Advances and Applications in Mathematical Sciences **ISSN 0974-6803**
Volume 22, Issue 6, April 2023, Pages 1079-1092

ANALYSIS OF IDENTITY GRAPH OF FINITE GROUP WITH REFERENCE TO VARIOUS TYPES OF RESOLVING SETS

K. ARUNA SAKTHI[1], R. RAJESWARI[2] and N. MEENAKUMARI[3]

[1]Research Scholar (Reg. No. 20212012092006)
A. P. C. Mahalaxmi College for Women, Thoothukudi
Affiliated to Manonmaniam Sundaranar University
Abhishekapatti, Tirunelveli-627012, Tamil Nadu, India
E-mail: arunasakthi9397@gmail.com

[2]Assistant Professor
[3]Associate Professor
PG and Research Department of Mathematics
A. P. C. Mahalaxmi College for Women
Thoothukudi-628002, Tamil Nadu, India
E-mail: rajimuthuram@gmail.com
meenakumari.n123@gmail.com

Abstract

The concept of resolving set was first introduced by Slater, Harary and Melter. After that various resolving set has been introduced and studied for various graphs by many mathematicians. In this paper, investigation of various types of resolving set and its dimensions for identity graph of finite groups has been discussed as a theorem in detail.

1. Introduction

The concept of resolving sets was first introduced by Slater [6] and Harary and Melter [2]. Resolving sets have many applications in network discovery and verification, chemistry and robot navigation. Neighborhood number was introduced by E. Sampathkumar and then neighborhood

2020 Mathematics Subject Classification: 05C12, 05C50.
Keywords: outer connected resolving set, neighborhood resolving set, co-equitable resolving set, identity graphs.
Received July 7, 2022; Accepted December 22, 2022

resolving sets was introduced [1] and analysed for various graphs by many mathematician. Outer connected resolving sets was introduced by J. Sivakumar, A. Wilson Baskar, R. Sundareswaran, P. Nataraj and V. Swaminathan [4]. Degree equitability for the graphs was introduced by E. Sampathkumar and then co-equitable resolving sets was introduced by J. Sivakumar, A. Wilson Baskar and V. Swaminathan [5]. In this paper we have investigated various resolving sets for the identity graphs [3] of finite groups.

2. Preliminaries

Definition 2.1 Identity Graph. Let g be a group. The identity graph $G = (V, E)$ with vertices as the elements of group and two elements $x, y \in g$ are adjacent or can be joined by an edge if $x \cdot y = e$, where e is the identity element of g and identity element is adjacent to every other vertices in G.

Definition 2.2 Resolving set. A set of vertices S in a graph G is called a resolving set for G if, for any two vertices u, v there exists $x \in S$ such that the distances $d(u, x) \neq d(v, x)$. The minimum cardinality of a resolving set of G is called the dimension of G and is denoted $\dim(G)$.

Definition 2.3 Outer connected resolving set. Let $G = (V, E)$ be a connected graph. A subset $S \subseteq V(G)$ is an outer connected resolving set if S is a resolving set of G and $\langle V - S \rangle$ is connected. The minimum cardinality of an outer connected resolving set (ocr-set) is called outer connected resolving number of G and is denoted by $\dim_{oc}(G)$.

Definition 2.4 Co-equitable resolving set. Let $G = (V, E)$ be a simple graph. A subset S of $V(G)$ is called co-equitable resolving set of G if S is a resolving set and $V - S$ is a degree equitable (i.e. for $u, v \in V - S$ $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality of a co-equitable resolving set of G is called the co-equitable dimension of G and is denoted by $\operatorname{coeqdim}(G)$.

