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ON COUNTABILITY IN MIXED FUZZY TOPOLOGICAL SPACES

G. C. RAY and H. P. CHETRI

Department of Mathematics
Central Institute of Technology
Kokrajhar, Assam, India
E-mail: agautomofcit@gmail.com

Morigaon Town Secondary School
Morigaon, Assam, India
E-mail: bhariprasadchetri1234@gmail.com

Abstract

In this article, given a mixed fuzzy topological space $(X, \tau_1(\tau_2))$, we investigate a few properties of countability using the concepts of quasi-coincidence in mixed fuzzy topological spaces and constructed a countable basis for $\tau_1(\tau_2)$, from a countable basis of τ_2.

1. Introduction

The concept of mixed topology is a technique of mixing two topologies on a non-empty set to get a third topology on that set. The notion of mixed topology first incepted and investigated by Alexiewicz and Semaden [1]. They have introduced this notion via two norm spaces. Thereafter, many researchers around the globe worked on mixed topology and established many interesting and applicable results on mixed topology. Some of the remarkable results are found in the research works due to Cooper [4], Buck [2], Wiweger [15] and many others.

In 1965, L. A. Zadeh [17] introduced the concept of fuzzy sets. Since the inception of the notion of fuzzy sets and fuzzy logics, fuzziness has been applied for the study in almost all branches of science and technology. The notion of fuzziness has been applied in topology for the first time by C. L.

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Chang [3] and the notion of fuzzy topological spaces has been introduced and investigated by many researchers on fuzzy topological spaces. The concept of strong separation and strong countability in fuzzy topological spaces was introduced and investigated by M. Macho. Stadler et al. [10]. In 1995, the concept of mixed fuzzy topological spaces is being investigated from different aspects by Das and Baishya [5]. Recently, in 2012, Tripathy and Ray [11] re-state the definition of mixed fuzzy topology as introduced by Das and Baishya [5] by replacing fuzzy point to fuzzy set. Tripathy and Ray [11] introduced the concepts of countability in the light of mixed fuzzy topology and proved the existence of three types of countability. But this concept of mixed fuzzy topology is not the generalization of the concept of crispy mixed topology due to Wiweger [15], Alexiewicz and Semadeni [1], Cooper [4] and others. In this article, we have established some results on countability.

2. Preliminaries

Let X be a non-empty set and I, the unit interval $[0, 1]$. A fuzzy set A in X is characterized by a function $\mu_A : X \to I$, where μ_A is called the membership function of A. $\mu_A(x)$ representing the membership grade of x in A. The empty fuzzy set is defined as $\mu_\phi(x) = 0$ for all $x \in X$. Also X can be regarded as a fuzzy set in itself defined $\mu_X(t) = 1$ for all $t \in X$. Further, an ordinary subset A of X can also be regarded as a fuzzy set in X if its membership function is taken as usual characteristic function of A that is $\mu_A(x) = 1$, for all $x \in X$ and $\mu_A(x) = 0$ for all $x \in X - A$. Two fuzzy set A and B are said to be equal if $\mu_A = \mu_B$. A fuzzy set A is said to be contained in a fuzzy set B, written as $A \subseteq B$, if $\mu_A \le \mu_B$. Complement of a fuzzy set A in X is a fuzzy set A in X defined as $\mu_{A^c} = 1 - \mu_A$. We write $A^c = coA$ if there is no confusion. Union and intersection of a collection $\{A_i : i \in I\}$ of fuzzy sets in X, to be written as $\underset{i=1}{\cup} A_i$ and $\underset{i=1}{\cap} A_i$ respectively, are defined as follows:

$$\mu_{\bigcup_{i \in I} A_i}(x) = \sup\{\mu_{A_i}(x) : i \in I\}, \text{ for all } x \in X$$

and $\mu_{\bigcap_{i \in I} A_i}(x) = \inf \{\mu_{A_i}(x) : i \in I\}$ for all $x \in X$.

Definition 2.1 [3]**.** Let $I = [0, 1]$, X be a non empty set, and I^X be the collection of all mappings from X into I, i.e. the class of all fuzzy sets in X.

