NEUTROSOPHIC GENERALISED REGULAR* CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract

Exploring a new type of neutrosophic set in neutrosophic topology is the major aim of our research. In this paper, the concepts “Neutrosophic generalised regular star closed sets” and “Neutrosophic generalised regular star open sets” are newly defined and their properties and some interesting theorems are introduced. We have analyzed the relationships between this newly introduced sets and the already existing neutrosophic sets.

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I. Introduction

In the past several decades, fuzzy set theory has played an important role in the research of mathematics. The research on the theory of fuzzy sets has been witnessing an exponential growth in mathematics. Zadeh established the fuzzy set as an extension of a classical notion of crisp set in 1965 [13]. K. Atanassov, extended the intuitionistic fuzzy set as a generalization of fuzzy set in 1983 [3]. Then Florentin Smarandache generalized the concept intuitionistic fuzzy sets as Neutrosophic sets in 1999 [5]. Later A. Salama and S. A. Alblowi studied the concept of neutrosophic topological spaces [9]. Wadei Al-Omeri and Saeid Jafari discovered the generalized closed sets and generalized pre-closed sets in neutrosophic topological space which belong to the important class of neutrosophic sets [12].

II. Preliminaries

Definition 2.1 [9]. Let $\mathcal{X}$ be a non-empty fixed set. A neutrosophic set (NS) $G$ is an object having the form $\{(x, \mu_G(x), \sigma_G(x), \nu_G(x)) : x \in \mathcal{X}\}$ where $\mu_G(x)$, $\sigma_G(x)$ and $\nu_G(x)$ represent the degree of membership, degree of indeterminacy and the degree of nonmembership respectively of each element $x \in \mathcal{X}$ to the set $G$. A Neutrosophic set $G = \{(x, \mu_G(x), \sigma_G(x), \nu_G(x)) : x \in \mathcal{X}\}$ can be identified as an ordered triple $(\mu_G, \sigma_G, \nu_G)$ in $[0, 1]^3$ on $\mathcal{X}$.

But in real life application, using Neutrosophic set with values from real standard or non-standard subset of $[0, 1]^3$ is very difficult. So that we used the Neutrosophic set which takes values from the subsets of $[0, 1]$. Here, the neutrosophic topological space is denoted by $(\mathcal{X}, \tau_N)$. Also the neutrosophic interior, neutrosophic closure of a neutrosophic set $G$ are denoted by $N_{int}(G)$ and $N_{cl}(G)$. The complement of a neutrosophic set $G$ is denoted by $(G)$ and the empty and whole sets are denoted by $0_N$ and $1_N$ respectively.

Definition 2.2 [1]. We have used the following definitions throughout this paper.

1. $G \subseteq H \iff \mu_G(x) \leq \mu_H(x)$, $\sigma_G(x) \leq \sigma_H(x)$ and $\nu_G(x) \geq \nu_H(x) \forall x \in \mathcal{X}$
2. \( G \cap H = \{ x, \mu_G(x) \land \mu_H(x), \sigma_G(x) \land \sigma_H(x), v_G(x) \lor v_H(x) \} \)

3. \( G \cup H = \{ x, \mu_G(x) \lor \mu_H(x), \sigma_G(x) \lor \sigma_H(x), v_G(x) \land v_H(x) \} \)

4. \( C(G) = \{(x, v_G(x), 1-\sigma_G(x), \mu_G(x)) : x \in \mathcal{X}\} \)

5. \( 0_N = \{(x, 0, 0, 1) : x \in \mathcal{X}\} \)

6. \( 1_N = \{(x, 1, 1, 0) : x \in \mathcal{X}\} \).

**Definition 2.3.** [2]. A subset \( G \) of a neutrosophic topological space \((\mathcal{X}, \tau_N)\) is called,

1. a neutrosophic semi open set (NSO set) if \( G \subseteq N_{cl}(N_{int}(G)) \) and a neutrosophic semi closed set (NSC set) if \( N_{int}(N_{cl}(G)) \subseteq G \).

2. a neutrosophic preopen set (NPO set) if \( G \subseteq N_{int}(N_{cl}(G)) \) and a neutrosophic pre closed set (NPC set) if \( N_{cl}(N_{int}(G)) \subseteq G \).

