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^1 (e^{-\sum_{i=1}^{n}\lambda_{i,\alpha}^{L}.t} + e^{-\sum_{i=1}^{n}\lambda_{i,\alpha}^{U}.t})d\alpha dt.$$

Proposition 4. *Let* ξ_i *be the intuitionistic fuzzy random lifetime of component* $i, i = (1, 2, 3, \cdots, n)$. *For any given* $\omega \in \Omega$ *and* $\beta \in [0, 1]$, *then the MTTF of series system is given by*,

$$MTTF = \frac{1}{2}\int_0^\infty \int_0^1 (e^{-\sum_{i=1}^n \lambda_{i,\beta}^L . t} + e^{-\sum_{i=1}^n \lambda_{i,\beta}^U . t}) d\alpha dt.$$

Proof. When ω varies all over Ω, $\omega \in \Omega$ and β is fixed than the MTTF of series system is defined as

$$MTTF = \int_0^\infty R(t) dt.$$

Using equations (10) and (11), the MTTF is

$$= \frac{1}{2}\int_0^\infty \int_0^1 (\Pr\{\omega \in \Omega \mid \xi_\beta^L(\omega) \geq t\} + \Pr\{\omega \in \Omega \mid \xi_\beta^U(\omega) \geq t\}) d\beta dt$$

$$= \frac{1}{2}\int_0^\infty \int_0^1 \left(\prod_{i=1}^n \Pr\{\omega \in \Omega \mid \xi_{i,\beta}^L(\omega) \geq t\} + \prod_{i=1}^n \Pr\{\omega \in \Omega \mid \xi_{i,\beta}^U(\omega) \geq t\} \right) d\beta dt$$

and let the intuitionistic fuzzy random variable ξ_i follow exponential distribution then from equation (13), we have

$$MTTF = \frac{1}{2}\int_0^\infty \int_0^1 (e^{-\sum_{i=1}^n \lambda_{i,\beta}^L . t} + e^{-\sum_{i=1}^n \lambda_{i,\beta}^U . t}) d\beta dt$$

Remark 1. If X_i, $i = 1, 2, 3, \cdots, n$, be the random variable of i^{th} component, then the reliability of the form

$$R(t) = \prod_{i=1}^n \Pr\{\xi_i \geq t\}$$

which is the result in the stochastic case.

3.3. Reliability of unrepairable parallel system

Consider a parallel system consisting n components as shown in Figure 2.

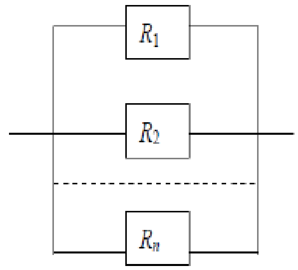

Figure 2. Parallel system.

Let ξ_i be the lifetime of the i^{th} component, which is a positive intuitionistic fuzzy random variable on the probability space, $i = 1, 2, 3, \cdots, n$. The lifetime of the parallel system is $\xi = \max\{\xi_1, \xi_2, \cdots, \xi_n\}$. Here ξ is an intuitionistic fuzzy random variable on the probability space $(\Omega, A, \Pr)$, where $\Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_n$, $A = A_1 \times A_2 \times \cdots \times A_n$ and $\Pr = \Pr_1 \times \Pr_2 \times \cdots \Pr_n$. Let $\alpha, \beta \in [0, 1]$, then $\xi^L_{i,\alpha}$ and $\xi^U_{i,\alpha}$ are infimum value and supremum value of α-cut and $\xi^L_{i,\beta}$, $\xi^U_{i,\beta}$ are infimum and supremum value of β-cut of intuitionistic fuzzy random variable.

Proposition 5. Let ξ_i be the lifetime of the $i^{th}(1, 2, 3, \cdots, n)$. component and $\omega \in \Omega$ varies all over Ω and α is fixed, then the reliability of system is given by,

$$R(t) = \frac{1}{2}\int_0^1 \Bigg(1 - \prod_{i=1}^{n} [1 - \Pr\{\omega \in \Omega \mid \xi^L_{i,\alpha}(\omega) \geq t\}$$

$$+ 1 - \prod_{i=1}^{n} [1 - \Pr\{\omega \in \Omega \mid \xi^U_{i,\alpha}(\omega) \geq t\}]\Bigg) d\alpha.$$

Proof. For any given $\alpha \in [0, 1]$, ξ^L_α and ξ^U_α are the infimum and supremum values of α-cut of ξ, which is intuitionistic fuzzy random variable then the probability of system is

$$\Pr\{\omega \in \Omega \mid \xi_{\alpha}^{L}(\omega) \geq t\} = 1 - \prod_{i=1}^{n} [1 - \Pr\{\omega \in \Omega \mid \xi_{i,\alpha}^{L}(\omega) \geq t\}],$$

$$\Pr\{\omega \in \Omega \mid \xi_{\alpha}^{U}(\omega) \geq t\} = 1 - \prod_{i=1}^{n} [1 - \Pr\{\omega \in \Omega \mid \xi_{i,\alpha}^{U}(\omega) \geq t\}]. \tag{14}$$

The reliability of unrepairable parallel system using α-cut and equations (2) is defined as:

$$R(t) = ch\{\xi \geq t\}$$

$$R(t) = \frac{1}{2}\int_{0}^{1} (\Pr\{\omega \in \Omega \mid \xi_{\alpha}^{L}(\omega) \geq t\} + \Pr\{\omega \in \Omega \mid \xi_{\alpha}^{U}(\omega) \geq t\}) d\alpha \tag{15}$$

By equation (14)-(15), the reliability is obtained as:

$$R(t) = \frac{1}{2}\int_{0}^{1} \left(1 - \prod_{i=1}^{n} [1 - \Pr\{\omega \in \Omega \mid \xi_{\alpha}^{L}(\omega) \geq t\}] \right.$$

$$\left. + 1 - \prod_{i=1}^{n} [1 - \Pr\{\omega \in \Omega \mid \xi_{\alpha}^{L}(\omega) \geq t\}] \right) d\alpha. \tag{16}$$

