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ANTI INTUITIONISTIC FUZZY GRAPH

M. MOHAMMED JABARULLA and K. ASMIYA BANU

Department of Mathematics
Jamal Mohamed College
Tiruchirappalli-620 020
Tamil Nadu, India
E-mail: m.md.jabarulla@gmail.com
asmiyabanu1995@gmail.com

Abstract

In this paper we define the concept of anti intuitionistic fuzzy graph is introduced. Some interesting properties of anti intuitionistic fuzzy graph are established with proofs and examples.

2. Preliminaries

In this section, we give some basic definitions.

Definition 2.1. Intuitionistic Fuzzy Graph (IFG). An Intuitionistic fuzzy graph is the form $G = (V, E)$ where,

(i) $V = \{v_1, v_2, v_3, \ldots, v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in V, i = (1, 2, \ldots, n)$.

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \le \min [\mu_1(v_i), \mu_1(v_j)]$, $\gamma_2(v_i, v_j) \le \max [\gamma_1(v_i), \gamma_1(v_j)]$ $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$. For every $(v_i, v_j) \in E, (i, j = 1, 2, \ldots, n)$.

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Definition 2.2. Strong Intuitionistic Fuzzy Graph (SIFG). An intuitionistic fuzzy graph, $G = (V, E)$ is said to be a Strong intuitionistic fuzzy graph if $\mu_2(v_i, v_j) = \min [\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) = \max [\gamma_1(v_i), \gamma_1(v_j)]$ for all $(v_i, v_j) \in E$.

Definition 2.3. Complement of intuitionistic fuzzy graph (CIFG). Let $G = \langle V, E \rangle$ be the intuitionistic fuzzy graph and it is said to be complement if $\overline{G} = \langle \overline{V}, \overline{E} \rangle$ where,

(i) $\overline{V} = V$

(ii) $\overline{\mu_{1i}} = \mu_{1i}$ and $\overline{\gamma_{1i}} = \gamma_{1i}$, for all $i = 1, 2, \ldots, n$

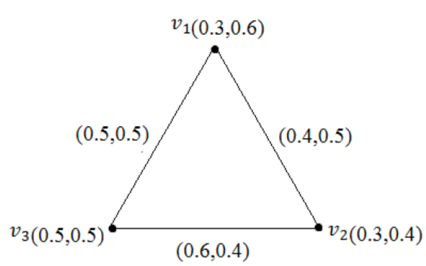
(iii) $\overline{\mu_{2ij}} = \mu_{2ij} - \min [\mu_1(v_i), \mu_1(v_j)]$, and $\overline{\gamma_{2ij}} = \gamma_{2ij} - \max [\gamma_1(v_i), \gamma_1(v_j)]$, for all $i, j = 1, 2, \ldots, n$.

3. Anti Intuitionistic Fuzzy Graph

Definition 3.1. Anti intuitionistic fuzzy graph (AIFG). An anti intuitionistic fuzzy graph is of the form $G_A = \langle V, E \rangle$ where,

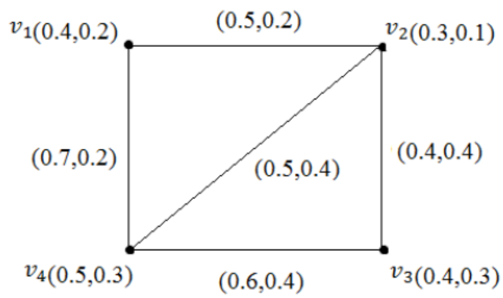
(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_1) + \gamma_1(v_1) \leq 1$ for every $v_i \in V$ $(i = 1, 2, \ldots, n)$.

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \geq \max [\mu_1(v_i), \mu_1(v_j)]$, $\gamma_2(v_i, v_j) \geq \min [\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$, $(i, j = 1, 2, \ldots, n)$.



G_A

Definition 3.2. Anti intuitionistic fuzzy subgraph (AIFSG). An anti intuitionistic fuzzy graph $H_A = \langle V', E' \rangle$ is said to be an anti intuitionistic fuzzy subgraph of the intuitionistic fuzzy graph, $G_A = \langle V, E \rangle$ if $V' \subseteq V$ and $E' \subseteq E$. In other words, if $\mu'_{1i} \leq \mu_{1i}$; $\gamma'_{1i} \geq \gamma_{1i}$ and $\mu'_{2ij} \leq \mu_{2ij}$; $\gamma'_{2ij} \geq \gamma_{2ij}$ for every $i, j = 1, 2, \ldots, n$.



