Advances and Applications in Mathematical Sciences
Volume 19, Issue 5, March 2020, Pages 341-352

A STUDY AND MATHEMATICAL MODELING FOR AUTOPILOT GUIDANCE AND CONTROL IN HOMING GUIDED MISSILES

MANVINDER SHARMA and ANUJ KUMAR GUPTA

Jaipur National University
Jaipur, Rajasthan, India
E-mail: manvinder.sharma@gmail.com

CGC College of Engineering
Landran, Punjab, India
E-mail: anuj.coecse@cgc.edu.in

Abstract

In the modern combat systems, the needs of intelligent combat weapons are essential for any country's defense system. This initiated the requirement and research on automatic and self-controlled weapons. One of the most important weapons for any country is guided missile. The self guided and controlled missiles have been a topic of research from decades. The intelligent autopilot control and guidance provide accurate calculation of target and précised target elimination. Homing missiles has guidance unit, self motion measurement and target tracking unit inbuilt in missiles so they are more accurate and effective than other types. The objective of autopilot design in missile is to provide better stability for miss distance, robustness and satisfactory performance, so that the missile only hit the target with more accuracy. Missile autopilots can command body rates, flight angle path, acceleration and incident angles. This paper provides study of various guidance control methods and their mathematical modeling for autopilots for homing missiles. Various autopilot methods are discussed to control and navigate the missile towards path of target.

I. Introduction

Missile is any object which can be fired, thrown or projected towards anything (target). With manual or automatic means, the projectile with payload (warhead) is guided towards target with a motive to provide damage to target. Depending upon orientation towards target, the missiles are

---

2010 Mathematics Subject Classification: 93C40.
Keywords: Missile guidance, Autopilot Control, Autopilot angle control, Missile.
Received September 23, 2019; Accepted December 16, 2019

classified into guided and unguided missiles [1-3]. Unguided missiles (rockets) can be aimed towards the direction of target only before firing, once fired they cannot be controlled and can be aimed towards the direction of moving target. They can be best used for stationary targets or short distance targets. For distant targets or moving targets the accuracy of these missiles are reduced due to curvature and rotation of earth, crosswinds and aiming errors. The guided missiles can also be controlled after being fired as well. The projectile can change the direction and speed towards the target [4-6]. The motion of guided missile can be tracked and flight path can be deviated with the commands. So guided missiles can be used for shorter as well as longer distances also for moving and stationary targets [7]. Figure 1 shows world's first liquid propelled guided missile V2 developed by NAZI Germany during world war 2. V2 was guided by radio controlled mechanical autopilot.

Figure 1. First guided missile.

Guided missiles can be categorized into two groups namely tactical missiles and strategic (ballistic or cruise) missiles. With the use of some sort of sensors the tactical missiles can be guided and can be used from short distance to medium distance. While strategic missiles are long range missiles which travel a long distance and generally the target is stationary location [8-10].

Homing guidance is a method by which a missile has self-contained guidance unit which can generate motion based commands to autopilot of missile by seeking the characteristics of target and autopilot steers the missile towards the target. A homing missile is more sophisticated than other missiles as it contains guidance unit, measuring unit for self-motion and unit to trace the target [11]. But also this type of missiles comes under 'Fire-and-Forget' category. As compared with Command to line of Sight (CLOS) and Beam Rider guidance (BRG) Guidance, the measurement accuracy of relative motion of target is more in homing guidance missile [12].

II. Homing Guidance

Homing guidance is a method by which a missile has self-contained guidance unit which can generate motion based commands to autopilot of missile by seeking the characteristics of target and autopilot steers the missile towards the target. A homing missile is more sophisticated than other missiles as it contains guidance unit, measuring unit for self-motion and unit to trace the target. But also this type of missiles comes under 'Fire-and-Forget' category. As compared with Command to line of Sight (CLOS) and Beam Rider guidance (BRG) Guidance, the measurement accuracy of relative motion of target is more in homing guidance missile. Homing guidance can be further divided into active homing, semi-active homing, passive homing and homing by map matching [13-14].

A. Active Homing

In this type of method, a missile carries a transmitter and receiver units for detection of target. The transmitter unit transmits some sort of radiations towards target and receiver unit receives the reflected radiations and calculates the path and provides the correction in autopilot for flight path. The homing radiation can be in form of light waves, IR waves, radio waves or sound waves [15].