Corollary 3. *Let the intuitionistic fuzzy random variable* ξ_i *exponentially distributed then the probability of parallel system using* (3) *is as follows*:

$$\Pr\{\lambda \in \Omega \mid \xi_{\alpha}^{L}(\lambda) \geq t\} = 1 - \prod_{i=1}^{n} [1 - e^{-\lambda_{i,\alpha}^{U} . t}],\ \Pr\{\lambda \in \Omega \mid \xi_{i,\beta}^{U}(\lambda) \geq t\} = 1$$

$$\prod_{i=1}^{n} [1 - e^{\lambda_{i,\alpha}^{L} . t}]. \tag{17}$$

The reliability of parallel system is:

$$R(t) = \frac{1}{2}\int_{0}^{1} \left(1 - \prod_{i=1}^{n} [1 - \Pr\{\lambda \in \Omega \mid \xi_{i,\alpha}^{L}(\lambda) \geq t\}] \right.$$

$$+1-\prod_{i=1}^{n}[1-\Pr\{\lambda\in\Omega\,|\,\xi_{i,\alpha}^{U}(\lambda)\geq t\}]\Bigg)d\alpha$$

using (17), the reliability is obtained as:

$$=\frac{1}{2}\int_{0}^{1}\left(1-\prod_{i=1}^{n}[1-e^{-\lambda_{i,\alpha}^{U}.t}]+1-\prod_{i=1}^{n}[1-e^{-\lambda_{i,\alpha}^{U}.t}]\right)d\alpha. \tag{18}$$

Proposition 6. *Let* ξ_i *be the lifetime of the* $i^{th}(1, 2, 3, \cdots, n)$ *component and* $\omega\in\Omega$ *varies all over* Ω *and* β *is fixed, then the reliability of system is given by,*

$$R(t)=\frac{1}{2}\int_{0}^{1}\Bigg(1-\prod_{i=1}^{n}[1-\Pr\{\omega\in\Omega\,|\,\xi_{i,\beta}^{L}(\omega)\geq t\}]$$

$$+1-\prod_{i=1}^{n}[1-\Pr\{\omega\in\Omega\,|\,\xi_{i,\beta}^{U}(\omega)\geq t\}]\Bigg)d\beta.$$

Proof. For any given $\beta\in[0, 1]$, ξ_{β}^{L} and ξ_{β}^{U} are the infimum and supremum values of β-cut of intuitionistic fuzzy random variable ξ then the probability of unrepairable parallel system is given by,

$$\Pr\{\omega\in\Omega\,|\,\xi_{\beta}^{L}(\omega)\geq t\}=1-\prod_{i=1}^{n}[1-\Pr\{\omega\in\Omega\,|\,\xi_{i,\beta}^{L}(\omega)\geq t\}],$$

$$\Pr\{\omega\in\Omega\,|\,\xi_{\beta}^{U}(\omega)\geq t\}=1-\prod_{i=1}^{n}[1-\Pr\{\omega\in\Omega\,|\,\xi_{i,\beta}^{U}(\omega)\geq t\}]. \tag{19}$$

The reliability of unrepairable parallel system using β-cut and equations (2) is defined as:

$$R(t)=ch\{\xi\geq t\}$$

$$R(t)=\frac{1}{2}\int_{0}^{1}(\Pr\{\omega\in\Omega\,|\,\xi_{\beta}^{L}(\omega)\geq t\}+\Pr\{\omega\in\Omega\,|\,\xi_{\beta}^{U}(\omega)\geq t\})d\beta. \tag{20}$$

Using equations (19)-(20), the reliability is obtained as:

$$R(t) = \frac{1}{2}\int_0^1 (1 - \prod_{i=1}^{n} [1 - \Pr\{\omega \in \Omega \mid \xi_{i,\beta}^{L}(\omega) \geq t\}$$

$$+ 1 - \prod_{i=1}^{n} [1 - \Pr\{\omega \in \Omega \mid \xi_{i,\beta}^{U}(\omega) \geq t\}])d\beta \qquad (21)$$

Corollary 4. *Assume the intuitionistic fuzzy random variable* ξ_i *exponentially distributed then the probability of parallel system using* (3) *is as follows*:

$$\Pr\{\lambda \in \Omega | \xi_{\beta}^{L}(\lambda) \geq t\} = 1 - \prod_{i=1}^{n} [1 - e^{-\lambda_{i,\beta}^{U}.t}],\ \Pr\{\lambda \in \Omega | \xi_{\beta}^{U}(\lambda) \geq t\} = 1 - \prod_{i=1}^{n} [1 - e^{\lambda_{i,\beta}^{L}.t}]. \quad (22)$$

The reliability of parallel system is:

$$R(t) = \frac{1}{2}\int_0^1 \left(1 - \prod_{i=1}^{n} [1 - \Pr\{\lambda \in \Omega | \xi_{i,\beta}^{L}(\lambda) \geq t\}] + 1 - \prod_{i=1}^{n} [1 - \Pr\{\lambda \in \Omega | \xi_{i,\beta}^{U}(\lambda) \geq t\}]\right) d\beta$$

using (22), *the reliability is obtained as*:

$$= \frac{1}{2}\int_0^1 \left(1 - \prod_{i=1}^{n} [1 - e^{-\lambda_{i,\beta}^{U}.t}] + 1 - \prod_{i=1}^{n} [1 - e^{-\lambda_{i,\beta}^{U}.t}]\right) d\beta \qquad (23)$$

3.4. MTTF of Unrepairable Parallel System:

Proposition 7. *Let* ξ_i *be the intuitionistic fuzzy random lifetime of any* i^{th} *component. For any given* $\omega \in \Omega$, ω *varies all over* Ω *and* $\alpha \in [0, 1]$, *then the MTTF of unrepairable parallel system is given by*,

$$MTTF = \frac{1}{2}\int_0^{\infty}\int_0^1 \left(1 - \prod_{i=1}^{n} [1 - e^{\lambda_{i,\alpha}^{U}.t}] + 1 - \prod_{i=1}^{n} [1 - e^{\lambda_{i,\alpha}^{U}.t}]\right) d\alpha dt.$$

Proof. When ω varies all over Ω, $\omega \in \Omega$ and α is fixed than the MTTF of parallel system is defined as

$$MTTF = \int_0^\infty R(t)dt.$$

Using equations (15) and (16), the MTTF is

$$= \frac{1}{2}\int_0^\infty\int_0^1 (\Pr\{\omega \in \Omega | \xi_\beta^L(\omega) \geq t\} + \Pr\{\omega \in \Omega | \xi_\beta^U(\omega) \geq t\}) d\beta dt \ 1$$

$$MTTF = \frac{1}{2}\int_0^\infty \left(\int_0^1 1 - \prod_{i=1}^n [1 - \Pr\{\omega \in \Omega | \xi_{i,\alpha}^L(\omega) \geq t\}] \right.$$

$$\left. + 1 - \prod_{i=1}^n [1 - \Pr\{\omega \in \Omega | \xi_{i,\beta}^U(\omega) \geq t\}] \right) d\alpha dt.$$