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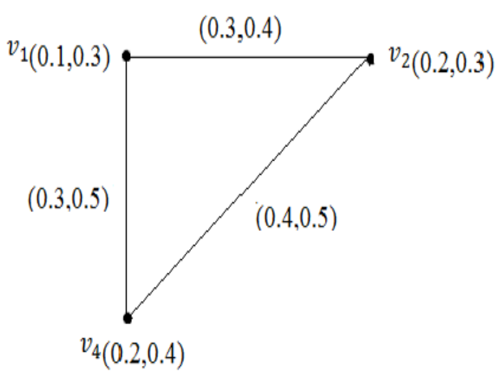


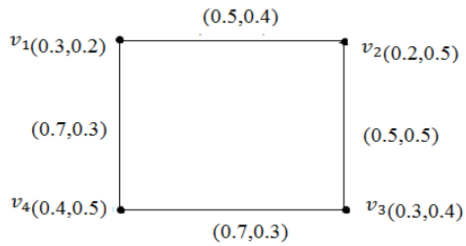
H_A

Definition 3.3. Complement of anti intuitionistic fuzzy graph (IVIFGST) Let $G_A = \langle V, E \rangle$ be the anti intuitionistic fuzzy graph and it is said to be complement if $\overline{G_A} = \langle \overline{V}, \overline{E} \rangle$ where,

(i) $\overline{V} = V$

(ii) $\overline{\mu_{1i}} = \mu_{1i}$ and $\overline{\gamma_{1i}} = \gamma_{1i}$, for all $i = 1, 2, \ldots, n$

(iii) $\overline{\mu_{2ij}} = \mu_{2ij} - \max\ [\mu_1(v_i), \mu_1(v_j)]$, and $\overline{\gamma_{2ij}} = \gamma_{2ij} - \max\ [\gamma_1(v_i), \gamma_1(v_j)]$, for all $i, j = 1, 2, \ldots, n$.



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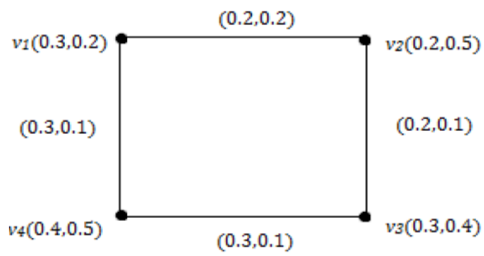
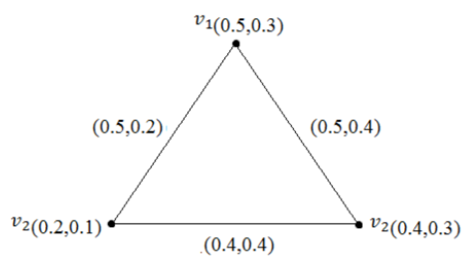


$\overline{G_A}$

Definition 3.4. Semi-μ strong anti intuitionistic fuzzy graph (SSAIFG) An anti intuitionistic fuzzy graph $G_A = \langle V, E \rangle$ is said to be a semi-μ strong if $\mu_2(v_i, v_j) = \max\ [\mu_1(v_i), \mu_1(v_j)]$, for every $(v_i, v_j) \in E$.



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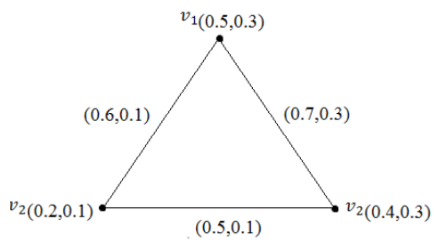
Definition 3.5. Semi- γ strong anti intuitionistic fuzzy graph (SSAIFG) An anti intuitionistic fuzzy graph $G_A = \langle V, E \rangle$ is said to be a semi-γ strong if $\gamma_2(v_i, v_j) = \min\ [\gamma_1(v_i), \gamma_1(v_j)]$, for every $(v_i, v_j) \in E$.



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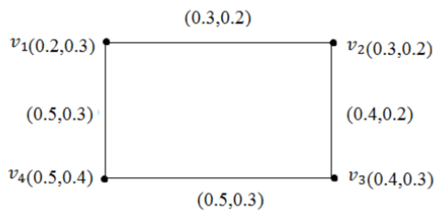
Definition 3.6. Strong anti intuitionistic fuzzy graph (SAIFG) A graph $G_A = \langle V, E \rangle$ is said to be a strong anti intuitionistic fuzzy graph if $\mu_2(v_i, v_j) = \max[\mu_1(v_i), \mu_1(v_j)]$, $\gamma_2(v_i, v_j) = \min[\gamma_1(v_i), \gamma_1(v_j)]$, for all $(v_i, v_j) \in E$.



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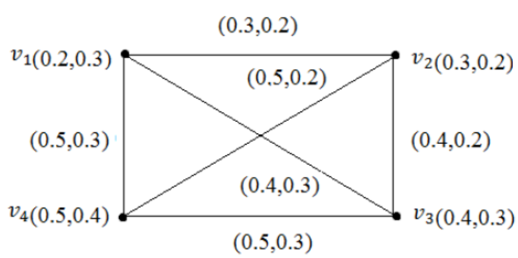
Definition 3.7. Complete anti intuitionistic fuzzy graph (CAIFG) A graph $G_A = \langle V, E \rangle$ is said to be a complete anti intuitionistic fuzzy graph if $\mu_2(v_i, v_j) = \max[\mu_1(v_i), \mu_1(v_j)]$, $\gamma_2(v_i, v_j) = \min[\gamma_1(v_i), \gamma_1(v_j)]$, for all $v_i, v_j \in V$.