B. Semiactive Homing

In this method, the missile has only receiver. The homing radiation is being transmitted by any other source (usually placed at launch site), and receiver on missile picks up the reflected radiations from the target and steers the autopilot towards the target. The high power radar illuminator or LASER illuminator can be used for transmitter [16-17].

C. Passive Homing

Passive homing is simple and mostly used homing missile method. In this the transmitter is the target itself, the missile has receiver unit (seeker) which picks the radiations from the target and changes flight path. The example is heat detector or radar detector [18].

D. Homing by Map Matching

In this method, the missile has a camera which detects the target by seeking and comparing view below with the preloaded image of target. When feature matches, the guidance unit homes the missile towards a specific terrain.

III. Missile Autopilot Control Algorithms

In order to control the path of missile in form of yaw, roll direction and its pitch, three different autopilots are needed which controls these motions.

A. Roll Autopilot

Due to forces during thrust and thrust misalignment moments, the missile achieves a motion in roll around axis. Within a period of time this roll motion is damped after boost phase. Depending upon the orientation of fixed tail fins the length of the period changes. The roll motion will last much longer if the tail fins are fixed at an orientation with zero angles. Prior to the control of missile's yaw and pitch motion, it is quite useful to compensate roll motion in autopilot design. This means at the beginning of control the roll rate and roll altitude must be nullified [19].

For roll altitude of missile the state feedback law can be written as

$$u_{\Gamma} = K_{\phi}(\phi_{1d} - \phi_1) - K_p p \tag{1}$$

where K_{ϕ} and K_p are controller gains.

The closed loop transfer function of roll motion can be determined as

$$\frac{\phi_1(s)}{\phi_d(s)} = \frac{1}{c_{\phi 2}s^2 + c_{\phi 1}s + 1}, \tag{2}$$

where $c_{\phi 1} = \dfrac{L_\delta K_p - L_{p1}}{L_\delta K_\phi}$ and $c_{\phi 2} = \dfrac{1}{L_\delta K_\phi}$.

The characteristic polynomial appears as

$$D_\Gamma(s) = c_{\phi 2}s^2 + c_{\phi 1}s + 1. \tag{3}$$

The second order buttterwoth polynomial can be used for controller gain K_p and K_ϕ which can be placed as two poles of closed loop system.

$$B_2(s) = \left(\frac{1}{\omega_c^2}\right)s^2 + \left(\frac{\sqrt{2}}{\omega_c}\right)s + 1 \tag{4}$$

ω_c is desired bandwidth for control system. Equating the polynomials, K_p and K_ϕ can be found as

$$K_\phi = \frac{\omega_c^2}{L_\delta} \tag{5}$$

$$K_p = \frac{\sqrt{2}\omega_c + L_{p1}}{L_\delta}. \tag{6}$$

During missile flight, aerodynamics stability derivatives vary in accordance with side-slip angle (β) angle of attack (α), mach number (M_∞), and spin angle (Φ_s). The roll control system must adapt to changes of these parameter to keep the roll stability of missile. The values of K_p and K_Φ should be updated as current values throughout flight.

B. Acceleration Autopilot

The guidance unit in pitch plane provides lateral acceleration commands. Considering classical proportional plus integral action with contribution of pitch damping, the control system can be designed [20]. The figure 2 shows pitch acceleration control system.

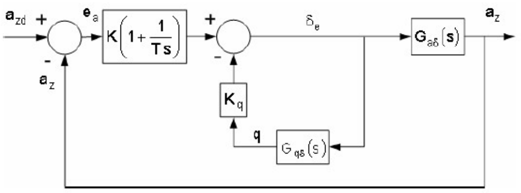

Figure 2. Pitch Acceleration control system.