Definition 2.5 Neighbourhood resolving set. Let $G = (V, E)$ be a simple connected graph. A subset S of V is called a neighbourhood set of G if

$G = \bigcup_{s \in S} \langle N[s] \rangle$, where $N[v]$ denotes the closed neighbourhood of the vertex v in G. Further for each ordered subset $S = \{s_1, s_2, \ldots, s_k\}$ of V and a vertex $u \in V$, we associate a vector $\Gamma(u/S) = (d(u, s_1), d(u, s_2), \ldots, d(u, s_k))$ with respect to S, where $d(u, v)$ denote the distance between u and v in G. A subset S is said to be resolving set of G if $\Gamma(u/S) \neq \Gamma(v/S)$ for all $u, v \in V - S$. A neighbouring set of G which is also a resolving set for G is called a neighbourhood resolving set (nr-set).

3. Co-equitable Dimension of Some Identity Graph of Finite Group

Theorem 3.1. *The identity graph* $(Z_n, \oplus_n)$ *for* $n > 3$ *odd number has* $\operatorname{coeqdim}(G)$ *as* $\frac{n+1}{2}$.

Proof. Let graph $G = (Z_n, \oplus_n)$ for $n > 3$ odd number.

The vertex set of G is $V(G) = \{0, 1, 2, \ldots, n-1\} = \{x_0, x_1, x_2, \ldots, x_{\frac{n-1}{2}}, x_{\frac{n+1}{2}}, \ldots, x_{n-1}\}$.

The edge set of G is $E(G) = \{x_0 x_i, x_1 x_{n-1}, x_2 x_{n-2}, \ldots, x_{\frac{n+1}{2}} x_{\frac{n+1}{2}}\}$, $1 \leq i \leq n-1$.

$|V(G)| = n, |E(G)| = \frac{3n-3}{2}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

If $|S| = 2$ i.e. subset S can take any of the vertex $\{x_i, x_j\}, i \neq j$ $0 \leq i, j \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = \frac{n+1}{2}$. The subset S is considered in such a way choose the vertex x_0 and $\frac{n-1}{2}$ odd vertices like $\{x_1, x_3, \ldots\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. Now $V - S$ set contains even vertices like $\{x_2, x_4, x_6, \ldots\}$ each of degree 2. Therefore $V - S$ is degree equitable i.e. $[| \deg(x_i) - \deg(x_j) | \leq 1]$. Therefore set S is a co-equitable resolving set. Also set S is of minimum cardinality of co-equitable resolving set of graph G. The cardinality of S is called as co-equitable metric dimension of G. i.e. $\text{coeqdim}(G) = \frac{n+1}{2}$.

Theorem 3.2. *The identity graph* $(Z_n, \oplus_n)$ *for* $n > 4$ *even number has* $\text{coeqdim}(G)$ *as* $\frac{n}{2}$.

Proof. Let graph $G = (Z_n, \oplus_n)$ for $n > 4$ even number.

The vertex set of G is $V(G) = \{0, 1, 2, \ldots, n-1\} = \{x_0, x_1, x_2, \ldots, x_{n2}, \ldots, x_{n-1}\}$.

The edge set of G is $E(G) = \{x_0 x_i, x_1 x_{n-2}, \ldots, /1 \leq i \leq n-1\}$

$| V(G) | = n, | E(G) | = \frac{3n-3}{2}$.

Let $S \subseteq V(G)$. If $| S | = 1$ i.e. subset S can take any one of the vertex $x_i, 0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

If $| S | = 2$ i.e. subset S can take any of the vertex $\{x_i, x_j\}, i \neq j$ $0 \leq i, j \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $| S | = \frac{n}{2}$. The subset S is considered in such a way choose the vertex x_0 and $\frac{n-2}{2}$ odd vertices like $\{x_1, x_3, \ldots, x_{n-2}\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. Now $V - S$ set contains even vertices like $\{x_2, x_4, x_6, \ldots\}$ each of degree 2. Therefore $V - S$ is degree equitable i.e. $[|\deg(x_i) - \deg(x_j)| \leq 1]$. Therefore set S is a co-equitable resolving set. Also set S is of minimum cardinality of co-equitable resolving set of graph G. The cardinality of S is called as co-equitable metric dimension of G. i.e. $\operatorname{coeqdim}(G) = \frac{n}{2}$.

Theorem 3.3. *The identity graph of klein-*4 *group under composition has* $\operatorname{coeqdim}(G)$ *as* 3.