3. a neutrosophic \( \alpha \)-open set (NaO set) if \( G \subseteq N_{int}(N_{cl}(N_{int}(G))) \) and a neutrosophic \( \alpha \)-closed (NaC set) if \( N_{cl}(N_{int}(N_{cl}(G))) \subseteq G \).

4. a neutrosophic semi pre open set (NSPO set) if \( G \subseteq N_{cl}(N_{int}(N_{cl}(G))) \) and a neutrosophic semi pre closed set (NSPC set) if \( N_{int}(N_{cl}(N_{int}(G))) \subseteq G \).

5. a neutrosophic regular open (NRO) set if \( G = N_{int}(N_{cl}(G)) \) and a neutrosophic regular closed (NRC) set if \( G = N_{cl}(N_{int}(G)) \).

**Definition 2.4.** [4]. A subset \( G \) of a neutrosophic topological space \((\mathcal{X}, \tau_N)\) is called a neutrosophic generalized closed set (NGC set) if \( N_{cl}(G) \subseteq H \) whenever \( G \subseteq H \) and \( H \) is NO in \((\mathcal{X}, \tau_N)\). The complement of a NGC set is called a NGO set.

**Definition 2.5.** [10]. A subset \( G \) of a neutrosophic topological space \((\mathcal{X}, \tau_N)\) is called a neutrosophic generalized semi closed set (briefly NGSC) if \( N_{cl}(N_{int}(G)) \subseteq H \) whenever \( G \subseteq H \) and \( H \) is NSO in \((\mathcal{X}, \tau_N)\). The complement of a NGSC set is called a NGSO set.

**Definition 2.6.** [11]. A subset \( G \) of a neutrosophic topological space \((\mathcal{X}, \tau_N)\) is called a neutrosophic generalized regular closed set (NGRC) if \( G = N_{cl}(N_{int}(G)) \).
(\mathcal{X}, \tau_N) is called a neutrosophic generalized \(\alpha\) closed set (briefly NG\(\alpha\)C) if \(N\alpha_{cl}(G) \subseteq H\) whenever \(G \subseteq H\) and \(H\) is NaO in \((\mathcal{X}, \tau_N)\). The complement of a NG\(\alpha\)C set is called a NG\(\alpha\)O set.

**Definition 2.7** [7]. A subset \(G\) of a neutrosophic topological space \((\mathcal{X}, \tau_N)\) is called a neutrosophic \(\alpha\) generalized closed set (briefly NG\(\alpha\)C) if \(N\alpha_{cl}(G) \subseteq H\) whenever \(G \subseteq H\) and \(G\) is NO in \((\mathcal{X}, \tau_N)\). The complement of a NaGC set is called a NaGO set.

**Definition 2.8** [6]. A subset \(G\) of a neutrosophic topological space \((\mathcal{X}, \tau_N)\) is called a neutrosophic regular generalized closed set (briefly NRGC) if \(N_{rg}(G) \subseteq H\) whenever \(G \subseteq H\) and \(G\) is NRO in \((\mathcal{X}, \tau_N)\). The complement of a NRGC set is called a NRGO set.

**Definition 2.9** [8]. A subset \(G\) of a neutrosophic topological space \((\mathcal{X}, \tau_N)\) is called a neutrosophic generalized pre regular closed set (briefly NGPRC) if \(N_{pr}(G) \subseteq H\) whenever \(G \subseteq H\) and \(G\) is NRO in \((\mathcal{X}, \tau_N)\). The complement of a NGPRC set is called a NGPRO set.

**III. Neutrosophic Generalized Regular* Closed Set**

**Definition 3.1.** A subset \(G\) of \((\mathcal{X}, \tau_N)\) is called neutrosophic generalized semi-preclosed (briefly NGSPC) if \(NS_{cl}(G) \subseteq U\) whenever \(G \subseteq U\) and \(U\) is NRO in \((\mathcal{X}, \tau_N)\). The complement of a NGSPC set is called a NGSPO set.