The closed loop transfer function between desired and actual lateral accelerations can be obtained as

$$\frac{a_z(s)}{a_{zd}(s)} = \frac{(T_p s + 1)(n_{p2}s^2 + n_{p1}s + 1}{a_{p3}s^3 + a_{p2}s^2 + a_{p1}s + 1}, \tag{7}$$

where,

$$n_{p1} = \frac{n_{z1}}{n_{z0}} \tag{8}$$

$$n_{p2} = \frac{n_{z2}}{n_{z0}} \tag{9}$$

$$a_{p1} = \frac{T_p(d_{p0} + K_q n_{q0} + K_p n_{z0}) + K_p n_{z1}}{K_p n_{z0}} \tag{10}$$

$$a_{p2} = \frac{T_p(d_{p1} + K_q n_{q1} + K_p n_{z1}) + K_p n_{z2}}{K_p n_{z0}} \tag{11}$$

$$a_{p3} = \frac{T_p(1 + K_p n_{z2})}{K_p n_{z0}}. \tag{12}$$

Where K_p, T_p and K_q are proportional, integral and pitch damping gains. For the above type 1 system, polynomial of transfer function can be written as

$$D_p(s) = a_{p3}s^3 + a_{p2}s^2 + a_{p1}s + 1. \tag{13}$$

In order to determine K_p, T_p and K_q, third order Butterworth polynomial is given as

$$B_3(s) = \left(\frac{1}{\omega_c^3}\right)s^3 + \left(\frac{2}{\omega_c^2}\right)s^2 + \left(\frac{2}{\omega_c}\right)s + 1 \tag{14}$$

$\sigma_P = \dfrac{T_P}{K_P}$ and $\eta_P = \dfrac{T_P K_q}{K_P}$, σ_P, η_P and T_P

can be obtained by matching equations

$$\begin{bmatrix} \sigma_P \\ \eta_P \\ T_P \end{bmatrix} = \widehat{M_p^{-1}}\,\overline{b_P}, \tag{15}$$

where

$$\widehat{M}_P = \begin{bmatrix} 1 & 0 & n_{z2} \\ d_{p1} & n_{q1} & n_{z1} \\ d_{p0} & n_{q0} & n_{z0} \end{bmatrix}, \tag{16}$$

$$\overline{b}_P = \begin{bmatrix} \dfrac{n_{z0}}{\omega_c^3} & \dfrac{2n_{z0}}{\omega_c^2}\, n_{z2} & \dfrac{2n_{z0}}{\omega_c} - n_{z1} \end{bmatrix}^T \tag{17}$$

In order for T_P to take finite values $\widehat{M_P^{-1}}$ must exist or determinant of matrix must be non zero.

C. Rate Autopilots

Similarly to acceleration autopilot, pitch (yaw) rate autopilot can be designed to control rate of flight path angle [21]. Figure 3 shows flight path angle rate control

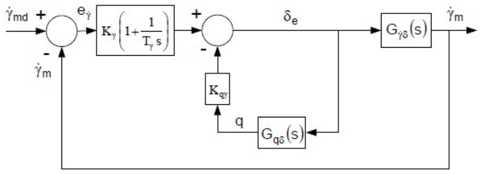

Figure 3. Flight Path Angle Rate Control

For type 1 control system, the closed loop transfer function from actual flight path angle rate $\dot{\gamma}_m$ and desired flight path rate $\dot{\gamma}_{md}$ can be obtained as

$$\frac{\dot{\gamma}_m(s)}{\gamma_{m\dot{d}}(s)} = \frac{(T_\gamma s + 1)(n_{\dot{\gamma}2}s^2 + n_{\dot{\gamma}1}s + 1)}{d_{\dot{\gamma}3}s^3 + d_{\dot{\gamma}2}s^2 + d_{\dot{\gamma}1}s + 1} \tag{18}$$

where, $n_{\dot{\gamma}1} = \frac{n_{\alpha q}}{n_{q0}}$ and $n_{\dot{\gamma}2} = -\frac{n_{\alpha 1}}{n_{q0}}$

$$d_{\dot{\gamma}1} = \frac{T_\gamma(d_{p0} + K_{q\gamma}n_{q0} + K_\gamma n_{\alpha q}) + K_\gamma n_{\alpha q}}{K_\gamma n_{q0}} \tag{19}$$

$$d_{\dot{\gamma}2} = \frac{T_\gamma(d_{p1} + K_{q\gamma}n_{q1} + K_\gamma n_{\alpha q}) - K_\gamma n_{\alpha q}}{K_\gamma n_{q0}} \tag{20}$$

$$d_{\dot{\gamma}3} = \frac{T_\gamma - K_\gamma T_\gamma n_{\alpha 1}}{K_\gamma n_{q0}}. \tag{21}$$

