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A REVIEW: THREE PHASE LAG THERMO-ELASTICITY

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Abstract

This paper throws a light on the field of thermos-elasticity which was studied under various media pertaining to surface waves. The article particularly focuses on reviewing the extensive work done on Three-Phase-Lag Model (3-PLM), as this model is of great relevance nowadays. The 3-PLM was employed and applied by many researchers to scrutinize the various effects and particularly for mechanical and temperature. The paper also put an emphasis on the squandering of energy or in simple terms energy dissipation while studying the thermos-elasticity. This paper will be of great interest for the budding researchers as this article deals with the research work starting from the pioneer work done by Biot, Lord and Shulman, Green and Naghdi and many more are reviewed in detail.

1. Introduction

The word Thermo-elasticity can be termed as a phenomenon in which there are various changes in the material, it may be the size or shape of the material. These changes on material happen due to the fact that there is addition of thermal energy or in simple terms the change in temperature is responsible for the said changes in the material. It is a well known fact that the elastic materials have the property which makes expansion or contraction in the material as reversible more while comparing with the so called inelastic material. It can also be defined as the dependence of the stress distribution of an elastic solid on its thermal state, or of its thermal conductivity on the stress distribution. Thermo-elastic stress analysis is used

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to identify defects in polymeric materials and it is also used for determining the temperature dependence of the unperturbed mean-square end-to-end distance of macro-molecules. The thermos-elasticity is an aspect which investigates about mainly 02 fields i.e. mechanical and thermal related to solid bodies. Both of these fields have a significant and remarkable impact on the expansion or contraction of an elastic as well as in-elastic material. Various types of thermos-elasticity include i.e. uncoupled, coupled and generalized thermos-elasticity. Biot [1-6] brought this remarkable theory about the thermos-elasticity in the world's arena and there on the researchers further enhanced and strengthens his theory. Biot developed a hypothesis which is related to thermos-elasticity (coupled) which can be read as that the elastic changes have a significant effect on the temperature. The equations which were developed in the work for calculating the thermal variables for uncoupled and coupled relates to diffusion type forecasting the propagation speed to a value which cannot be counted i.e. infinity, for waves which have thermal content in nature and this is exactly opposite to the remarks physically. Hetnarski and Ignaczack [7] acquired significant knowledge in the field of thermos-elasticity. The work is of great importance as they examined the 05 generalization related to the basic theory of thermos-elasticity (coupled). The Fourier law which was termed as standard for the researchers is substituted by a hypothesis which was then called a new law related to thermal conduction. In this law, substitution of a single parameter is done i.e. relaxation time and this term was brought in preview of world by Lord and Shulman [8]. These researchers were also considered as the generators of 1st generalization theory related to thermos-elasticity. This hypothesis not only engulfs the thermal flux which is in vector form but also its time derivatives. This theory also ensures a limited value for the propagation speed for elastic as well as thermal waves resulting from the thermal equation which are of wave type. The controlling equations remain identical for the thermos-elasticity theories (uncoupled and coupled). The credit of 2nd generalization for the thermos-elasticity theory (coupled) goes to Muller [9] who introduced the concept of two relaxation time. The approach used by Muller allows not only for second sound effect but also explains that the usual heat conduction tensor associated with Fourier's law is symmetric for linearized theory by putting the restrictions on the class of basic variables, by the help of entropy inequality involving an entropy flux vector. A more explicit version was then introduced by Green and Laws [10], Green and Lindsay [11] and independently by Suhubi [12]. By considering the time rate between the basic variables and also predicts the finite propagation speed for elastic and thermal waves similar to the Lord-Shulman theory without violating the Fouriers law of heat conduction. For the anisotropic media, Lord and Shulman had developed a theory which is extensively known as LS-Theory and this theory was given stronger and wider scope by Dhaliwal and Sherief [13]. Thereafter, Chandrasekharaiah [14] presented a concept of 2nd sound which is nothing but an interruption similar to a wave. Hetnarski and Ignaczack [15] introduced the 3rd generalization of thermos-elasticity (coupled) which is known as low temperature nonlinear thermos-elastic model and is also named as H-I theory. The 4th generalization of the thermos-elasticity is based on thermos-elasticity without energy dissipation (T-WO-ED) which is also termed as GN-Theory (TYPE-II) [16, 17] is based on thermos-elastic behaviour (WO-ED), although it concerned with both linear and non linear theories but the significance is given to the linearized theory because in this the heat flow does not involve energy dissipation, constitutive equations for entropy flux vector and stress are determined by the same potential function, and finally this theory allows the transmission of heat as thermal waves at finite speed. The 4th generalization of the thermo-elasticity is also based on with energy dissipation (T-W-ED) which was introduced by Green and Naghdi which is also termed as GN-Theory (TYPE-III) [16,17] and is concerned with undamped thermal waves in an elastic solid accommodating the finite wave speed. The problem relating to GN-Theory (TYPE-II) and GN-Theory (TYPE-III) have been considered by many researchers. Dual phase lag thermos-elasticity (DPLT) developed by Tzou [18] and Chandrasekharian [19] was considered as the 5th generalization to the thermos-elasticity theory. Tzou proposed the universal model of heat conduction by considering the microstructural effect into the delayed response in time in the macroscopic formulation by taking into account the increased lattice temperature which is delayed due to phonon-electron interaction on the macroscopic level. Two Phase Lag to both heat flux vector and temperature gradient is also introduced. According to Dual phase lag model (DPLM), the classical Fourier law $q = -K\nabla T$ is replaced by $q(s, t + \tau_q) = -K\nabla T(s, t + \tau_T)$, where the temperature gradient ∇T at point

