Advances and Applications in Mathematical Sciences
Volume 22, Issue 1, November 2022, Pages 289-293


# MORE RESULTS ON 2-ODD LABELING OF GRAPHS

**P. ABIRAMI[1], N. SRINIVASAN[2] and A. PARTHIBAN[3]**

[1,2]Department of Mathematics
St. Peter's Institute of Higher Education and Research
Avadi, Chennai - 600 054, Tamil Nadu, India
E-mail: sriabi1@yahoo.co.in
sri24455@yahoo.com

[3]Division of Mathematics
School of Advanced Sciences
Vellore Institute of Technology
Vellore- 632 014, India
Email: parthiban.a@vit.ac.in

## Abstract

A 2-odd labeling of $G(V, E)$ is an injection '$f$' from $V$ to $Z$ (the set of all integers) such that the absolute difference between the labels of the adjacent nodes is either an odd number or exactly 2. If $G$ admits 2-odd labeling, then it is called 2-odd graph. This paper discusses 2-odd labeling of some graphs and highlights a few notable applications of graph labeling in manufacturing.

## 1. Introduction

Only finite, simple, connected, and undirected graphs are studied in this article. Let $G(V, E)$ be a graph with set $V$ and the line set $E$. For graph theory concepts, we refer to [3]. According to Laison et al. [4] $G$ is 2-odd if there exists an injective labelling $h : V(G) \to Z$ such that for any two nodes $x$ and $y$ which are adjacent, the integer $|h(x) - h(y)|$ is either an odd or exactly 2. It is also defined that $h(st) = |h(s) - h(t)|$ and called $h$ 2-odd labelling of $G$. So $G$ is 2-odd graph iff there exists 2-odd labelling of $G$.

2020 Mathematics Subject Classification: 05C78.
Keywords: Graph, Graph labeling, 2-odd labeling, 2-odd graph.
Received May 27, 2022; Accepted June 1, 2022

Moreover, $h(xy)$ may still be either 2 or odd if $xy$ is not an line of $G$. For more results on 2-odd graphs, one can refer to [1, 2, and 4].

## 2. Main Results

This section is devoted for deriving 2-odd labeling of various classes of graphs, besides recalling few relevant concepts for the study undertaken.

**Definition 1.** [4] The graph $Pl_n = (V, E)$, where $V\{1, 2, \ldots, n\}$ and $E = (K_n)/\{(k, l)3 \leq n-2$ and $k+2 \leq l \leq n\}$ is a planar graph having maximum number of lines, with $n$ nodes. The planar graph $Pl_n$ having maximum number of lines with $n$ nodes is obtained by removal of $[(n-4)(n-3)]/2$ lines from $K_n$, The number of lines in $Pl_n : n \geq 5$ is $3(n-2)$. One such example $Pl_{10}$ class graph is shown in Figure 1.

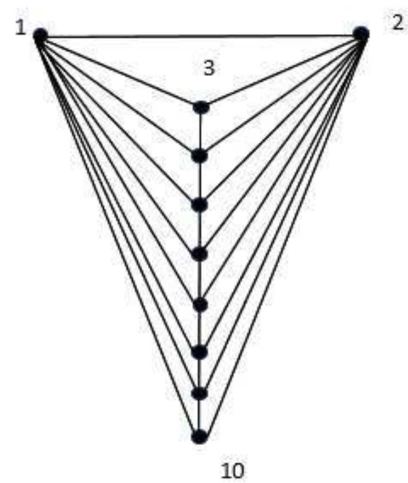


**Figure 1.** Planar graph $Pl_{10}$.

**Theorem 1.** *The planar graph* $Pl_n$ *allows* 2-*odd labeling*.

**Proof.** Let $Pl_n$ be the planar graph with $V(Pl_n) = V_1 U V_2$, where $V_1 = \{u_1, u_2\}$ and $V_2 = \{v_i; 1 \leq i \leq n\}$. Now define an injective function $f : V(Pl_n) \to Z$ as given: let $f(u_1) = 1$ and $f(u_1) = -1$. Then $f(v_i) = 2i; 1 \leq i \leq n$. One can see that $|f(u_1) - f(u_2)| = 2$ and $|f(u_1) - f(v_i)|$

are odd numbers and $| f(u_2) - f(v_i) |$ are also odd numbers for all $1 \leq i \leq n$. Hence $f$ is the required $2-$odd labelling of $Pl_n$.

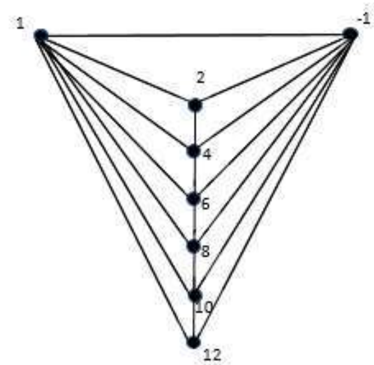


**Figure 2.** Planar graph $Pl_8$.

**Definition 2** [6]. The friendship graph $F_{r_n}^{(k)}$ is a planar undirected graph with $(k+1)n+1$ nodes and $kn$ lines connected by connecting $n$ copies of the $C_k$ with a common node.