Let the intuitionistic fuzzy random variable ξ_i follow exponential distribution then from equation (18), we have

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^1 \left(1 - \prod_{i=1}^n [1 - e^{\lambda_{i,\alpha}^U . t}] + 1 - \prod_{i=1}^n [1 - e^{\lambda_{i,\alpha}^U . t}]\right) d\beta dt.$$

Proposition 8. *Let* ξ_i *be the intuitionistic fuzzy random lifetime of any* i^{th} *component. For any given* $\omega \in \Omega$, ω *varies all over* Ω *and* $\beta \in [0, 1]$, *then the MTTF of unrepairable parallel system is given by*,

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^1 \left(1 - \prod_{i=1}^n [1 - e^{\lambda_{i,\beta}^U . t}] + 1 - \prod_{i=1}^n [1 - e^{\lambda_{i,\beta}^U . t}]\right) d\beta dt.$$

Proof. When ω varies all over Ω, $\omega \in \Omega$ and β is fixed than the MTTF of parallel system is defined as

$$MTTF = \int_0^\infty R(t)dt$$

Using equations (20) and (21), the MTTF is

$$= \frac{1}{2}\int_0^\infty\int_0^1 (\Pr\{\omega \in \Omega \mid \xi_\beta^L(\omega) \ge t\} + \Pr\{\omega \in \Omega \mid \xi_\beta^U(\omega) \ge t\}) d\beta dt$$

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^1 \left(1 - \prod_{i=1}^n [1 - \Pr\{\omega \in \Omega \mid \xi_{i,\alpha}^L(\omega) \ge t\}]\right.$$

$$\left. + 1 - \prod_{i=1}^n [1 - \Pr\{\omega \in \Omega \mid \xi_{i,\alpha}^U(\omega) \ge t\}]\right) d\alpha dt.$$

Consider the intuitionistic fuzzy random variable ξ_i follow exponential distribution then from equation (23), we have

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^1 \left(1 - \prod_{i=1}^n [1 - e^{-\lambda_{i,\beta}^U . t}] + 1 - \prod_{i=1}^n [1 - e^{-\lambda_{i,\beta}^U . t}]\right) d\beta dt.$$

Remark 2. If X_i, $i = 1, 2, 3, \cdots, n$, be the random variable of i^{th} component, then the reliability of the form

$$R(t) = 1 - \prod_{i=1}^n [1 - \Pr\{X_i \ge t\}]$$

which is the result in the stochastic case.

3.5. Reliability of Unrepairable Parallel-Series System

Consider a parallel-series system which is a parallel system of m subsystems; each subsystem which is composed of n series components as shown in Figure 3.

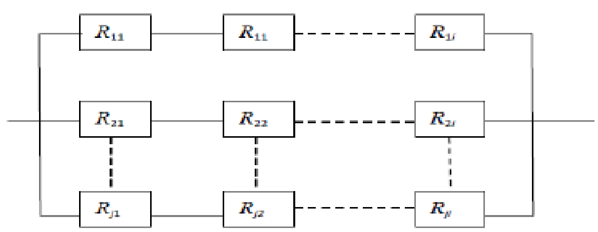

Figure 3. Parallel-Series System.

Let ξ_{ij} be the lifetime of j^{th} component in the i^{th} subsystem, which is a positive intuitionistic fuzzy random variable on the probability space $(\Omega_{ij}, A_{ij}, \Pr_{ij})$, $i = 1, 2, 3, \cdots, m$, $j = 1, 2, \cdots, n$. The lifetime of the parallel system is $\xi = \max_{1 \le i \le m}(\min_{1 \le j \le n} \xi_{ij})$, where ξ_{ij} is an intuitionistic fuzzy random variable on the probability space $(\Omega, A, \Pr)$, where $\Omega = \Omega_{11} \times \Omega_{12} \times \cdots \times \Omega_{mn}$, $A = A_{11} \times A_{12} \times \cdots \times A_{mn}$ an $\Pr = \Pr_{11} \times \Pr_{12} \times \cdots \times \Pr_{mn}$. Let $\alpha, \beta \in [0, 1]$, then $\xi^L_{ij, \alpha}$ and $\xi^U_{ij, \alpha}$ are infimum value and supremum value of α-cut and $\xi^L_{ij, \beta}$, $\xi^U_{ij, \beta}$ are infimum and supremum value of β-cut of intuitionistic fuzzy random variable ξ_{ij} respectively.

Proposition 9. *Let* ξ_{ij} *be the lifetime of the* $i^{th} = (i = 1, 2, 3, \cdots, n)$ *component and* $\omega \in \Omega$ *varies all over* Ω *and* α *is fixed, then the reliability of system is given by*,

$$R(t) = \frac{1}{2}\int_0^1 \left(1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega | \xi^L_{ij,\alpha}(\omega) \ge t\}]\right.$$

$$\left. + 1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega | \xi^U_{ij,\alpha}(\omega) \ge t\}]\right) d\alpha.$$

Proof. For any given $\alpha \in [0, 1]$, ξ^L_α and ξ^U_α are infimum and supremum values of α-cut of ξ, which is intuitionistic fuzzy random variable then the probability of system is defined as,

$$\Pr\{\omega \in \Omega \,|\, \xi^L_\alpha(\omega) \ge t\} = 1 - \prod_{i=1}^{n}\left[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \,|\, \xi^L_{ij,\alpha}(\omega) \ge t\}\right],$$

$$\Pr\{\omega \in \Omega \,|\, \xi^U_\alpha(\omega) \ge t\} = 1 - \prod_{i=1}^{n}\left[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \,|\, \xi^U_{ij,\alpha}(\omega) \ge t\}\right]. \quad (24)$$

The reliability of unrepairable parallel-series system using α-cut and equation (2) is

$$R(t) = ch\{\xi \geq t\}$$

$$R(t) = \frac{1}{2}\int_0^1 (\Pr\{\omega \in \Omega \mid \xi_\alpha^L(\omega) \geq t\} + \Pr\{\omega \in \Omega \mid \xi_\alpha^U(\omega) \geq t\}) d\alpha \tag{25}$$

By equations (24)-(25), the reliability is obtained as:

$$R(t) = \frac{1}{2}\int_0^1 \left(1 - \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\alpha}^L(\omega) \geq t\}\right],\right.$$

$$\left. + 1 - \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\alpha}^U(\omega) \geq t\}\right]\right) d\alpha. \tag{26}$$