The characteristic polynomial of the closed loop unction can be written as

$$\begin{bmatrix} \sigma_\gamma \\ \eta_\gamma \\ T_\gamma \end{bmatrix} = \widehat{M_\gamma^{-1}}\,\overline{b_\gamma} \tag{22}$$

where

$$\widehat{M}_P = \begin{bmatrix} 1 & 0 & -n_{\alpha 1} \\ d_{p1} & n_{q1} & n_{\alpha q} \\ d_{p0} & n_{q0} & n_{q0} \end{bmatrix}, \tag{23}$$

$$\overline{b_K} = \begin{bmatrix} \frac{n_{q0}}{\omega_c^3} & \frac{2n_{q0}}{\omega_c^2} + n_{\alpha 1} & \frac{2n_{q0}}{\omega_c} - n_{\alpha q} \end{bmatrix}^T \tag{24}$$

$$\sigma_\gamma = \frac{T_\gamma}{K_\gamma} \text{ and } \eta_\gamma = \frac{T_\gamma K_{q\gamma}}{K_\gamma}.$$

D. Angle Autopilot

To track flight path angle commands the equation of motion of missile should be rearranged in a compatible form. Pitch and yaw motion can be treated as planar motion [22].

Taking gravity effect as an external disturbance, conceding roll free motion in pitch plane, the component of acceleration in $\vec{u}_3^{(b)}$ direction.

Time derivative of pitch rate can be obtained as

$$a_z = \frac{Z}{m}$$

$$\dot{q} = \frac{M}{I_t'}.$$

In pitch plane motion, a_z is equal to multiplication of flight path angle rate and missile velocity.

$$a_z = V_M \dot{\gamma}_m \tag{25}$$

$$\dot{\gamma}_m = -Z_\alpha \gamma_m + Z_\alpha \theta + Z_q q + Z_\delta \delta_e \tag{26}$$

$$\dot{q} = -M_\alpha \gamma_m + M_\alpha \theta + M_q q + M_\delta \delta_e \tag{27}$$

where

$$\theta = \theta_1,$$

$$Z_\alpha = \frac{q_\infty S_M}{m V_M} C_{z_\alpha},$$

$$Z_q = \frac{q_\infty S_M d_M}{m V_M^2} C_{z_q},$$

$$Z_\delta = \frac{q_\infty S_M}{m V_M} C_{z_\delta},$$

$$M_\alpha = \frac{q_\infty S_M d_M}{I_t'} C_{M_\alpha},$$

$$M_q = \frac{q_\infty S_M d_M^2}{I_t'} C_{M_q}, \text{ and}$$

$$M_\delta = \frac{q_\infty S_M d_M}{I_t'} C_{M_\delta}.$$

The equation can be expressed in matrix as

$$\begin{bmatrix} \dot{\gamma_m} \\ \theta \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -Z_\alpha & Z_\alpha & Z_q \\ 0 & 0 & 1 \\ -M_\alpha & M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \gamma_m \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} Z_\delta \\ 0 \\ M_\delta \end{bmatrix} \delta_e. \tag{28}$$

Figure 4 shows flight path angle control system. The state feedback control law can be designed to control flight path control

$$u = \delta_e = K_\gamma(\gamma_{md} - \gamma_m) - K_\theta \theta - K_q q + K_i x_i \tag{29}$$

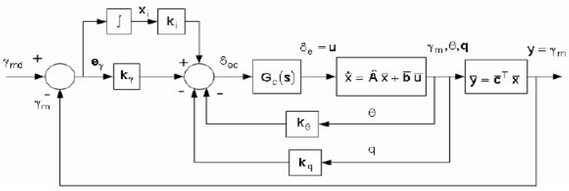

Figure 4. Flight path angle control.

In this paper different type of homing missile methods and various models to control and steer autopilot of missiles has been discussed with mathematical modeling. In the modern era, to increase the accuracy for any missile the guidance system is mostly automatic. The target should be identified accurately so that the missile can be guided towards it to provide better defense. The mathematical modeling is structured using the basic equations of kinematics and dynamic modeling. With the use of mathematical model, closed loop models of acceleration, angle flight control, roll and rate autopilots has been structured and discussed in paper.