**Theorem 2.** $F_{r_n}^{(k)}$ *allows* $2-$*odd labelling* $\forall\, k \geq 3$.

**Proof.** Let $F_{r_n}^{(k)}$ be on $(k+1)n+1$ nodes and $kn$ lines. Let $v_0$ be the central node and $v_i^j : 1 \leq i \leq k, 1 \leq j \leq n$ be the nodes on the first cycle, second cycle, and so on up to $n^{\text{th}}$ cycle. Now we define a one- to- one labelling $f : V(F_{r_n}^{(k)}) \to Z$ as follows: without loss of generality, let the central node $v_0$ be labelled with 0, i.e., $f(v_0) = 0.$ There arise two cases.

**Case (i).** The cycle $C_k$, when $k$ is odd.

Without loss of generality, we label the nodes on the first cycle, starting with the second node, say $v_1$ (as the first node $v_0$ is already labelled) by giving 1, for $v_2$ assign 2, and consecutively up to $(n-1)^{th}$ node assign $n-1.$ Finally for the $n^{th}$ node in the first cycle, we label by adding 2 with $(n-1)^{th}$

label. That is $f(v_n^1) = f(v_{n-1}^1) + 2.$ Similarly, we label the second node of the second cycle with $f(v_n^1) + 2,$ the third node with $f(v_2^2) + 1,$ and so on up to the $(n-1)^{th}$ node of the second cycle. Finally, the nth node of the second cycle is labeled with $f(v_{n-1}^2) + 2.$ Proceeding the same for the remaining cycles, see that *f* is the 2 – odd labeling of $F_{r_n}^{(k)}.$

**Case (ii).** The cycle $C_k,$ when *k* is even

The graph thus obtained is a bipartite graph and the result follows from the fact that every bipartite graph is a 2-odd graph.

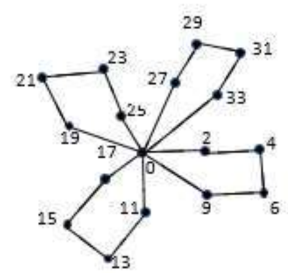


**Figure 3.** 2- Odd Labeling of friendship graph.

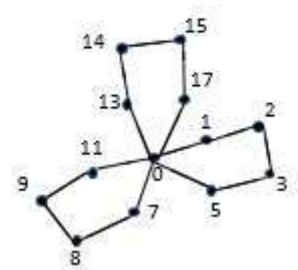


**Figure 4.** 2-odd Labeling of friendship graph.

## 3. Applications of Graph Labeling

Graph theory is used to support selection of materials for designs in engineering and the identification of a system's suitability for remanufacturing. Graph theory might be a more appropriate tool to use the advantages of a product's state-based representation of a manufacturing program's data and information gives. The consequent states along with the manufacturing program and the (inter-) relations between the state characteristics believed to provide an important foundation for graph theory without requiring huge adaptation. For a complete study, one can refer to [7].

## 4. Conclusion

In this paper 2-odd labeling of some special graphs such as a planar graph, and friendship graph is derived, besides recalling interesting applications of graph theory in manufacturing. A complete characterization of 2-odd graphs is still an open problem for further study.

## References

[1] P. Abirami, A. Parthiban and N. Srinivasan: On 2-Odd Labeling of Graphs, European Journal of Molecular and Clinical Medicine 07(07) (2020), 3914-3918.

[2] P. Abirami, A. Parthiban, and N. Srinivasan, Some Results on 2-odd Labeling of Graphs, International Journal of Recent Technology and Engineering (IJRTE) 8(5) (2020), 5644-5646.

[3] V. Govindan and S. Dhivya, Difference Labelling of Jewel Graph, International Journal of Mathematics Trends and Technology (IJMTT) 65(4) (2019), 64-68.

[4] J. Baskar Babujee, Planar Graphs with Maximum Lines – Antimagic Property, The Mathematics Education 37(04) (2003), 194-198.

[5] S. M. Lee, Graceful Labelling of Mongolian tents and Related Graphs, Congr. Number 50 (1985), 85-96.

[6] S. Meena and K. Vaithilingam, Prime Labelling of Friendship Graphs, International Journal of Engineering Research and Technology (IJERT) 1(10) (2012), 1-13.

[7] Thorsten Wuestab, Benjamin Knokea and Klaus-Dieter Thoben, Applying Graph Theory and the Product State Concept in Manufacturing, Procedia Technology 15 (2014), 349-358.