Corollary 5. *Assume the intuitionistic fuzzy random variable* ξ_i *exponentially distributed then the probability of parallel-series system using* (3) *is as follows*:

$$\Pr\{\lambda \in \Omega \mid \xi_{ij,\alpha}^L(\lambda) \geq t\} = 1 - \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n} e^{-\lambda_{ij,\alpha}^L . t}\right],$$

$$\Pr\{\lambda \in \Omega \mid \xi_{ij,\alpha}^L(\lambda) \geq t\} = 1 - \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n} e^{-\lambda_{ij,\alpha}^U . t}\right], \tag{27}$$

and the reliability of parallel-series system using (27) is obtained as:

$$R(t) = \frac{1}{2}\int_0^1 \left(1 - \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n} \Pr\{\lambda \in \Omega \mid \xi_{ij,\alpha}^L(\lambda) \geq t\}\right]\right.$$

$$\left. + 1 - \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n} \Pr\{\lambda \in \Omega \mid \xi_{ij,\alpha}^U(\lambda) \geq t\}\right]\right) d\alpha$$

$$= \frac{1}{2}\int_0^1 \left(1 - \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n} e^{-\lambda_{ij,\alpha}^L . t}\right] + 1 - \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n} e^{-\lambda_{ij,\alpha}^U . t}\right]\right) d\alpha. \tag{28}$$

Proposition 10. *Let* ξ_i *be the lifetime of the* $i^{th} (i = 1, 2, 3, \cdots, n)$ *component and* $\omega \in \Omega$ *varies all over* Ω *and* α *is fixed, then the reliability of system is given by,*

$$R(t) = \frac{1}{2}\int_0^1 (1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\beta}^{L}(\omega) \geq t\}]$$

$$+ 1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\beta}^{U}(\omega) \geq t\}])d\beta.$$

Proof. For any given $\beta \in [0, 1], \xi_\beta^L$ and ξ_β^U are infimum and supremum values of β-cut of intuitionistic fuzzy random variable ξ then the probability of parallel-series system is defined as,

$$\Pr\{\omega \in \Omega \mid \xi_\beta^L(\omega) \geq t\} = 1 - \prod_{i=1}^{n}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\beta}^{L}(\omega) \geq t\}],$$

$$\Pr\{\omega \in \Omega \mid \xi_\beta^U(\omega) \geq t\} = 1 - \prod_{i=1}^{n}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\beta}^{U}(\omega) \geq t\}]. \quad (29)$$

The reliability of unrepairable parallel-series system using β-cut and equation (24) is obtained as:

$$R(t) = ch\{\xi \geq t\}$$

$$R(t) = \frac{1}{2}\int_0^1 (\Pr\{\omega \in \Omega \mid \xi_\beta^L(\omega) \geq t\} + \Pr\{\omega \in \Omega \mid \xi_\beta^U(\omega) \geq t\})d\beta. \quad (30)$$

Using equations (29)-(30), the reliability of above system is obtained as:

$$R(t) = \frac{1}{2}\int_0^1 (1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\beta}^{L}(\omega) \geq t\}]$$

$$+ 1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\beta}^{U}(\omega) \geq t\}])d\beta.$$

Corollary 6. *Assume that the intuitionistic fuzzy random variable* ξ_i *is exponentially distributed then the probability of parallel-series system using* (3) *is as follows*:

$$\Pr\{\lambda \in \Omega \mid \xi_\beta^L(\lambda) \geq t\} = 1 - \prod_{i=1}^{m} [1 - \prod_{j=1}^{n} [e^{-\lambda_{ij,\beta}^L . t}],$$

$$\Pr\{\lambda \in \Omega \mid \xi_\beta^U(\lambda) \geq t\} = 1 - \prod_{i=1}^{m} [1 - \prod_{j=1}^{n} e^{-\lambda_{ij,\beta}^U . t}] \tag{31}$$

and the reliability of parallel system is:

$$R(t) = \frac{1}{2}\int_0^1 (\Pr\{\omega \in \Omega \mid \xi_\beta^L(\omega) \geq t\} + \Pr\{\omega \in \Omega \mid \xi_\beta^U(\omega) \geq t\}) d\beta$$

$$R(t) = \frac{1}{2}\int_0^1 (1 - \prod_{i=1}^{m} [1 - \prod_{j=1}^{n} \Pr\{\lambda \in \Omega \mid \xi_{ij,\beta}^L(\lambda) \geq t\}]$$

$$+ 1 - \prod_{i=1}^{m} [1 - \prod_{j=1}^{n} \Pr\{\lambda \in \Omega \mid \xi_{ij,\beta}^U(\omega) \geq t\}]) d\beta$$

using (31), the reliability is obtained as

$$R(t) = \frac{1}{2}\int_0^1 \left(1 - \prod_{i=1}^{m} [1 - \prod_{j=1}^{n} e^{-\lambda_{ij,\beta}^L . t}] + 1 - \prod_{i=1}^{m} [1 - \prod_{j=1}^{n} e^{-\lambda_{ij,\beta}^U . t}] \right) d\beta \tag{32}$$

3.6. MTTF of Unrepairable Parallel-Series System:

Proposition 11. *Let* ξ_i *be the intuitionistic fuzzy random lifetime of any* i^{th} *component. For any given* $\omega \in \Omega, \omega$ *varies all over* Ω *and* $\alpha \in [0, 1]$, *then the MTTF of unrepairable parallel-series system is given by*,

$$MTTF = \frac{1}{2}\int_0^\infty \int_0^1 \left(1 - \prod_{i=1}^{m} [1 - \prod_{j=1}^{n} e^{-\lambda_{ij,\alpha}^L . t}] + 1 - \prod_{i=1}^{m} [1 - \prod_{j=1}^{n} e^{-\lambda_{ij,\alpha}^U . t}] \right) d\alpha dt.$$

Proof. When ω varies all over Ω, $\omega \in \Omega$ and α is fixed than the MTTF of parallel system is defined as

$$MTTF = \int_0^\infty R(t)dt.$$

Consider the intuitionistic fuzzy random variable ξ_i follow exponential distribution then from equation (28), we have

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^\infty (1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\alpha}^{U}(\omega) \geq t\}]$$

$$+ 1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\alpha}^{U}(\omega) \geq t\}])$$

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^1 \left(1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} e^{\lambda_{i,\alpha}^{L}.t}] + 1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} e^{\lambda_{i,\alpha}^{U}.t}]\right) d\alpha dt.$$

Proposition 12. *Let* ξ_i *be the intuitionistic fuzzy random lifetime of any* i^{th} *component. For any given* $\omega \in \Omega$, ω *varies all over* Ω *and* $\beta \in [0, 1]$, *then the MTTF of unrepairable parallel-series system is given by*,