A fuzzy topology on X is a family τ of member of I^X such that

(i) $\overline{1}, \overline{0} \in \tau$,

(ii) τ is closed under formation of arbitrary union and closed under formation of finite intersection in the fuzzy sense.

The pair (X, τ) is called a fuzzy topological space (fts in short) and members of τ are called τ-open fuzzy sets.

Definition 2.2 [5]**.** The closure of a fuzzy set A in a fts (X, τ) is defined as the intersection of all closed supersets of A, i.e. $\overline{A} = \bigcap\{F : A \subset F, F \text{ is closed fuzzy set}\}$. One can prove that $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$ for any two subsets A, B of (X, τ).

Definition 2.3 (Definition 2.1 [8])**.** A fuzzy set X is called a fuzzy point iff it takes the value 0 for all $y \in X$ except one, say $x \in X$. If its value at x is $\alpha(0 < \alpha \leq 1)$ then we denote this fuzzy point by x_α and we call the point x its support.

Definition 2.4 (Definition 2.2 [8])**.** A fuzzy point x_λ is said to be contained in a fuzzy set A or belongs to A, denoted by $x_\alpha \in A$ iff $\alpha \leq A(x)$. Evidently, every fuzzy set A can be expressed as the union of all the fuzzy points which belong to A.

Definition 2.5 (Definition 2.3 [8])**.** Two fuzzy sets A and B in X are said to be intersecting iff there exists a point $x \in X$ such that $(A \cap B)(x) \neq 0$. In this case, we say that A and B intersect at x.

Definition 2.6 (Definition 1.6 [8])**.** A collection B of open fuzzy sets in a fts X is said to be an open base for X if every open fuzzy sets in X is a union of members of B.

Definition 2.7 (Definition 2.4 [8]). A fuzzy set A in a fuzzy topological space (X, τ) is called a neighbourhood of a point $x_\lambda \in X$ if and only if there exists $B \in \tau$ such that and $x_\lambda \in B \subseteq A$; neighbourhood A is called open neighbourhood iff A is open. The family consisting of all the neighbourhood of x_λ called system of neighbourhoods of x_λ.

Definition 2.8 (Definition 2.2 [8]). A fuzzy point x_α is said to be quasi-coincident with A, denoted by $x\alpha q A$, if and only if $\alpha + A(x) > 1$ or $\alpha > (A(X))^c$.

Definition 2.9 (Definition 2.3 [8]). A fuzzy set A is said to be quasi-coincident with B and is denoted by AqB, if and only if there exists a $x \in X$ such that $A(x) + B(x) > 1$.

It is clear that if A and B are quasi-coincident at x both $A(x)$ and $B(x)$ are not zero at x and hence A and B intersect at x.

Definition 2.10 (Definition 2.4 [8]). A fuzzy set A in an fts (X, τ) is called a quasi-neighbourhood (Q-neighbourhood) of x_λ iff $\exists A_1 \in \tau$ such that $A_1 \subseteq A$ and $x_\lambda q A_1$. The family $U_{x\lambda}$ consisting of all the Q-neighbourhoods of x_λ. is called the system of Q-neighbourhood system of x_λ. Intersection of two quasi-neighbourhood of x_λ is a quasi-neighbourhood.

Ming and Ming [8] established the following necessary and sufficient conditions for existence of a fuzzy topology. This result is applied to ensure the existence of the mixed topology.

Proposition 2.1 (Proposition 2.2 [8]). *Let U be the family of quasi-neighbourhood of a fuzzy point x in* (X, τ). *Then.*

(i) x_λ *is quasi-coincident with A for every* $A \in U_{x_\lambda}$

(ii) *If* $A, B \in U_{x_\lambda}$ *then* $A \cap B \in U_{x_\lambda}$.

(iii) *If* $A \in U_{x_\lambda}$ *and* $A \subset B$, *then* $B \in U_{x_\lambda}$.

Conversely, for each fuzzy point x_λ in X if U_{x_λ} is the family of fuzzy sets in X satisfying the conditions (i), (ii) and (iii) then the family τ of all fuzzy sets A, such that $A \in U_{x_\lambda}$ whenever $x\lambda q A$ is a fuzzy topology for X.