s of the material at time $t + \tau_T$, corresponds to heat flux q at the same point with time $t + \tau_q \cdot \tau_T$, τ_q is the phase lag of the temperature gradient and heat flux respectively, caused by microstructural interactions and fast transient effect of the thermal inertia respectively. The propagation of waves have got quite a big attention from the researcher which are dealing with astronautics, earth-quake engineering, rocket engineering and many more engineering areas.

Many researchers have worked on many interesting problems in context of generalized thermos-elasticity which in-turns brought out these various generalizations of thermos-elasticity theories, for examining the plane harmonic waves which are in unbounded media. Nayfeh and Nemat-Nasser [20] deployed the Maxwell heat conduction equation (modified form) and also examined the Rayleigh-Surface wave with these mentioned Maxwell equations. The materials which were isotropic in character were examined by McCarthy [21] in relation to wave propagation, postulated the fact that principle waves which are 04 in number may propagate. The medium which is transversely isotropic are critically examined for the propagation of waves by Sharma and Singh [22]. Chadwick [23] studied the problem of wave propagation for homogeneous plane elastic waves for getting a basic level of understanding of behavior of surface waves. The investigation of transverse wave propagation having an amplitude which is limited in number was credited to Haddow and Erbay [24]. Sharma et al. [25] investigated the Rayleigh wave propagation for the various factors which are free from heat, stress and charge. Thereafter, Sharma and Sharma [26] critically investigated the characteristics of propagation for Lamb-wave which is of an elasto-thermo-diffusive type. The orthotropic plates which are free from stress and having a condition that the temperature is constant throughout were examined by Yu et al. [27] for guided wave propagation. The researcher has made this hypothesis while considering zero dissipation of energy in the framework of famous GN-Theory for thermos-elasticity and the plates in this work were laminated. The anisotropic media which is high in order or order of fractional type incorporating the time rate of displacement (having voids) was looked into by Abd-Alla et al. [28] for a case where surface wave propagation was studied. The researcher for examining the results of gaps and rotation on surface waves also obtained the general surface wave speed.

It was found in the literature that the researchers were having a keen interest for the case which has zero squandering of energy and in this context; Singh [29] investigated about the wave propagation particularly for plane waves for the conditions that were 2-temperature, isotropic (transverse) and lastly the rotating. The researcher also focused on the fact that there are 03 waves which are known as plane waves. For this, the researcher critically evaluated and solved the critical equations which were also termed as controlling equations. The researcher also examined the image of the discussed wave for a condition where temperature is constant and during this 03 equations were also gathered which were of non-homogeneous by character for the mirror imaged wave.