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^1 \left(1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} e^{\lambda_{i,\beta}^{L}.t}] + 1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} e^{\lambda_{i,\beta}^{U}.t}]\right) d\alpha dt.$$

Proof. When ω varies all over Ω, $\omega \in \Omega$ and β is fixed than the MTTF of parallel system is defined as

$$MTTF = \int_0^\infty R(t)dt.$$

Consider the intuitionistic fuzzy random variable ξ_i follow exponential distribution then from equation (32), we have

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^\infty (1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\beta}^{U}(\omega) \geq t\}]$$

$$+ 1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi_{ij,\beta}^{U}(\omega) \geq t\}])d\beta$$

$$MTTF = \frac{1}{2}\int_0^\infty\int_0^1 \left(1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} e^{\lambda_{i,\beta}^{L}.t}] + 1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} e^{\lambda_{i,\beta}^{U}.t}]\right) d\beta dt.$$

Remark 3. If ξ_{ij}, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$, be the random variable of j^{th} component in the i^{th} subsystem, then the reliability in the stochastic case is as follows:

$$R(t) = \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} \Pr\{\xi_{ij} \geq t\}]$$

3.7. Reliability of Unrepairable Series-Parallel System

Consider a series-parallel system which is a series system of m subsystems; each subsystem which is composed of n parallel components as shown in Fig. 4.

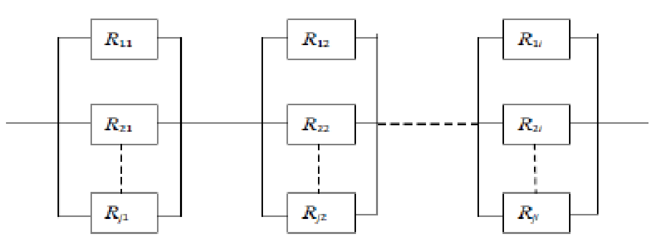

Figure 4. Series-Parallel System.

Let ξ_{ij} be the lifetime of j^{th} component in the i^{th} subsystem, which is a positive intuitionistic fuzzy random variable on the probability space $(\Omega_{ij}, A_{ij}, \Pr_{ij})$, $i = 1, 2, 3, \cdots, m$, $j = 1, 2, \cdots, n$. The lifetime of the series-parallel system is $\xi = \max_{1 \leq i \leq m}(\min_{1 \leq j \leq n} \xi_{ij})$, where ξ_{ij} is an intuitionistic fuzzy random variable on the probability space $(\Omega, A, \Pr)$, where $\Omega = \Omega_{11} \times \Omega_{12} \times \cdots \times \Omega_{mn}$, $A = A_{11} \times A_{12} \times \cdots \times A_{mn}$ and $\Pr = \Pr_{11} \times \Pr_{12} \times \cdots \times \Pr_{mn}$. Let $\alpha, \beta \in [0, 1]$, then $\xi^L_{ij,\alpha}$ and $\xi^U_{ij,\alpha}$ are infimum value and supremum value of α-cut and $\xi^L_{ij,\beta}, \xi^U_{ij,\beta}$ are infimum and supremum value of β-cut of intuitionistic fuzzy random variable ξ_{ij} respectively.

Proposition 13. *Let* ξ_i *be the lifetime of the* $i^{th} = (i = 1, 2, 3, \cdots, n)$ *component and* $\omega \in \Omega$ *varies all over* Ω *and* α *is fixed, then the reliability of series-parallel system is given by*,

$$R(t) = \frac{1}{2}\int_0^1 \{\prod_{i=1}^{m}(1 - \prod_{j=1}^{n} \Pr\{\omega \in \Omega \mid \xi^L_{ij,\alpha}(\omega) \geq t\}])$$

$$+ \prod_{i=1}^{m}(1 - \prod_{j=1}^{n}[1 - \Pr\{\omega \in \Omega \mid \xi^L_{ij,\alpha}(\omega) \geq t\}])d\alpha.$$

Proof. For any given $\alpha \in [0, 1]$, ξ^L_α and ξ^U_α are infimum and supremum values of α-cut of intuitionistic fuzzy random variable ξ, when $\omega \in \Omega$ varies and α is fixed, then the probability of system is defined by,

$$\Pr\{\omega \in \Omega \mid \xi^L_\alpha(\omega) \geq t\} = \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n}[1 - \Pr\{\omega \in \Omega \mid \xi^L_{ij,\alpha}(\omega) \geq t\}]\right),$$

$$\Pr\{\omega \in \Omega \mid \xi^U_\alpha(\omega) \geq t\} = \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n}[1 - \Pr\{\omega \in \Omega \mid \xi^L_{ij,\alpha}(\omega) \geq t\}]\right). \quad (33)$$

The reliability of series-parallel system using α-cut of intuitionistic fuzzy random lifetimes and equations (2) is:

$$R(t) = ch\{\xi \geq t\}$$

$$R(t) = \frac{1}{2}\int_0^1 (\Pr\{\omega \in \Omega \mid \xi^L_\alpha(\omega) \geq t\} + \Pr\{\omega \in \Omega \mid \xi^U_\alpha(\omega) \geq t\})d\alpha. \quad (34)$$

Using equations (33)-(34), the reliability of above system is obtained as:

$$R(t) = \frac{1}{2}\int_0^1 \{\prod_{i=1}^{m}(1 - \prod_{j=1}^{n}[1 - \Pr\{\omega \in \Omega \mid \xi_{ij,\alpha}^{L}(\omega) \geq t\}])$$

$$+ 1 - \prod_{i=1}^{m}(1 - \prod_{j=1}^{n}[1 - \Pr\{\omega \in \Omega \mid \xi_{ij,\alpha}^{L}(\omega) \geq t\}])\} d\alpha.$$

Corollary 7. *Assume that the intuitionistic fuzzy random variable* ξ_i *follow exponential distribution then the probability of series-parallel system using* (3) *is*

$$\Pr\{\lambda \in \Omega \mid \xi_{ij,\alpha}^{L}(\lambda) \geq t\} = \prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(1 - e^{-\lambda_{ij,\alpha}^{L}.t})],$$

$$\Pr\{\lambda \in \Omega \mid \xi_{ij,\alpha}^{U}(\lambda) \geq t\} = \prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(1 - e^{-\lambda_{ij,\alpha}^{U}})]. \tag{35}$$