Proof. Let graph G = Identity graph of Klein-4 group

The vertex set of G is $V(G) = \{x_0, x_1, x_2, x_3\} = \{e, a, b, ab\}$.

The edge set of G is $E(G) = \{x_0 x_i / 1 \leq i \leq 3\}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = 3$. The subset S is considered in such a way choose the vertex x_0 and any two vertices from $\{x_1, x_2, x_3\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. Now $V - S$ set contains only one vertex like $\{x_i\}$ of degree 1. Therefore $V - S$ is degree equitable i.e. $[|\deg(x_i) - \deg(x_j)| \leq 1]$. Therefore set S is a co-equitable resolving set. Also set S is of minimum cardinality of co-equitable resolving set of graph G. The cardinality of S is called as co-equitable metric dimension of G. i.e. $\operatorname{coeqdim}(G) = 3$.

Theorem 3.4. For the identity graph of *quaternion group co-equitable dimension is* 4.

Proof. Let graph G = identity graph of Q_8.

The vertex set of G is $V(G) = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$.

The edge set of G is $E(G) = \{x_0 x_i, x_2 x_3, x_4 x_5, x_6 x_7 : 1 \leq i \leq 7\}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = 4$. The subset S is considered in such a way choose the vertex x_0 and any three odd or even numbered vertices from $\{x_1, x_2, x_3, x_4, x_5, x_6\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. Now $V - S$ set contains only remaining four vertex like either $\{x_2, x_4, x_6, x_7\}$ or $\{x_1, x_3, x_5, x_7\}$ of degree 2 and $\{x_7\}$ of degree 1. Therefore $V - S$ is degree equitable i.e. $[|\deg(x_i) - \deg(x_j)| \leq 1]$. Therefore set S is a co-equitable resolving set. Also set S is of minimum cardinality of co-equitable resolving set of graph G. The cardinality of S is called as co-equitable metric dimension of G. i.e. $\text{coeqdim}(G) = 4$.

Theorem 3.5. *The co-equitable dimension for the identity graph of dihedral group* D_{2n}, $n > 2n$ *even is* $\frac{3n}{2}$.

Proof. Let graph $G =$ Identity graph of D_n n is even.

The vertex set of G is $V(G) = \{1, a, b : a^2 = b^n = 1\ bab = 1\}$

$= \{1, a, ab, ab^2, ab^3, \ldots, ab^{n-1}, b, b^2, b^3, \ldots, b^{n-1}\}$

$= \{x_0, x_1, y_1, y_2, \ldots, y_{n-1}, z_1, z_2, z_3, \ldots, z_{n-1}\}$.

The edge set of G is

$E(G) = \{x_0x_1, x_0y_i, x_0z_j, z_1z_{n-1}, z_2z_{n-2}, \ldots, z_{\frac{n}{2}}x_0 : 1 \leq i \leq n-1,$

$1 \leq j \leq n-1$ and $j \neq \frac{n}{2}\}$

$|V(G)| = 2n, |E(G)| = \frac{5n-4}{2}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex

x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = \frac{3n}{2}$. The subset S is considered in such a way choose the vertex x_0 and $\frac{3n-4}{2}$ vertices of degree one and $\frac{n-2}{2}$ vertices of degree two. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. Now $V - S$ set contains only one degree 1 vertex and one two degree vertex. Therefore $V - S$ is degree equitable i.e. $[|\deg(x_i) - \deg(x_j)| \leq 1]$. Therefore set S is a co-equitable resolving set. Also set S is of minimum cardinality of co-equitable resolving set of graph G. The cardinality of S is called as co-equitable metric dimension of G, i.e. $\text{coeqdim}(G) = \frac{3n}{2}$.

Theorem 3.5. *The co-equitable dimension for the identity graph of dihedral group* D_{2n}, $n \geq 3$ *n is odd is* $\frac{3n-1}{2}$.

Proof. Let graph G = Identity graph of D_n n is odd.