**Example 3.1.** Let \(\mathcal{X} = \{a, b\}, \tau_N = \{0_N, A, 1_N\}\) where \(A = \{(a, 0.4, 0.3, 0.7), (b, 0.6, 0.4, 0.9)\}\). Then \(\tau_N\) is a NT and consider \(G = \{(a, 0.5, 0.5, 0.6), (b, 0.7, 0.8, 8)\}\). Let \(U\) be any NRO such that \(G \subseteq U\). Then \(NS_{cl}(G) \subseteq U\). Hence \(G\) is NGSPC.

**Definition 3.2.** A subset \(G\) of \((\mathcal{X}, \tau_N)\) is called neutrosophic semi-generalized closed (NSGC) if \(NS_{cl}(G) \subseteq U\) whenever \(G \subseteq U\) and \(U\) is NSO in \((\mathcal{X}, \tau_N)\). The complement of NSGC is called NSGO.

**Example 3.2.** Let \(\mathcal{X} = \{a, b\}, \tau_N = \{0_N, A, 1_N\}\) where \(A = \{(a, 0.3, 0.3, 0.6), (b, 0.5, 0.5, 0.7)\}\). Then \(\tau_N\) is a NT and consider
\[ G = \{(a, 0.6, 0.4, 0.7), (b, 0.6, 0.5, 0.8)\} \]. Let \( U \) be any NSO such that \( G \subseteq U \). Then \( NS_{cl}(G) \subseteq U \). Hence \( G \) is NSGC.

**Definition 3.3.** A subset \( G \) of \( (X, \tau_N) \) is called **neutrosophic generalized regular closed** (NGRC) if \( NR_{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is NO in \( (X, \tau_N) \). The complement of a NGRC set is called a NGRO set.

**Example 3.3.** Let \( X = \{a, b\}, \tau_N = \{0_N, A, B, 1_N\} \) where \( A = \{(a, 0.2, 0.3, 0.7), (b, 0.4, 0.4, 0.8)\}, B = \{(a, 0.6, 0.5, 0.5), (b, 0.7, 0.6, 0.5)\}. \) Then \( \tau_N \) is a NT and consider \( G = \{(a, 0.4, 0.5, 0.6), (b, 0.6, 0.5, 0.7)\}. \) Let \( U \) be any NO such that \( G \subseteq U \). Then \( NR_{cl}(G) = G \cup C(B) = C(B) \subseteq U \). Hence \( G \) is NGRC.

**Definition 3.4.** A subset \( G \) of a neutrosophic topological space \( (X, \tau_N) \) is called **neutrosophic generalized*semi closed** (NG*SO) if \( NS_{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is NGO in \( (X, \tau_N) \). The complement of a NG*SO set is called a NG*SO set.

**Example 3.4.** Let \( X = \{a, b\}, \tau_N = \{0_N, A, B, 1_N\} \) where \( A = \{(a, 0.6, 0.7, 0.3), (b, 0.7, 0.9, 0.2)\}, B = \{(a, 0.5, 0.6, 0.4), (b, 0.6, 0.8, 0.3)\}. \) Then \( \tau_N \) is a NT and consider \( G = \{(a, 0.2, 0.3, 0.9), (b, 0.2, 0.1, 0.9)\}. \) Let \( U \) be any NGO such that \( G \subseteq U \). Then \( NS_{cl}(G) = N_{int}(C(A)) \cup G = O_N \cup G \subseteq U \). Hence \( G \) is NG*R*C.

**Definition 3.5.** A subset \( G \) is called **neutrosophic generalized regular star closed** (NG*R*C) if \( NR_{cl}(G) \subseteq U \) whenever \( G \subseteq U \) and \( U \) is NGO in \( (X, \tau_N) \).

**Example 3.5.** Let \( X = \{a, b\}, \tau_N = \{0_N, A, 1_N\} \) where \( A = \{(a, 0.4, 0.4, 0.4), (b, 0.4, 0.5, 0.7)\}. \) Then \( \tau_N \) is a NT and consider \( G = \{(a, 0.5, 0.5, 0.4), (b, 0.6, 0.8, 0.3)\}. \) Let \( U \) be any NGO such that \( G \subseteq U \). Then \( NR_{cl}(G) \subseteq U \). Hence \( G \) is NG*R*C.

**Definition 3.6.** A neutrosophic space \( (X, \tau_N) \) is called a neutrosophic partition space, if every NO set is NC and every NC set is NO.
Remark 3.1. \(0_N\) and \(1_N\) are \(NGR^*C\) subset of \((\chi, \tau_N)\).