2. Literature Survey Related to Three-Phase-Lag Model (3-PLM)

One of the theory related to 3-PLM was brought in the researcher's preview by Choudhuri [30] which is also called the sixth generalization of thermos-elasticity. In this model Fourier law of heat conduction is replaced by an approximation to a modification of the Fourier law with the introduction of three different phase lags for the heat flux vector, the temperature gradient and the thermal displacement gradient i.e. heat conduction equation is

$$\vec{q}(s, t + \tau_q) = -[k\vec{\nabla}(s, t + \tau_T) + K^*\vec{\nabla}v(s, t + \tau_v)]. \tag{1}$$

The coupled system of equation is

$$\left(1 + \tau_q \frac{\partial}{\partial t}\right)(\rho c_v \ddot{T} + \gamma T_0 \ddot{\Delta} - \rho\dot{Q}) = \tau_v^* \nabla^2 \dot{T} + k\tau_T \nabla^2 \ddot{T} + k^* \nabla^2 T \text{ and}$$

$$\mu\nabla^2\vec{u} + (\lambda + \mu)\nabla(\nabla \cdot \vec{u}) - \gamma\nabla T + \rho\vec{F} = \rho\ddot{\vec{u}}. \tag{2}$$

These equations represent the equation of three phase lag thermos-elasticity (3-PLT) for isotropic material and also admitting damped solution due to the existence of temperature rate term. If $k >> k^*$ the above mentioned system reduces to that of Chandrasekharaiah [19] whereas for $\tau_q = 0, \tau_T = 0$ and $\tau_v = 0$ the above coupled system of equation reduces to Green and Naghdi [16] and for $\tau_q = 0, \tau_T = 0$ and $\tau_v = 0$ and $\tau_v^* = k$ the above coupled system of equation reduces to Green and Naghdi [17]. Also for hyperbolic thermos-elasticity with three phase lag model (3-PLM) the coupled equation is

$$\left(1+\tau_q \frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2 \frac{\partial}{\partial t^2}\right)(\rho c_v \ddot{T}+\gamma T_0 \ddot{\Delta}-\rho \dot{Q})=\tau_v^* \nabla^2 \dot{T}+k\tau_T \nabla^2 \ddot{T}+k^* \nabla^2 T \text{ and}$$

$$\mu \nabla^2 \vec{u}+(\lambda+\mu)\nabla(\nabla \cdot \vec{u})-\gamma \nabla T+\rho \vec{F}=\rho \ddot{\vec{u}}. \tag{3}$$