The vertex set of G is $V(G) = \{1, a, b : a^2 = b^n = 1\ bab = 1\}$

$= \{1, a, ab, ab^2, ab^3, \ldots, ab^{n-1}, b, b^2, b^3, \ldots, b^{n-1}\}$

$= \{x_0, x_1, y_1, y_2, \ldots, y_{n-1}, z_1, z_2, z_3, \ldots, z_{n-1}\}$.

The edge set of G is

$$E(G) = \{x_0x_1, x_0y_i, x_0z_i, z_1z_{n-1}, z_2z_{n-2}, \ldots, z_{\frac{n-1}{2}}z_{\frac{n+1}{2}} : 1 \leq i \leq n-1\}$$

$$|V(G)| = 2n, |E(G)| = \frac{5n-3}{2}.$$

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = \frac{3n}{2}$. The subset S is considered in such a way choose the vertex x_0 and $\frac{3n-5}{2}$ vertices of degree one and $\frac{n-1}{2}$ vertices of degree two. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. Now $V - S$ set contains only one degree 1 vertex and one two degree vertex. Therefore $V - S$ is degree equitable i.e. $[|\deg(x_i) - \deg(x_j)| \leq 1]$. Therefore set S is a co-equitable resolving set. Also set S is of minimum cardinality of co-equitable resolving set of graph G. The cardinality of S is called as co-equitable metric dimension of G, i.e. $\text{coeqdim}\,(G) = \frac{3n-1}{2}$.

4. Outer Connected Dimension of Some Identity Graph of Finite Group

Theorem 4.1. *The outer connected dimension of identity graph* $(Z_n, \oplus_n)$, *for* $n > 3$ *odd number is* $\dim_{oc}(G) = \frac{n-1}{2}$.

Proof. Let graph $G = (Z_n, \oplus_n)$ for $n > 3$ odd number

The vertex set of G is $V(G) = \{0, 1, 2, \ldots, n-1\} = \{x_0, x_1, x_2, \ldots, x_{\frac{n-1}{2}}, x_{\frac{n+1}{2}}, \ldots, x_{n-1}\}$.

The edge set of G is $E(G) = \{x_0 x_i, x_1 x_{n-1}, x_2 x_{n-2}, \ldots, x_{\frac{n-1}{2}}\, x_{\frac{n+1}{2}}\}$, $1 \leq i \leq n-1$.

$|V(G)| = n, |E(G)| = \frac{3n-3}{2}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = \frac{n-1}{2}$. The subset S is considered in such a way choose the $\frac{n-1}{2}$ odd vertices like $\{x_1, x_3, \ldots\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. Now $V - S$ set contains even vertices like $\{x_2, x_4, x_6, \ldots\}$. $\langle V - S \rangle$ is also connected. S is also outer connected resolving set. The minimum cardinality of outer connected resolving set is called as ocr-set of G, $\dim_{oc}(G) = \frac{n-1}{2}$.

Theorem 4.2. *The outer connected dimension of identity graph* $(Z_n, \oplus_n)$ *for* $n > 4$ *even number is* $\dim_{oc}(G) = \frac{n-2}{2}$.

Proof. Let graph $G = (Z_n, \oplus_n)$ for $n > 4$ even number.

The vertex set of G is $V(G) = \{0, 1, 2, \ldots, n-1\} = \{x_0, x_1, x_2, \ldots, x_{n2}, \ldots, x_{n-1}\}$.

The edge set of G is $E(G) = \{x_0 x_i, x_1 x_{n-1}, x_2 x_{n-2}, \ldots, /1 \leq i \leq n-1\}$

$|V(G)| = n, |E(G)| = \frac{3n-3}{2}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = \frac{n-2}{2}$. The subset S is considered in such a way choose the $\frac{n-2}{2}$ odd vertices like $\{x_1, x_3, \ldots\}$ and x_n. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. Now $V - S$ set contains even vertices like $\{x_2, x_4, x_6, \ldots\}$. $\langle V - S \rangle$ is also connected. S is also outer connected resolving set. The minimum cardinality of outer connected resolving set is called as ocr-set of G,

$\dim_{oc}(G) = \frac{n-2}{2}$.