**Theorem 3.1.** Every NC set in \((\chi, \tau_N)\) is \(NGR^*C\) in \((\chi, \tau_N)\).

**Proof.** Let \(A\) be NC in \((\chi, \tau_N)\). Let \(U\) be any NGO such that \(A \subseteq U\). Since \(A\) is NC, we get \(N_{cl}(A) = A\). Therefore, \(A \subseteq U \Rightarrow N_{cl}(A) \subseteq U\). Therefore \(NR_{cl}(A) \subseteq U\), since \(N_{cl}(A) \subseteq NR_{cl}(A)\). Hence \(A\) is \(NGR^*C\) set.

**Theorem 3.2.** Every NRC set in \((\chi, \tau_N)\) is \(NGR^*C\) in \((\chi, \tau_N)\).

**Proof.** Let \(A\) be NRC in \((\chi, \tau_N)\). Let \(U\) be NGO such that \(A \subseteq U\). Since \(A\) is NRC, we have \(NR_{cl}(A) = A \subseteq U\). Hence \(A\) is \(NGR^*C\) set in \((\chi, \tau_N)\).

**Example 3.6.** The converse of the above theorem need not be true. For, let \(\chi = \{a, b\}\), \(\tau_N = \{0_N, A, 1_N\}\) where \(A = \{(a, 0.4, 0.4, 0.4), (b, 0.4, 0.5, 0.7)\}\). Then \(\tau_N\) is a NT. Consider \(G = \{(a, 0.5, 0.5, 0.4), (b, 0.6, 0.8, 0.3)\}\). Let \(U\) be any NGO such that \(G \subseteq U\). Then \(NR_{cl}(G) \subseteq U\). Hence \(G\) is \(NGR^*C\). But \(G\) is not NC. Hence \(G\) is not NRC.

**Theorem 3.3.** For a neutrosophic topological space \((\chi, \tau_N)\), the following conditions are hold.

- Every \(NGR^*C\) set is NGSC set.
- Every \(NGR^*C\) set is NSGC set.
- Every \(NGR^*C\) set is NGPC set.
- Every \(NGR^*C\) set is NGSPC set.
- Every \(NGR^*C\) set is NRGC set.
- Every \(NGR^*C\) set is NGPRC set.
- Every \(NGR^*C\) set is NaGC set.
- Every \(NGR^*C\) set is NGC set.
Proof.

(1) Let $A$ be a $N^*GR$ set in $(X, \tau_N)$. Let $U$ be NO such that $A \subseteq U$. Since every NO is NGO and $A$ is $N^*GR$, we have $NS_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore $A$ is NGSC set in $(X, \tau_N)$.

(2) Let $A$ be $N^*GR$, in $(X, \tau_N)$. Let $U$ be any NSO such that $A \subseteq U$. Since every NSO is NGO and $A$ is $N^*GR$, we have $NS_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore $A$ is NSGC set in $(X, \tau_N)$.

(3) Let $A$ be $N^*GR$, in $(X, \tau_N)$. Let $U$ be any NO such that $A \subseteq U$. Since every NO set is NGO and $A$ is $N^*GR$, we have $NP_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore $A$ is NGPC in $(X, \tau_N)$.

(4) Let $A$ be a $N^*GR$, set in $(X, \tau_N)$. Let $U$ be any NO set such that $A \subseteq U$. Since every NO set is NGO and $A$ is $N^*GR$, we have $NS_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore $A$ is NGPC in $(X, \tau_N)$.

(5) Let $A$ be $N^*GR$ set in $(X, \tau_N)$. Let $U$ be a NRO set such that $A \subseteq U$. Since every NRO is NGO and $A$ is $N^*GR$, we have $N_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore $A$ is NRG set in $(X, \tau_N)$.

(6) Let $A$ be $N^*GR$ set in $(X, \tau_N)$. Let $U$ be a NRO set such that $A \subseteq U$. Since every NRO set is NSO and $A$ is $N^*GR$, we have $NP_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore $A$ is NGPRC set in $(X, \tau_N)$.