For $k^* = 0$ the above equation of 3-PL hyperbolic thermos-elasticity reduces to Chandrasekharaiah [19] and when $\tau_q^2 = 0$, $\tau_T = 0$, $k^* = 0$ and $\tau_v = 0$ and hence, $\tau_v^* = k$. The above coupled equations reduce to L-S Theory. Quintanilla and Racke [31] further emphasized on the system which deals with the problems related to thermal conduction having 3-PLM for having a better understanding of the discussed problem. They considered 02-cases and put forward Lyapunov function, which can be used to examine the properties qualitatively. The researcher also confirms the stability of the solution exponentially by gathering suitable conditions for the factors related to materials. As the interest of the researchers grows in this 3-PLM and its allied theory, Kar and Kanoria [32] showcased the crux of the different available theories related to TE-GN-III (with energy squandering). During the literature survey it was found that Kanoria and Mallik [33] examined the interrelationship for the temperature, visco-elastic medium in alike, yet conducting agent thermally. This examination was done for the reason that there is existence of periodic heat source which change w.r.t. time. These researchers also differentiated the extracts of various theories such as 3-PLM, GN-II,-III. While working and analyzing the field of Thermo-elasticity Mukhopadhyay et al. [34] studied the research work of Roy-Choudhuri and then described the G-K-type result for the thermos-elasticity which falls in the category of linear theory. While carrying out the research, a general proposition from the results were gathered after solving the equations formed through G-K type. Thereafter, a system was developed in which relations related to oscillations were developed and these were time independent. Then, the final result for the relations which were homogeneous in nature was showcased for time independent oscillations. Kumar and Chawla [35] analyzed the propagation of waves which were plane in character in the framework of 2-PLM and most importantly the 3-PLM. All these examinations were done in a medium which is of anisotropic type. During this literature work the researchers came across and presented in their postulation that there are 03-types of waves which can be written as quasi-longitudinal wave which has temperature in consideration and finally 02 waves which were of transverse type. A particular manifestation was deduced from the controlling equation for iso-tropic transverse 3-PLM which was of homogeneous type. The researcher showcased their work in the form of various graphs as well as in the form of values for the piercing distance, specific loss, velocity (phase) and many more. During the literature survey of 3-PLM a significant work of Das and Kanoria [36] came across the researchers in the framework of thermos-elasticity with 02-cases i.e. with and without squandering of energy and 3-PLM. They also brought in light the complications that aroused due to the interactions which were of magneto-thermo-elastic type. The controlling equations were set up for thermos-elasticity under the application of the field which is magnetic in-character for the studied model. The researchers got their result for the problem by applying the method i.e. Laplace-Fourier transformation. The researcher showcased their results by using various types of graphs and they also displayed the numerical values for the terms i.e. strain and stress, temperature and displacement. Kumar and Chawla [37] critically examined the phenomenons which were of great interest i.e. refraction and mirror image (reflection) which occur due to the waves which were incident on a interface of a plane in oblique fashion. This study was also carried out under the study of 3-PLM and that too in half space. This study also validated the conservation of energy principle at the plane interface and this was very significant for the budding researchers to understand. Ezzat et al. [38] worked in the field of thermos-elasticity and presented a numerical model for the conduction of heat and in which the researcher deduced 3-PLM by applying a special technique of fractional calculus. During this research the numerical model was of great interest as the heat source was changing w.r.t. time and this model suits for the thermos-elastic interactions. Deswal and Kalkal [39] studied the 3-PLM in different contexts i.e. 02 temperature. The researcher also throws light on the complications which arises due to the interaction of thermo-visco-elastic in a 3-D medium with special focus on temperature shock. The researcher used a special method i.e. commonly known as analysis of normal mode to find out the values resulting from the equation for the variables such as strain and stress, temperature and displacement. Kumar and Gupta [40] put forward the heat transfer which is of 3-PLM type for the thermos-elastic substances. During this examination the researcher applied the fractional order hypothesis of thermos-elasticity. During analysis a particular situation was deduced from the controlling equation for the 3-PLM which is isotropic and transverse in character. The researcher pictorially showcased and mathematically calculated the wave phenomenon. Othman [41] critically scrutinized a complex situation of 2-D in the field of micro-polar thermos-elasticity. During this examination, the researcher put forward 3-PLM to examine the result of stress (initial) and rotation for the said situation. The analysis using normal mode is applied so as to find the numerical value coming out from the equation to find out the strain and stress, temperature, displacement and finally the components related to rotation. The researcher also showcased these variables pictorially and in the form of numerical values. This hypothesis put forward an idea that the said variables have a significant effect due to stress (initial) and rotation. Said [42] reviewed the propagation of waves under the effect of 02-fields i.e. magnetic and gravitational. The researcher used GN-Theory and 3-PLM having zero squandering of energy to examine the situation of general thermo-elasticity, this case is special as the heat source is constantly moving with a uniform speed. The analysis using normal mode was used to find out the values of strain and stress, displacement and finally the temperature. In the pictorial representation the variables were showcased during activation and de-activation of the 02-fields i.e. gravitational and magnetic. Othman et al. [43] investigated on the thermo-elastic interaction for a surface which is loaded thermally as well as mechanically this investigation also put light in the context of a field which is magnetic in character with stress (initial). The analysis using normal mode was applied to find out the values and equations for the various components. The researcher also presented his solution in the framework of activation and de-activation of the magnetic field and stress (initial), while comparing with the available theories in literature such as 3-PLM, GN-03 and L-S.

3. Nomenclature

λ, μ : constants of Lame's

α_t : Linear thermal expansion coefficient

e_{ij} : Strain tensor

σ_{ij} : Stress tensor

T : Absolute Temperature

T_0 : Temperature in natural state (un-deformed and unstressed state)

$\bar{u}$: Displacement vector

ρ : Density

δ_{ij} : Kroneckers delta

K : Thermal conductivity coefficient

K^* : Material constant coefficient

C^* : Specific heat

∇ : Gradient

∇^2 : Laplacian operator

τ_T : Temperature gradient- PHL

τ_q : Heat Flux-PHL

τ_v : Displacement gradient (thermal)-PHL

t : Time

F_i : Component of Body force F

k: Diffusivity coefficient

τ_0 : Relaxation Time

q_i : Heat-Flux (vector)

q: The rate of generation of heat/unit volume.