Theorem 4.3. *The outer connected dimension for the identity graph of klein-*4 *group under composition is* 2.

Proof. Let graph G = Identity graph of Klein-4 group

The vertex set of G is $V(G) = \{x_0, x_1, x_2, x_3\} = \{e, a, b, ab\}$.

The edge set of G is $E(G) = \{x_0 x_i / 1 \leq i \leq 3\}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = 2$. The subset S is considered in such a way choose any two vertices from $\{x_1, x_2, x_3\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. Now $V - S$ set contains even vertices like $\{x_0\}$ and any one vertex from $\{x_1, x_2, x_3\}$. $\langle V - S \rangle$ is also connected. S is also outer connected resolving set. The minimum cardinality of outer connected resolving set is called as ocr-set of G, $\dim_{oc}(G) = 2$.

Theorem 4.4. *The outer connected dimension of the identity graph of quaternion group neighborhood dimension is* 3.

Proof. Let graph G = identity graph of Q_3.

The vertex set of G is $V(G) = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$.

The edge set of G is $E(G) = \{x_0 x_i, x_2 x_3, x_4 x_5, x_6 x_7 : 1 \leq i \leq 7\}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = 3$. The subset S is considered in such a way choose any three odd or even numbered vertices from $\{x_2, x_3, x_4, x_5, x_6, x_7\}$.

Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. Now $V - S$ set contains vertices like $\{x_0, x_1\}$ and any three even or odd numbered vertex accordingly as set S is chosen.

$\langle V - S\rangle$ is also connected. S is also outer connected resolving set. The minimum cardinality of outer connected resolving set is called as ocr-set of G, $\dim_{oc}(G) = 3$.

5. Neighbourhood Dimension of Some Identity Graph of Finite Group

Theorem 5.1. *For identity graph* $(Z_n, \oplus_n)$ *for* $n > 3$ *odd number has neighborhood dimension as* $nmd(G) = \frac{n+1}{2}$.

Proof. Let graph $G = (Z_n, \oplus_n)$ for $n > 3$ odd number

The vertex set of G is $V(G) = \{0, 1, 2, \ldots, n-1\} = \{x_0, x_1, x_2, \ldots, x_{\frac{n-1}{2}}, x_{\frac{n+1}{2}}, \ldots, x_{n-1}\}$.

The edge set of G is $E(G) = \{x_0x_i, x_1x_{n-1}, x_2x_{n-2}, \ldots, x_{\frac{n-1}{2}}x_{\frac{n+1}{2}}\}$, $1 \leq i \leq n-1$.

$|V(G)| = n, |E(G)| = \frac{3n-3}{2}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

If $|S| = 2$ i.e. subset S can take any of the vertex $\{x_i, x_j\}, i \neq j$ $0 \leq i, j \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = \frac{n+1}{2}$. The subset S is considered in such a way choose the vertex x_0 and $\frac{n-1}{2}$ odd vertices like $\{x_1, x_3, \ldots\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. And $\bigcup_{s \in S} \langle N[s] \rangle = G$. S is also a neighborhood set. Also set S is of minimum cardinality of neighborhood resolving set of graph G. The cardinality of S is called as neighborhood metric dimension of G, $nmd(G) = \frac{n+1}{2}$.

Theorem 5.2. *For the identity graph* $(Z_n, \oplus_n)$ *for* $n > 4$ *even number has neighborhood dimension to be* $nmd(G) = \frac{n}{2}$.

Proof. Let graph $G = (Z_n, \oplus_n)$ for $n > 4$ even number.

The vertex set of G is $V(G) = \{0, 1, 2, \ldots, n-1\} = \{x_0, x_1, x_2, \ldots, x_{n2}, \ldots, x_{n-1}\}$.