Theorem 2.1 (Theorem 3.1 [5]). *Let (X, τ_1) and (X, τ_2) be two fuzzy topological spaces. Consider the collection of fuzzy sets $\tau_1(\tau_2) = \{A \in I^X :$ for any $x\alpha q A$, there exists τ_2-Q-neighbourhood A_α of x_α such that τ_1-closure $\overline{A}_\alpha \subseteq A\}$. Then this family of fuzzy sets will form a topology on X and this topology is called a mixed fuzzy topology on X.*

Lemma 2.1 (Lemma 3.2 [5]). *Let τ_1 and τ_2 be two fuzzy topological spaces on a set X. If every τ_1-quasi-neighbourhood of x_α is τ_2-quasi-neighbourhood of x_α for all fuzzy point x_α then τ_1 is coarser than τ_2.*

Theorem 2.2 (Theorem 3.3 [5]). *Let τ_1 and τ_2 be two fuzzy topological spaces on a set X. Then mixed fuzzy topology $\tau_1(\tau_2)$ is coarser than τ_2. In symbol $\tau_1(\tau_2) \subseteq \tau_2$.*

Theorem 2.3 (Theorem 3.1 [11]). *Let (X, τ_1) and (X, τ_2) be two fuzzy topological spaces. Consider the collection of fuzzy sets $\tau_1(\tau_2) = \{A \in I^X :$ For any fuzzy set B in X with AqB, there exists τ_2-open set A_1 such that $A_1 qB$ and τ_1-closure $\overline{A}_1 \subseteq A\}$. Then this family of fuzzy sets will form a topology on X and it is called mixed fuzzy topology on X.*

Theorem 2.4 (Theorem 4.1 [11]). *If $(X, \tau_1(\tau_2))$ is a C_I-space, then it is a QC_I-space.*

Proposition 2.1 (Proposition 4.2 [11]). *If $(X, \tau_1(\tau_2))$ is a C_{II}-space, then it is also QC_I-space.*

Proposition 2.2 (Proposition 4.3 [11]). *Let τ_1 and τ_2 be two fuzzy topology on a non-empty set X and if the mixed fuzzy topology $\tau_1(\tau_2)$ is Q-first countable, then τ_2 is also Q-first countable.*

3. Main Results

We have produced an alternative and stronger way to prove the collection of fuzzy sets $\tau_1(\tau_2) = \{A \in I^X$: for any fuzzy set B in X with AqB, there exists τ_2-open set A_1 such that $A_1 qB$ and τ_1-closure $\overline{A_1} \subseteq A\}$ is a mixed fuzzy topology that is already proved in (Theorem 3.1) [11] and we have also investigated some results on countability in mixed fuzzy topological spaces.

Throughout this article, we shall use this definition of mixed fuzzy topological space.

Theorem 3.1. *Let (X, τ_1) and (X, τ_2) be two fuzzy topological spaces. Consider the collection of fuzzy sets $\tau_1(\tau_2) = \{A \in I^X$: for any fuzzy set B in X with AqB, there exists τ_2-open set A_1 such that $A_1 qB$ and τ_1-closure $\overline{A_1} \subseteq A\}$. Then this family of fuzzy sets will form a topology on X and this topology we call mixed fuzzy topology on X.*

Proof. $\overline{0} \in \tau_1(\tau_2)$, vacuously true.

$\overline{1} \in \tau_1(\tau_2)$, since for any fuzzy set $Bq\overline{1}$, $\overline{1} \in \tau_2$ and $\overline{1}qB$ and τ_1-closure of $\overline{1} = \overline{1} \subseteq \overline{1}$.

(T_2) Let $A_1, A_2 \in \tau_1(\tau_2)$. To show that $A_1 \cap A_2 \in \tau_1(\tau_2)$.

Let B be any fuzzy set such that $(A_1 \cap A_2)qB \Rightarrow A_1 qB$ and $A_2 qB \Rightarrow \exists O_1$ and O_2 open in τ_2 such that τ_1-closure of $\overline{O_2} \subseteq A_2$. and τ_1-closure of $\overline{O_2} \subseteq A_2$. [Since $A_1, A_2 \in \tau_1(\tau_1)$].