The reliability of series-parallel system using (35) is obtained as:

$$R(t) = \frac{1}{2}\int_0^1 (\prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(1 - \Pr\{\lambda \in \Omega \mid \xi_{ij,\alpha}^{L}(\lambda) \geq t\}])$$

$$+ \prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(1 - \Pr\{\lambda \in \Omega \mid \xi_{ij,\alpha}^{U}(\omega) \geq t\})]\} d\alpha$$

$$= \frac{1}{2}\int_0^1 \left(\prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(e^{-\lambda_{ij,\beta}^{L}.t})] + \prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(e^{-\lambda_{ij,\beta}^{U}.t})]\right) d\alpha. \tag{36}$$

Proposition 14. *Assume that the intuitionistic fuzzy random lifetime* ξ_i *of any component follow exponential distribution then the probability of series-parallel system using* (3) *is as follows*:

$$R(t) = \frac{1}{2}\int_0^1 \{\prod_{i=1}^{m}(1 - \prod_{j=1}^{n}[1 - \Pr\{\omega \in \Omega \mid \xi_{ij,\beta}^{L}(\omega) \geq t\}]),$$

$$+\prod_{i=1}^{m}(1-\prod_{j=1}^{n}[1-\Pr\{\omega \in \Omega \,|\, \xi_{ij,\beta}^{L}(\omega) \geq t\}])\}d\beta.$$

Proof. For any given $\beta \in [0, 1]$, ξ_{β}^{L} and ξ_{β}^{U} are infimum and supremum values of β-cut of intuitionistic fuzzy random variable ξ, when $\omega \in \Omega$ varies and β is fixed, then the probability of system is as follows:

$$\Pr\{\omega \in \Omega \,|\, \xi_{\beta}^{L}(\omega) \geq t\} = \prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}[1-\Pr\{\omega \in \Omega \,|\, \xi_{ij,\beta}^{L}(\omega) \geq t\}]\right),$$

$$\Pr\{\omega \in \Omega \,|\, \xi_{\beta}^{U}(\omega) \geq t\} = \prod_{i=1}^{n}\left(1-\prod_{j=1}^{n}[1-\Pr\{\omega \in \Omega \,|\, \xi_{ij,\beta}^{U}(\omega) \geq t\}]\right). \quad (37)$$

The reliability of series-parallel system using β-cut of intuitionistic fuzzy random lifetimes and equations (2) is defined as:

$$R(t) = ch\{\xi \geq t\}$$

$$R(t) = \frac{1}{2}\int_{0}^{1}(\Pr\{\omega \in \Omega \,|\, \xi_{\beta}^{L}(\omega) \geq t\} + \Pr\{\omega \in \Omega \,|\, \xi_{\beta}^{U}(\omega) \geq t\})d\beta. \quad (38)$$

The reliability of above system using equations (37)-(38) is obtained as:

$$R(t) = \frac{1}{2}\int_{0}^{1}\{\prod_{i=1}^{m}(1-\prod_{j=1}^{n}[1-\Pr\{\omega \in \Omega \,|\, \xi_{ij,\beta}^{L}(\omega) \geq t\}])$$

$$+\prod_{i=1}^{m}(1-\prod_{j=1}^{n}[1-\Pr\{\lambda \in \Omega \,|\, \xi_{ij,\beta}^{L}(\omega) \geq t\})]\}d\beta.$$

Corollary 8. *Assume that the intuitionistic fuzzy random variable* ξ_i *follow exponential distribution then the probability of series-parallel system using* (3) *is as follows*:

$$\Pr\{\lambda \in \Omega \,|\, \xi_{ij,\beta}^{L}(\lambda) \geq t\} = \prod_{i=1}^{m}[1-\prod_{j=1}^{n}(1-e^{-\lambda_{ij,\beta}^{L}.t})],$$

$$\Pr\{\lambda \in \Omega \,|\, \xi^{U}_{ij,\beta}(\lambda) \geq t\} = \prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(1 - e^{-\lambda^{U}_{ij,\beta}.t})]. \quad (39)$$

The reliability of series-parallel system using (39) is given by:

$$R(t) = \frac{1}{2}\int_{0}^{1}(\prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(1 - \Pr\{\lambda \in \Omega \,|\, \xi^{L}_{ij,\beta}(\lambda) \geq t\}])$$

$$+ \prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(1 - \Pr\{\lambda \in \Omega \,|\, \xi^{U}_{ij,\beta}(\lambda) \geq t\})]\}d\beta$$

$$= \frac{1}{2}\int_{0}^{1}\left(1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(e^{-\lambda^{L}_{ij,\beta}.t})] + 1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(e^{-\lambda^{U}_{ij,\beta}.t})]\right)d\beta. \quad (40)$$

3.8. MTTF of Unrepairable Series-Parallel System

Proposition 15. *Let* ξ_i *be the intuitionistic fuzzy random lifetime of any* i^{th} *component. For any given* $\omega \in \Omega$, ω *varies all over* Ω *and* $\alpha \in [0, 1]$, *then the MTTF of unrepairable series-parallel system is given by*,

$$MTTF = \frac{1}{2}\int_{0}^{\infty}\int_{0}^{1}\left(\prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(1 - e^{\lambda^{L}_{i,\alpha}.t})] + \prod_{i=1}^{m}[1 - \prod_{j=1}^{n}(1 - e^{\lambda^{U}_{i,\alpha}.t}]\right)d\alpha dt.$$

Proof. When ω varies all over Ω, $\omega \in \Omega$ and α is fixed than the MTTF of series-parallel system is defined as

$$MTTF = \int_{0}^{\infty} R(t)dt.$$

Consider the intuitionistic fuzzy random variable ξ_{ij} follow exponential distribution then from equation (36), we have

$$MTTF = \frac{1}{2}\int_{0}^{\infty}\int_{0}^{1}\{\prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n}[1 - \Pr\{\lambda \in \Omega \,|\, \xi^{L}_{ij,\alpha}(\lambda) \geq t\}]\right)$$

$$+\prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}[1-\Pr\{\lambda\in\Omega \mid \xi_{ij,\beta}^{U}(\lambda)\geq t\}]\right)d\alpha dt$$

$$MTTF=\frac{1}{2}\int_{0}^{\infty}\int_{0}^{1}\left(\prod_{i=1}^{m}[1-\prod_{j=1}^{n}(1-e^{\lambda_{i,\alpha}^{L}.t})]+\prod_{i=1}^{m}[1-\prod_{j=1}^{n}(1-e^{\lambda_{i,\alpha}^{U}.t})]\right)d\beta dt.$$