4. Basic Equations related to the Thermo-elasticity

The main equation of isotropic thermos-elasticity are as follows:

(i) The relationship between the displacement and strain is:

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{4}$$

(ii) Equation of Compatibility:

$$e_{ij,kl} + e_{kl,ij} - e_{ik,jl} - e_{jl,ik} - 0 \quad (i, j, k, l) = (1, 2, 3), \tag{5}$$

(iii) The equation of motion or equilibrium:

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i \tag{6}$$

(iv) Stress-Strain relations:

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \gamma T)\delta_{ij} \tag{7}$$

(v) Heat conduction equations: For the uncoupled and unsteady case, when the parameters related to thermal are independent of temperature or position; the heat conduction equation can be written as :

$$K \nabla^2 T = \frac{\partial T}{\partial t} - W$$

$$\nabla^2 = \frac{\partial^2}{\partial x_i \partial x_j}; \; K = \frac{k}{\rho c}; \; W = \frac{q}{\rho c}. \tag{8}$$

When the coupling between strain and temperature fields is considered, the coupled equation of heat conduction is :

$$K \nabla^2 T = \frac{\partial T}{\partial t} + \beta \frac{\partial e_{kk}}{\partial t} - W, \tag{9}$$

where $\beta = \frac{(3\lambda + 2\mu)\alpha T_0}{\rho c} = \frac{\gamma T_0}{\rho c}$.

A wave type of heat conduction is given by Lord and Shulman (1967)

$$k \nabla^2 T = \frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} + \beta\left(\frac{\partial e_{kk}}{\partial t} + \frac{\tau_0 \partial^2 e_{kk}}{\partial t^2}\right) - W', \tag{10}$$

where $W' = \frac{Q + \tau_0 \dot{Q}}{\rho c} = W + \tau_0 \frac{\dot{Q}}{\rho c} = W + \tau_0 \dot{W}$.

Equation (7) is based upon the generalized Fourier law (for isotropic materials)

$$q_i + \tau_0 \dot{q}_i = -kT_{,i}. \tag{11}$$

Using the equations by Choudhari [30], the fundamental equations in homogenous Isotropic thermo-elastic medium with 3-PLM

The equation of Motion with 3-PLM is:

$$(\lambda + \mu)\nabla(\nabla \cdot u) + \mu\nabla^2 u - \beta\nabla T = \rho\frac{\partial^2 u}{\partial t^2}. \tag{12}$$

The generalized equation for the heat conduction with 3-PLM is:

$$k\left(1 + \tau_T \frac{\partial}{\partial t}\right)\nabla^2 T + k^*\left(1 + \tau_v \frac{\partial}{\partial t}\right)\nabla^2 T = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)$$

$$(\rho C^* T + \beta T_0 e_{kk}). \tag{13}$$

The Constitutive law of Thermo-elasticity (generalized) with 3-PLM is:

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) - \beta T\delta_{ij} \tag{14}$$

and $\beta = (3\lambda + 2\mu)\alpha_t$.

5. Conclusion

The theory of Thermo-elasticity and Three-Phase Lag Model (3-PLM) under various media pertaining to surface waves was reviewed in this article. The fundamental equations of 3-PLM were reviewed in the frame work of L-S Theory, G-N Theory and many more. It can be concluded after this detailed review that the results which were gathered after solving the 3-PLM were on the same lines as that of the results which came out from the famous G-N Model (II and III). It was also found that researchers used 3-PLM to examine the effects such as mechanical and thermal and these effects are of keen interest for the researchers nowadays. In this review work it was found that a number of theories related to thermo-elasticity and 3-PLM have been evolved and developed over the years and these theories were being used to solve a number of problems such as heat flow. This paper will enable the future researchers to have in-sight of thermo-elasticity and 3-PLM in detail.

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