Therefore, $O_1 \cap O_2 \in \tau_2$ and $\overline{O_1} \cap \overline{O_2} \subseteq A_1 \cap A_2$. So, $\overline{O_1 \cup O_2} \subseteq \overline{O_1} \cap \overline{O_2}$ $\Rightarrow A_1 \cap A_2 \in \tau_1(\tau_2)$.

(T_3) Let $\{A_\lambda : \lambda \in \Delta\}$ be a family of open sets in $\tau_1(\tau_2)$.

We want to show that $\underset{\lambda \in \Delta}{U} A_\lambda \in \tau_1(\tau_2)$.

Let B be any fuzzy set such that $Bq \underset{\lambda \in \Delta}{U} A_\lambda$

$\Rightarrow B(x) + \underset{\lambda \in \Delta}{U} A_\lambda(x) > 1,$ for some $x \in X.$

$\Rightarrow B(x) + \sup_{\lambda \in \Delta} \{A_\lambda(x)\} > 1$

$\Rightarrow B(x) + A_{\lambda_0}(x) > 1,$ for some $\lambda_0 \in \Delta.$

$\Rightarrow BqA_{\lambda 0}$

$\Rightarrow \exists A_1 \in \tau_2$ such that $A_1 qB$ and that τ_1-closure of $\overline{A}_1 \subseteq A_{\lambda_0} \subseteq A_{\lambda_0}$ $\subseteq \underset{\lambda \in \Delta}{U} A_\lambda$

$\Rightarrow \underset{\lambda \in \Delta}{U} A_\lambda \in \tau_1(\tau_2).$

Therefore, $\tau_1(\tau_2)$ is a fuzzy topology and we called it a mixed fuzzy topology.

Theorem 3.2. *Let τ_1 and τ_2 be two fuzzy topological spaces on a set X. Then the mixed fuzzy topology $\tau_1(\tau_2)$ is coarser than τ_2. In symbol, $\tau_1(\tau_2) \subseteq \tau_2$.*

Proof. Let U_{x_λ} be a $\tau_1(\tau_2)$ quasi-neighbourhood of x_λ. Then $\exists A \in \tau_1(\tau_2)$ such that $x\lambda qA$ and $A \subseteq U_{x_\lambda}$. (Since every fuzzy point is a fuzzy singleton set). So $\exists \tau_2$-open set A_1 such that $A_1 q x_\lambda$ and τ_1-closure $\overline{A}_1 \subseteq A$. Therefore, $x\lambda q A_1$ and $A_1 \subseteq \overline{A}_1 \subseteq A \subseteq U_{x_\lambda}$. Thus U_{x_λ} is also a τ_2-quasi-neighbourhood of x_λ. Hence, $\tau_1(\tau_2)$ is coarser than τ_2. That is $\tau_1(\tau_2) \subseteq \tau_2$.

Now, we construct two examples of mixed fuzzy topological space.

Example 3.1. Let $X = \{x, y\}$. Consider the fuzzy sets $A = \{(x, 0.4), (y, 0.6)\}$ and $B = \{(x, 0.6), (y, 0.4)\}$.

Now, we consider two topologies $\tau_1 = \{\overline{0}, \overline{1}, B\}$ and $\tau_2\{\overline{0}, \overline{1}, A\}$ on X.

If we consider a fuzzy set $B_1 = \{(x, 0.7), (y, 0.3)\}$, then $B_1 qA$ because $B_1(x) + A(x) = 0.7 + 0.4 = 1.1 > 1$.

Also, there exists $A \in \tau_2$ such that AqB_1 and $\tau_1\text{-}cl(\overline{A}) \subseteq A$. Since $\tau_1\text{-}cl(\overline{A}) = B^C = A$ is a closed set.

Therefore, $A \in \tau_1(\tau_2)$. Since is $\tau_1(\tau_2) \subseteq \tau_2$ (Theorem 3.2). So $\tau_1(\tau_2) = \{\overline{0}, \overline{1}, A\} = \tau_2$.

Example 3.2. Let $X = \{x, y\}$.

Consider the fuzzy sets $A = \{(x, 0.4), (y, 0.6)\}$, $B = \{(x, 0.4), (y, 0.4)\}$ and $C = \{(x, 0.6), (y, 0.6)\}$ on X.