Proposition 16. *Let ξ_i be the intuitionistic fuzzy random lifetime of any i^{th} component. For any given $\omega\in\Omega$, ω varies all over Ω and $\beta\in[0, 1]$, then the MTTF of unrepairable series-parallel system is given by,*

$$MTTF=\frac{1}{2}\int_{0}^{\infty}\int_{0}^{1}\left(\prod_{i=1}^{m}[1-\prod_{j=1}^{n}(1-e^{\lambda_{i,\beta}^{L}.t})]+\prod_{i=1}^{m}[1-\prod_{j=1}^{n}(1-e^{\lambda_{i,\beta}^{U}.t})]\right)d\beta dt$$

Proof. When ω varies all over Ω, $\omega\in\Omega$ and α is fixed than the MTTF of series-parallel system is defined as

$$MTTF=\int_{0}^{\infty}R(t)dt.$$

Consider the intuitionistic fuzzy random variable ξ_{ij} follow exponential distribution then from equation (40), we have

$$MTTF=\frac{1}{2}\int_{0}^{\infty}\int_{0}^{1}\{\prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}[1-\Pr\{\lambda\in\Omega \mid \xi_{ij,\beta}^{L}(\lambda)\geq t\}]\right),$$

$$+\prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}[1-\Pr\{\lambda\in\Omega \mid \xi_{ij,\beta}^{U}(\lambda)\geq t\}]\right)d\beta dt$$

$$MTTF=\frac{1}{2}\int_{0}^{\infty}\int_{0}^{1}\left(\prod_{i=1}^{m}[1-\prod_{j=1}^{n}(1-e^{-\lambda_{i,\beta}^{L}.t})]+\prod_{i=1}^{m}[1-\prod_{j=1}^{n}(1-e^{-\lambda_{i,\beta}^{U}.t})]\right)d\beta dt.$$

Remark 4. If ξ_{ij}, 1, 2, $\cdots$, m, $j=1, 2, \cdots, n$, be the random variable of j^{th} component in the i^{th} subsystem, then the reliability in the stochastic case is as follows

$$R(t)\prod_{i=1}^{m}\left(1-\prod_{j=1}^{n}[1-\Pr(\xi_{ij}\geq t)]\right).$$

Numerical Example. Consider a stereo hi-fi system consisting four components (1) FM tuner; (2) speaker A, (3) speaker B and (4) amplifier as shown in Figure 5. The lifetimes of these components denoted by ξ_1, ξ_2, ξ_3 and ξ_4 respectively and the lifetime of the whole system is denoted by ξ. We assume that intuitionistic fuzzy random variables ξ_1, ξ_2, ξ_3 and ξ_4 are exponentially distributed, $\xi_i(\lambda_i)\exp(\lambda_i), i=1,2,3,$ where $\lambda_1=(0.3,0.5,0.7;0.2,0.5,0.8)$, $\lambda_2=(0.2,0.5,0.8;0.1,0.5,0.9)$, $\lambda_3=(0.2,0.4,0.6;0.1,0.4,0.7)$ and $\lambda_4(0.4,0.6,0.8;0.2,0.6,0.9)$ then we can obtain the α-cut and β-cut of λ_i, $i=1, 2, 3, 4.$ α-cut is obtained as:

$$\lambda_1[\alpha]=[0.3+0.2\alpha,\ 0.7-0.2\alpha],\ \lambda_2[\alpha]=[0.2+0.3\alpha,\ 8-3\alpha],$$

$$\lambda_3[\alpha]=[0.2+0.2\alpha,\ 0.6-0.2\alpha],\ \lambda_4[\alpha]=[0.4+0.2\alpha,\ 0.8-0.2\alpha].$$

β-cut is obtained as:

$$\lambda_1[\beta]=[0.5-0.3\beta,\ 0.5+0.3\beta],\ \lambda_2[\beta]=[0.5-0.4\beta,\ 0.5+4\beta],$$

$$\lambda_3[\beta]=[0.4-0.3\beta,\ 0.4+0.3\beta],\ \lambda_4[\beta]=[0.6-0.4\alpha,\ 0.5+4\beta].$$

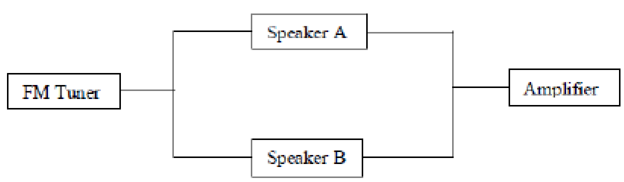

Figure 5. hi-fi system.

The probability of the system using α-cut of intuitionistic fuzzy random variables is given by,

$$\Pr(\lambda\in\Omega|\xi_\alpha^L(\lambda)\geq t)\}=\Pr(\lambda_1\in\Omega|\xi_{1,\alpha}^U(\lambda_1)\geq t)\times\{1-\Pr(\lambda_2\in\Omega|\xi_{2,\alpha}^U(\lambda_2)\geq t))$$

$$\times(1-\Pr(\lambda_3\in\Omega|\xi_{3,\alpha}^U(\lambda_3)\geq t)\}\times\Pr(\lambda_4\in\Omega|\xi_{4,\alpha}^U(\lambda_4)\geq t)$$

$$= [e^{-(1.7-0.1\alpha)t} + e^{-(1.7-2\alpha)t} - e^{-(2.9+0.1\alpha)t}] \quad (41)$$

$$\Pr(\lambda \in \Omega | \xi_{\alpha}^{U}(\lambda) \geq t)\} = \Pr(\lambda_1 \in \Omega | \xi_{1,\alpha}^{U}(\lambda_1) \geq t) \times \{1 - \Pr(\lambda_2 \in \Omega | \xi_{2,\alpha}^{L}(\lambda_2) \geq t))$$

$$\times (1 - \Pr(\lambda_3 \in \Omega | \xi_{3,\alpha}^{L}(\lambda_3) \geq t)\} \times \Pr(\lambda_4 \in \Omega | \xi_{4,\alpha}^{U}(\lambda_4) \geq t)$$

$$= [e^{-(1.5+0.1\alpha)t} + e^{-(1.3+0.2\alpha)t} - e^{-(2.1-0.1\alpha)t}]. \quad (42)$$

The reliability of the system using (41) and (42) is obtained as:

$$R(t) = \frac{1}{2}\int_0^1 [e^{-(1.7-0.1\alpha)t} + e^{-(1.7+0.2\alpha)t} - e^{-(1.9-0.1\alpha)t}$$

$$+ e^{-(1.5+0.1\alpha)t} + e^{-(1.3+0.2\alpha)t} - e^{-(2.1-0.1\alpha)t}]d\alpha$$

$$= \frac{5}{2t}[2e^{-1.5t} + e^{-1.3t} + 2e^{-21t} - 2e^{-1.9t} - 3e^{-1.7t}]. \quad (43)$$