The edge set of G is $E(G) = \{x_0 x_i, x_1 x_{n-1}, x_2 x_{n-2}, \ldots, /1 \leq i \leq n-1\}$

$| V(G) | = n, | E(G) | = \frac{3n-3}{2}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex x_i, $0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

If $|S| = 2$ i.e. subset S can take any of the vertex $\{x_i, x_j\}, i \neq j$ $0 \leq i, j \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = \frac{n}{2}$. The subset S is considered in such a way choose the vertex x_0 and $\frac{n-2}{2}$ odd vertices like $\{x_1, x_3, \ldots, x_{n-2}\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. And $\bigcup_{s \in S} \langle N[s] \rangle = G$. S is also a neighborhood set. Also set S is of minimum cardinality of neighborhood resolving set of graph G. The cardinality of S is called as neighborhood metric dimension of G, $nmd(G) = \frac{n}{2}$.

Theorem 5.3. *The neighborhood dimension for the identity graph of klein*-4 *group under composition is* 3.

Proof. Let graph G = Identity graph of Klein-4 group

The vertex set of G is $V(G) = \{x_0, x_1, x_2, x_3\} = \{e, a, b, ab\}$.

The edge set of G is $E(G) = \{x_0 x_i / 1 \leq i \leq 3\}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex $x_i, 0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = 3$. The subset S is considered in such a way choose the vertex x_0 and any two vertices from $\{x_1, x_2, x_3\}$. Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. And $\bigcup_{s \in S} \langle N[s] \rangle = G$. S is also a neighborhood set. Also set S is of minimum cardinality of neighborhood resolving set of graph G. The cardinality of S is called as neighborhood metric dimension of G, $nmd(G) = 3$.

Theorem 5.4. *For the identity graph of quaternion group neighborhood dimension is* 4.

Proof. Let graph G = identity graph of Q_8.

The vertex set of G is $V(G) = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$.

The edge set of G is $E(G) = \{x_0 x_i, x_2 x_3, x_4 x_5, x_6 x_7 : 1 \leq i \leq 7\}$.

Let $S \subseteq V(G)$. If $|S| = 1$ i.e. subset S can take any one of the vertex $x_i, 0 \leq i \leq n-1$. If this subset is consider each vertex has no distinct codes. Therefore S is not a resolving set.

Continuing like this if $|S| = 4.$ The subset S is considered in such a way choose the vertex x_0 and any three odd or even numbered vertices from $\{x_1, x_2, x_3, x_4, x_5, x_6\}.$ Consider these vertices as one set. Clearly each vertex receives distinct codes i.e. distance between the vertex x_i to the set S is distinct. Therefore set S is a resolving set. And $\bigcup_{s \in S} \langle N[s] \rangle = G.$ S is also a neighbourhood set. Also set S is of minimum cardinality of neighborhood resolving set of graph G. The cardinality of S is called as neighborhood metric dimension of G, $nmd(G) = 4.$

Reference

[1] B. Sooryanarayana and A. S. Suma, On classes of neighborhood resolving sets of a graph, Electronic Journal of Graph Theory and Applications 6(1) (2018), 29-36.

[2] F. Harary and R. Melter, On the metric dimension of a graph, Ars Combinatoria 2 (1976), 191-195.

[3] W. B. V. Kandasamy and F. Smarandache, Groups as Graphs, Editura CuARt2009, https://www.gallup.unm.edu/~smarandache/eBooks-otherformats.htm

[4] J. Sivakumar, A. Wilson Baskar, R. Sundareswaran, P. Nataraj and V. Swaminathan, Outer connected resolving sets in a graph, Adv. in Mathematics: Scientific Journal 10(3) (2021), 1367-1376.

[5] J. Sivakumar, A. Wilson Baskar and V. Swaminathan, Co-equitable resolving sets of a graphs, International Journal of Mathematics and Computer Research, ISSN:2320-7167 07(05) (2019), 1970-1972. doi:10.31142/ijmcr/v7i5.03

[6] P. J. Slater, Leaves of trees, Proc. 6th Southeastern Conference on Combinatorics, Graph Theory, and Computing (Florida Atlantic Univ., Boca Raton, Fla, 1975), Congressus Numerantium, no. 14, Utilitas Math., Winnipeg, Manitoba, 1975, pp. 549-559.