Also consider the collection of fuzzy sets $\tau_1 = \{\overline{0}, \overline{1}, A\}$ and $\tau_2 = \{\overline{0}, \overline{1}, B, C\}$.

Then, obviously τ_1 and τ_2 are fuzzy topology on X.

Now, we shall construct the mixed fuzzy topology $\tau_1(\tau_2)$ on X.

Possible quasi-coincident fuzzy sets to the set B are

(i) $B_1 = \{(x, \alpha > 0.6), (y, \beta \leq 0.6)\}$

(ii) $B_2 = \{(x, \alpha \leq 0.6), (y, \beta \leq 0.6)\}$

(iii) $B_3 = \{(x, \alpha > 0.6), (y, \beta \leq 0.6)\}$

For the fuzzy set B_1,

$B_1 qB$, and for $B \in \tau_2$, $\tau_1\text{-}cl(\overline{B}) = \{(x, 0.6), (y, 0.4)\}$. Therefore, $\tau_1\text{-}cl(\overline{B}) \not\subset B$.

Also for $C \in \tau_2$, CqB_1 and $\tau_1\text{-}cl(\overline{C}) = \overline{1} \not\subset B$.

Similarly for the cases B_2 and B_3, $\tau_1\text{-}cl(\overline{B}) \not\subset B$, $\tau_1\text{-}cl(\overline{C}) \not\subset B$.

Therefore, $B \notin \tau_1(\tau_2)$.

Again for the open set $C \in \tau_2$.

Let $B_4 = \{(x, 0.7), (y, 0.4)\} \in I^X$

CqB_4 and $\exists B \in \tau_2$ and BqB_4.

Also $\tau_1\text{-}cl(\overline{B}) = \{(x, 0.6), (y, 0.4)\} \subseteq C$.

But for $B_5 = \{(x, 0.5)\}$, CqB_5 and $C \in \tau_2$ and CqB_4.

But $\tau_1\text{-}cl(\overline{C}) = \overline{1} \not\subset B$.

Hence, $C \notin \tau_1(\tau_2)$ and so $\tau_1(\tau_2) = \{\overline{0}, \overline{1}\} \neq \tau_2$.

Now, we include following definitions:

Definition 3.1 (Definition 3.1 [8]). Let U_{x_λ} be a neighbourhood system of a fuzzy point x_λ. A sub-family B_{x_λ} of U_{x_λ} is called a Q-neighbourhood base of x_λ iff for each $A \in U_{x_\lambda}$ there exist a member B of B_{x_λ} such that $B < A$.

Definition 3.2 (Definition 3.1 [8]). Let $U_{x_\lambda Q}$ be a Q-neighbourhood system of a fuzzy point x_λ in (X, τ). A sub-family $B_{x_\lambda Q}$ of $U_{x_\lambda Q}$ is called a Q-neighbourhood base of $U_{x_\lambda Q}$ iff for each $A \in U_{x_\lambda Q}$ there exist a member $B \in B_{x_\lambda Q}$ such that $B < A$.

Definition 3.3 (Definition 3.1 [8]). A fuzzy topological space (X, τ) is said to satisfy the first axiom of countability if every fuzzy point x_λ in (X, τ) has a countable neighbourhood base. A topological space (X, τ) which satisfy first axiom of countability is called C_I- space.

Definition 3.4 (Definition 3.1 [8]). A fuzzy topological space (X, τ) is said to satisfy the Q-first axiom of countability or to be $Q - C_I$ iff every fuzzy point x_λ in (X, τ) has a countable Q-neighbourhood base. A topological space (X, τ) which satisfy Q-first axiom of countability is called $Q - C_I$ space.

Definition 3.5 (Definition 3.4 [16]). A topological space (X, τ) is said to satisfy the second axiom of countability or said to be C_{II} space iff τ has a countable base.

Theorem 3.3. *If* (X, τ_2) *is a* C_I*- space then* $(X, \tau_1(\tau_2))$ *is a* C_I*- space.*

Proof. Let x_λ be a fuzzy point in $(X, \tau_1(\tau_2))$.

Let U be any open neighbourhood system (without loss of generality) of x_λ in $\tau_1(\tau_2)$.