(7) Let $A$ be $N^*GR$ set in $(X, \tau_N)$. Let $U$ be a NO set such that $A \subseteq U$. Since every NO set is NGO and $A$ is $N^*GR$, we have $N_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore $A$ is $NaGC$ in $(X, \tau_N)$.

It is obvious.

The converse of the above need not be true.

Example 3.7. Let $X = \{a, b\}$, $\tau_N = \{0_N, A, 1_N\}$ where

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A = \{\{a, 0.3, 0.4, 0.7\}, \{b, 0.5, 0.4, 0.8\}\}. Then \(\tau_N\) is a NT. Consider
\(G = \{\{a, 0.5, 0.5, 0.7\}, \{b, 0.6, 0.8, 0.8\}\}\) and let \(V = \{\{a, 0.6, 0.6, 0.5\}, \{b, 0.7, 0.8, 0.4\}\}\), then \(V\) is NGO. Now \(G \subseteq V\), but \(\text{NR}_{cl}(G) = C(A) \not\subseteq V\).
Hence \(G\) is not a \(\text{NGR}^C\) set.

(1) For any NO set \(U\) such that \(G \subseteq U\). then \(\text{N}_{cl}(G) = C(A) \subseteq U\). Hence \(G\) is NGC.

(2) For any NO set \(U\) such that \(G \subseteq U\), \(\text{NS}_{cl}(G) = C(A) \subseteq U\), Hence \(G\) is NGSC.

(3) For any NO set \(U\) such that \(G \subseteq U\), \(\text{NP}_{cl}(G) = C(A) \subseteq U\), Since \(1_N\) is the only open set containing \(G\). Hence \(G\) is NGPC.

(4) For any NO set \(U\) such that \(G \subseteq U\), \(\text{NP}_{cl}(G) = A \subseteq U\). Hence \(G\) is NGSPC.

(5) Let \(U\) be any NRO set such that \(G \subseteq U\). Then \(U\) is a NO set. Hence \(\text{N}_{cl}(G) = C(A) \subseteq U\). Hence \(G\) is NRGC.

(6) Let \(U\) be any NRO set such that \(G \subseteq U\). Then \(U\) is a NO set. Hence \(\text{NP}_{cl}(G) \subseteq U\). Hence \(G\) is NGPRC.

(7) Let \(U\) be any NSO set such that \(G \subseteq U\). Then \(\text{NS}_{cl}(G) \subseteq U\). Hence \(G\) is NSGC.

(8) Let \(U\) be any NO set such that \(\text{No}_{cl}(G) \subseteq U\). Hence \(G\) is NoGC.

**Theorem 3.4.** Every \(\text{NGR}^C\) set is NGRC set but not conversely.

**Proof.** Let \(A\) be \(\text{NGR}^C\) set in \((\chi, \tau_N)\). Let \(U\) be a NO set such that \(A \subseteq U\). Since every NO set is NGO and \(A\) is \(\text{NGR}^C\) We have \(\text{NR}_{cl}(A) \subseteq U\). Therefore \(A\) is NGRC set in \((\chi, \tau_N)\).

**Example 3.8.** Let \(\chi = \{a, b\}, \tau_N = \{0_N, A, 1_N\}\) where
\(A = \{\{a, 0.2, 0.4, 0.7\}, \{b, 0.1, 0.5, 0.6\}\}\). Then \(\tau_N\) is a NT. Consider \(G\)
\(G = \{\{a, 0.5, 0.5, 0.3\}, \{b, 0.4, 0.8, 0.2\}\}\). Let \(U\) be any NO such that \(G \subseteq U\).
Then \(\text{NR}_{cl}(G) = C(A) \subseteq U\). Hence \(G\) is NGRC. Consider
$V = \{(a, 0.6, 0.6, 0.2), (b, 0.6, 0.7, 0.2)\}$. Then $V$ is NGO and $G \subseteq U$. but $NR_G = G \nsubseteq V$. Hence $G$ is not $NG^*C$. Hence NGRC set need not be a $NG^*C$. set.