References

[1] Yanushevsky and Rafael, Modern missile guidance, CRC Press, 2018.

[2] Seo, Min-Guk, Chang-Hun Lee and Min-Jea Tahk, New design methodology for impact angle control guidance for various missile and target motions, IEEE Transactions on Control Systems Technology 26(6) (2017), 2190-2197.

[3] Zarchan and Paul, ed. Advanced Tactical and Strategic Missile Guidance, Volume 2. American Institute of Aeronautics and Astronautics, Inc., 2019.

[4] Wang and Zheng, Adaptive smooth second-order sliding mode control method with application to missile guidance, Transactions of the Institute of Measurement and Control 39(6) (2017), 848-860.

[5] Lin, Li-Gang and Ming Xin, Missile Guidance Law Based on New Analysis and Design of SDRE Scheme, Journal of Guidance, Control, and Dynamics 42(4) (2018), 853-868.

[6] Garcia, Eloy, David W. Casbeer, Zachariah E. Fuchs and Meir Pachter, Cooperative missile guidance for active defense of air vehicles, IEEE Transactions on Aerospace and Electronic Systems 54(2) (2017), 706-721.

[7] Khan, A. Kabir and Robert S. Kerbel, A CD276 antibody guided missile with one warhead and two targets: the tumor and its vasculature, Cancer cell 31(4) (2017), 469-471.

[8] Park, Woong and Jae-Chon Lee, Model-Based Approach to Flight Test System Development to Cope with Demand for Simultaneous Guided Missile Flight Tests, Journal of the Korea Academia-Industrial cooperation Society 20(1) (2019), 268-277.

[9] Hampshire and Edward, Soviet Cruise Missile Submarines of the Cold War. Bloomsbury Publishing, 2018.

[10] Chwa and Dongkyoung, Robust Nonlinear Disturbance Observer Based Adaptive Guidance Law Against Uncertainties in Missile Dynamics and Target Maneuver, IEEE Transactions on Aerospace and Electronic Systems 54(4) (2018), 1739-1749.

[11] Jeon, In-Soo, Jin-Ik Lee and Min-Jea Tahk, Homing guidance law for cooperative attack of multiple missiles, Journal of guidance, control, and dynamics 33(1) (2010), 275-280.

[12] Song, Taek Lyul and Tae Yoon Um, Practical guidance for homing missiles with bearings-only measurements, IEEE Transactions on Aerospace and Electronic Systems 32(1) (1996), 434-443.

[13] Zarchan and Paul, Tactical and strategic missile guidance, American Institute of Aeronautics and Astronautics, Inc., 2012.

[14] Roy L. Nichols, Active optical terminal homing, U.S. Patent 4 (1983), 383-663.

[15] Wei, Wen-jian, Shi-qiao Qin, De-Jun Zhan, Bao-dong Zhang, and Hua Li, Research on Laser Encoding in Laser Semi-active Homing Guidance [J], Laser & Infrared 12 (2008).

[16] W. A. N. G. Kuang-biao, Latest development and trends of semi-active laser guidance technology, Infrared and Laser Engineering 37 (2008), 275-279.

[17] Jason L. Speyer, Kevin D. Kim and Minjea Tahk, Passive homing missile guidance law based on new target maneuver models, Journal of Guidance, Control, and Dynamics 13(5) (1990), 803-812.

[18] S. E. Talole, A. A. Godbole, J. P. Kolhe and S. B. Phadke, Robust roll autopilot design for tactical missiles, Journal of guidance, control, and dynamics 34(1) (2011), 107-117.

[19] Li, Keyong, Shuxing Yang and Liangyu Zhao, Stability of spinning missiles with an acceleration autopilot, Journal of Guidance, Control, and Dynamics 35(3) (2012), 774-786.

[20] Ching-Yaw Tzeng, An internal model control approach to the design of yaw-rate-control ship-steering autopilot, IEEE Journal of Oceanic Engineering 24(4) (1999), 507-513.

[21] Zhou, Di, Pingping Qu and Sheng Sun, A guidance law with terminal impact angle constraint accounting for missile autopilot, Journal of Dynamic Systems, Measurement, and Control 135(5) (2013), 051009.