$\Rightarrow$ U is also an open neighbourhood system of x_λ in τ_2. (Since $\tau_1(\tau_2) \subseteq \tau_2$.)

$\Rightarrow$ There exists a countable sub collection of U say B which is also a base for U.

(Since τ_2 is a C_1- space)

$\Rightarrow$ For every open neighbourhood A in U, there exists a neighbourhood $B \in B$ such that $x_\lambda \in B < A$.

$\Rightarrow$ B is a countable neighbourhood base for U in $\tau_1(\tau_2)$.

$\Rightarrow$ $(X, \tau_1(\tau_2))$ is a C_I- space.

Theorem 3.4. *If* (X, τ_2) *is a* C_I*- space then* $(X, \tau_1(\tau_2))$ *is a* Q*-*C_I*- space.*

Proof. Let (X, τ_2) is a C_I- space.

$\Rightarrow$ $(X, \tau_1(\tau_2))$ is a C_I- space. (By Theorem 3.3)

$\Rightarrow$ $(X, \tau_1(\tau_2))$ is a Q-C_I- space. (By Theorem 2.4)

Theorem 3.5. *If* (X, τ_2) *is a* C_{II}*- space, then* $(X, \tau_1(\tau_2))$ *is also* C_{II}*- space.*

Proof. Let $B = \{B_i : i \in \Delta\}$ be a countable base for τ_2.

$\Rightarrow$ For every open set $A_1 \in \tau_2$, $A_1 = \cup\{C_j : C_j \in B\}$.

$\Rightarrow$ For every open set $B \in \tau_1(\tau_2)$, $B = \cup\{D_j : D_j \in B\}$. (Since $\tau_1(\tau_2) \subseteq \tau_2$)

Now we will construct a countable base for $\tau_1(\tau_2)$. First we make sub-collection $B_1 = \{B_k \in B : B_k \in \tau_1(\tau_2)\}$. Therefore, $B_1 \subseteq B$ and so B_1 is countable.

Let us consider, $A \in \tau_1(\tau_2)$ which is not in B_1. Otherwise B_1 is countable basis for $\tau_1(\tau_2)$. Therefore, $A = \bigcup\{B_j : B_j \in B$ and at least one $B_j \notin B_1\}$. Now, the candidate for a base is $B_1^* = B_1 \cup \{A\}$. Again, if we consider another open set $C \in \tau_1(\tau_2)$ which cannot be expressible by the elements of B_1^*, then we continue as the previous step, otherwise B_1^* is a countable basis for $\tau_1(\tau_2)$. The process can be continued countably to get a countable basis for $\tau_1(\tau_2)$. or have to exhausted all $B_i \in B$ after countable numbers of step to get a countable base for $\tau_1(\tau_2)$. Therefore, $(X, \tau_1(\tau_2))$ is C_{II}.

Proposition 3.1. *If* $(X, \tau_1(\tau_2))$ *is a* C_I*- space, then* (X, τ_2) *is also* Q-C_{I}*-space.*

Proof. Let $(X, \tau_1(\tau_2))$ is a C_I- space.

$\Rightarrow (X, \tau_1(\tau_2))$ is a Q-C_{I}- space. (By Theorem 2.4)

$\Rightarrow (X, \tau_1)$ is Q-C_{I}- space. (Proposition 2.2)

Proposition 3.2. *If* (X, τ_2) *is a* C_{II}*- space, then* $(X, \tau_1(\tau_2))$ *is also* Q-C_{I}*- space.*

Proof. Let (X, τ_2) is a C_{II}- space.

$\Rightarrow (X, \tau_1(\tau_2))$ is C_{II}- space. (Theorem 3.5)

$\Rightarrow (X, \tau_1(\tau_2))$ is Q-C_{I}- space. (Proposition 2.1)

Proposition 3.3. *If* $(X, \tau_1(\tau_2))$ *is a* C_{II}*- space, then it is also* Q-C_{I}*- space.*

Proof. Let $(X, \tau_1(\tau_2))$ is a C_{I}- space.

$\Rightarrow (X, \tau_1(\tau_2))$ is a Q-C_{I}- space. (Theorem 2.4)

$\Rightarrow (X, \tau_2)$ is Q-C_{I}- space. (Proposition 3.2)

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