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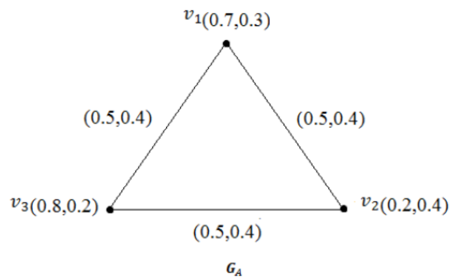
Definition 3.8. Degree of a vertex Let $G_A = \langle V, E \rangle$ be an anti intuitionistic fuzzy graph. Then the degree of a vertex v is defined by $d_{G_A}(v) = (d_\mu(v_i), d_\gamma(v_i))$ where,

$$d_{\mu}(v_i) = \sum_{u \neq v} \mu_{2(v,\, u)}$$

$$d_{\gamma}(v_i) = \sum_{u \neq v} \gamma_{2(v,\, u)}.$$



Degree of vertices:

$$d_{\mu}(v_1) = 1 \qquad d_{\gamma}(v_1) = 0.8$$

$$d_{\mu}(v_2) = 1 \qquad d_{\gamma}(v_2) = 0.8$$

$$d_{\mu}(v_3) = 1 \qquad d_{\gamma}(v_3) = 0.8.$$

Theorem 3.9. *If* $G_A(\mu, \gamma)$ *is an anti intuitionistic fuzzy graph with* $G_A(V, E)$. *Then*

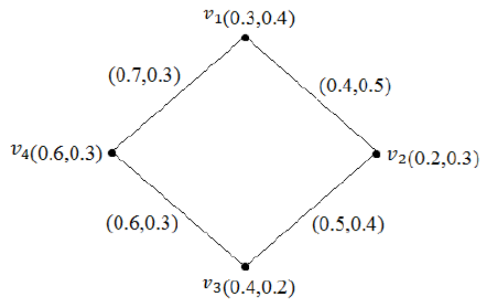
$$\sum_{i=1}^{n} d(\mu_1(v_i)) = 2 \sum_{i=1}^{n} \mu_{2ij}(v_i, v_j)$$

$$\sum_{i=1}^{n} d(\gamma_1(v_i)) = 2 \sum_{i=1}^{n} \gamma_{2ij}(v_i, v_j).$$

Proof. Consider G_A be an anti intuitionistic fuzzy graph with $G_A(V, E)$. Let us consider G_A has *n* vertices such as $v_1, v_2, \ldots v_n$.

We know that each edge is incident with two vertices. Then by the definition of degree of vertex of an anti intuitionistic fuzzy graph, the sum of the degree of vertices of membership of an anti intuitionistic fuzzy graph is twice the sum of membership of edges in G_A.

Similarly, the sum of degree of vertices of non-membership of an anti intuitionistic fuzzy graph is twice the sum of non-membership of edges in G_A.



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Degree of vertices:

$$d_\mu(v_1) = 1.1 \quad d_\gamma(v_1) = 0.8$$

$$d_\mu(v_2) = 0.9 \quad d_\gamma(v_2) = 0.9$$

$$d_\mu(v_3) = 1.1 \quad d_\gamma(v_3) = 0.7$$

$$d_\mu(v_4) = 1.3 \quad d_\gamma(v_4) = 0.6$$

$$\sum_{i=1}^{4} d(\mu_1(v_i)) = 1.1 + 0.9 + 1.1 + 1.3 = 4.4$$

$$2\sum_{i=1}^{4} \mu_{2ij}(v_i, v_j) = 2(0.4 + 0.5 + 0.6 + 0.7) = 2(2.2)$$

$$2\sum_{i=1}^{4} \mu_{2ij}(v_i, v_j) = 4.4.$$

Therefore,

$$\sum_{i=1}^{4} d(\mu_1(v_i)) = 2\sum_{i=1}^{4} \mu_{2ij}(v_i, v_j).$$

Similarly,

$$\sum_{i=1}^{4} d(\gamma_1(v_i)) = 0.8 + 0.9 + 0.7 + 0.6 = 3$$

$$2\sum_{i=1}^{4} \gamma_{2ij}(v_i, v_j) = 2(0.5 + 0.3 + 0.4 + 0.3) = 2(1.5)$$

$$2\sum_{i=1}^{4} \gamma_{2ij}(v_i, v_j) = 3.$$

Therefore,

$$\sum_{i=1}^{4} d(\gamma_1(v_i)) = 2\sum_{i=1}^{4} d(\gamma_{2ij}(v_i, v_j)).$$

Hence proved.

5. Conclusion

In this paper, we defined a new concepts called anti intuitionistic fuzzy graph (AIFG) and discussed some operations with suitable examples and theorems are proved.

In future we are planning to extend our investigation to some more operations and applications of anti intuitionistic fuzzy graph.

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