The MTTF of the above system using (43) is obtained as:

$$MTTF = \int_0^\infty R(t)dt = \int_0^\infty \frac{5}{2t}[2e^{-1.5t} + e^{-1.3t} + 2e^{-2.1t} - 2e^{-1.9t} - 3e^{-1.7t}]dt$$

$$= 0.796058388$$

The probability of the system using ß–cut of intuitionistic fuzzy random variables is given by,

$$\Pr(\lambda \in \Omega | \xi_{\beta}^{L}(\lambda) \geq t)\} = \Pr(\lambda_1 \in \Omega | \xi_{1,\beta}^{L}(\lambda_1) \geq t) \times \{1 - (1 - \Pr(\lambda_2 \in \Omega | \xi_{2,\beta}^{U}(\lambda_2) \geq t))$$

$$\times (1 - \Pr(\lambda_3 \in \Omega | \xi_{3,\beta}^{U}(\lambda_3) \geq t)\} \times \Pr(\lambda_4 \in \Omega | \xi_{4,\beta}^{L}(\lambda_4) \geq t)$$

$$= [e^{-(1.6-0.1\beta)t} + e^{-(1.5-0.4\beta)t} - e^{-(2.0)t}] \quad (44)$$

$$\Pr(\lambda \in \Omega | \xi_{\beta}^{U}(\lambda) \geq t)\} = \Pr(\lambda_1 \in \Omega | \xi_{1,\beta}^{U}(\lambda_1) \geq t) \times \{1 - (1 - \Pr(\lambda_2 \in \Omega | \xi_{2,\beta}^{L}(\lambda_2) \geq t))$$

$$\times (1 - \Pr(\lambda_3 \in \Omega | \xi_{3,\beta}^{L}(\lambda_3) \geq t)\} \times \Pr(\lambda_4 \in \Omega | \xi_{4,\beta}^{U}(\lambda_4) \geq t)$$

$$= [e^{-(1.6-0.2\beta)t} + e^{-(1.5+0.3\beta)t} - e^{-(2.0-0.1\beta)t}] \quad (45)$$

The reliability of the system using (44) & (45) is obtained as:

$$R(t) = \frac{1}{2}\int_0^1 [e^{-(1.6-0.3\beta)t} + e^{-(1.5-0.4\beta)t} - e^{-(2.0)t}$$

$$+ e^{-(1.6+0.2\beta)t} + e^{-(1.5+0.3\beta)t} - e^{-(2.0-0.1\beta)t}]d\beta$$

$$= \frac{5}{12t}[4e^{-1.3t} + 2e^{-1.6t} + 10e^{-1.8t} + e^{-1.5t} + 3e^{-1.1t} - 12e^{1.9t}$$

$$+ 12e^{-2t}] - \frac{e^{-2t}}{2}. \tag{46}$$

The MTTF of the above system using (46) is obtained as:

$$MTTF = \int_0^\infty \{\frac{5}{12t}[4e^{-1.3t} + e^{-1.6t} + 10e^{-1.8t} + e^{-1.5t} + 3e^{-1.1t} - 12e^{1.9t}$$

$$+ 12e^{-2t}] - \frac{e^{-2t}}{2}\}dt$$

$$= 0.85890738$$

Conclusion

In this study, we assume the lifetimes of unrepairable systems as intuitionistic fuzzy random variables. Based on that, we establish four basic mathematical models of unrepairable systems, including series system, parallel system, parallel-series system and series-parallel system. The expression of reliability and MTTF are obtained for above unrepairable systems. In this study, we also assume that the intuitionistic fuzzy random variables ξ_i of unrepairable systems follow exponential distribution, and then the expression of reliability and MTTF of systems are also evaluated. A numerical example is also taken to illustrate the methodology to calculate the reliability and MTTF characteristics of above unrepairable systems.

Reference

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986), 87-96.

[2] K. Cai, C. Wen and M. Zhang, Fuzzy states as a basis for a theory of fuzzy reliability,

Microelectronics and Reliability 33 (1993), 2253-2263.

[3] K. Cai, C. Wen and M. Zhang, Fuzzy variables as a basis for a theory of fuzzy reliability in the possibility context, Fuzzy Sets and Systems 42 (1991), 145-172.

[4] K. Cai, C. Wen and M. Zhang, Posbist reliability behaviour of fault tolerant systems, Microelectronics and Reliability 35 (1995), 49-56.

[5] Shshank Chaube and S. B. Singh, Fuzzy Reliability evaluation using Conflicting Bifuzzy Approach, ICDMIC-2014 proceedings to appear in IEEE Xplore DOI:10.1109/ICDMIC.2014.6954241 (Print ISBN: 978- 1-4799-4675-4) (2014).

[6] S. M. Chen, Fuzzy system reliability analysis using fuzzy number arithmetic operations, Fuzzy Sets and Systems 64 (1994), 31-38.

[7] H. Huang, Reliability analysis method in the presence of fuzziness attached to operating time, Microelectronics and Reliability 35 (1995), 1483-1487.

[8] A. Kaufmann, Introduction to the theory of fuzzy subsets, Vol. I: Fundamental theoretical elements, Academic Press, New York, NY, USA, (1975).

[9] H. Kwakernaak, Fuzzy random Variables, Part I: Definition and theorem, Information Sciences 15 (1978), 1-29.

[10] H. Kwakernaak, Fuzzy random Variables, Part II: Algorithms and examples for the discrete case, Information Sciences 17 (1979), 253-278.

[11] R. Parvathi and C. Radhika, Intuitionistic fuzzy random variable, Notes on Intuitionistic Fuzzy Sets 21 (2015), 69-80.

[12] D. Singer, A fuzzy set approach to fault tree and reliability analysis, Fuzzy Sets and Systems 34 (1990), 145-155.

[13] Liu Ying, Tang Wansheng and Zhao Ruiqing, Reliability and Mean Time to Failure of Unrepairable systems with Fuzzy Random Lifetimes, IEEE Transactions on Fuzzy Systems 15 (2007), 1009-1026.

[14] Liu Ying, Li Xiaozhong and Li Jianbin, Reliability Analysis of random fuzzy unrepairable systems, Discrete Dynamics in Nature and Society, (2014).

[15] L. A. Zadeh, Fuzzy sets, Informatics and Control 8 (1965), 338-353.

[16] Z. Zainali, M. G. Akbari and H. Noughabi, Intuitionistic fuzzy random variable and testing hypothesis about its variance, Soft Computing, vol. 19 (2015), 2681-2689.