**Remark 3.2.** $NG^*C$ sets and NGaC sets are independent of each other. It is shown by the following example.

**Example 3.9.** Let $X = \{a, b\}$, $\tau_N = \{0_N, A, 1_N\}$ where $A = \{(a, 0.7, 0.3, 0.5), (b, 0.8, 0.5, 0.6)\}$. Then $\tau_N$ is a NT. Consider $G = \{(a, 0.6, 0.3, 0.5), (b, 0.7, 0.5, 0.6)\}$. Let $U$ be any NGO such that $G \subseteq U$. Then $NR_G = C(A) \cup G = G \subseteq U$. Hence $G$ is $NG^*C$. Now $G \subseteq A$ and $A$ is NaO, but $NGaC(G) = G \cup 1_N = 1_N \nsubseteq A$. Hence $G$ is not NGaC. Therefore $NG^*C$ set need not be a NGaC set.

**Example 3.10.** Consider the Example 3.7. In this example, $0_N A$ and $1_N$ are the only NaO sets and $G$ is not $NG^*C$. Since $G \nsubseteq A$, we consider $1_N$ as the only NaO set such that $G \subseteq 1_N \Rightarrow N\alpha_G(G) = G \cup N_{cl}(N_{int}(N_{cl}(G))) = A^c \subseteq 1_N$. Hence $G$ is NGaC. Hence NGaC set need not be a $NG^*C$ set.

**Remark 3.3.** $NG^*C$ sets and $NG^*SC$ sets are independent of each other. It is shown by following example.

**Example 3.11.** Let $X = \{a, b\}$, $\tau_N = \{0_N, A, B, 1_N\}$ where $A = \{(a, 0.4, 0.5, 0.8), (b, 0.4, 0.4, 0.9)\}$ and $B = \{(a, 0.6, 0.5, 0.8), (b, 0.6, 0.7, 0.9)\}$. Then $\tau_N$ is a NT. Consider $G = \{(a, 0.5, 0.5, 0.7), (b, 0.4, 0.5, 0.8)\}$. Then NGO $(X, \tau_N) = \{U/U \subseteq \{(a, 0.6, 0.5, 0.8), (b, 0.6, 0.7, 0.9)\}$ and $U = 1_N\}$. Let be any NGO such that $G \subseteq U$. Then $NS_{cl}(G) = G \cup N_{int}(N_{cl}(G)) = G \cup A = G \subseteq U$. Hence $G$ is $NG^*SC$. For $G \subseteq B$, $NR_{G} = A^c \nsubseteq B$ and $B$ is NGO. Hence $G$ is not $NG^*SC$. Hence $NG^*SC$ set need not be a $NG^*C$ set.

**Example 3.12.** Let $X = \{a, b\}$, $\tau_N = \{0_N, A, 1_N\}$ where $A = \{(a, 0.7, 0.4, 0.5), (b, 0.9, 0.4, 0.5)\}$. Then $\tau_N$ is a NT. Consider
$G = \{ (a, 0.6, 0.2, 0.7), (b, 0.6, 0.1, 0.6) \}$. Let $U$ be any NGO such that $G \subseteq U$. Then $NR_{cl}(G) = G \cup 0_N \subseteq U$. Hence $G$ is $NGR^*C$. Now $G \subseteq A$ and $A$ is NGO. But $NS_{cl}(G) = G \cup 1_N \subseteq U$. Hence $G$ is not $NG^*SC$. Hence $NGR^*C$ set need not be a $NG^*SC$ set.

**Theorem 3.5.** Let $A$ be a NGO. Then $A$ is NRC if $A$ is $NGR^*C$.

**Proof.** It is obvious.

**Theorem 3.6.** The finite union of the $NGR^*C$ sets is $NGR^*C$.

**Proof.** Let $A$ and $B$ be $NGR^*C$ sets in $(\chi, \tau_N)$. Let $U$ be a NGO in $(\chi, \tau_N)$ such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since $A$ and $B$ are $NGR^*C$ sets, $NR_{cl}(A) \subseteq U$ and $NR_{cl}(B) \subseteq U$. Hence $NR_{cl}(A \cup B) = NR_{cl}(A) \cup NR_{cl}(B) \subseteq U$. Therefore $A \cup B$ is $NGR^*C$.

**Theorem 3.7.** The finite intersection of two $NGR^*C$ sets is $NGR^*C$.

**Proof.** The proof is obvious.

**Theorem 3.8.** The intersection of a $NGR^*C$ set and a NC set is an NGC set.

**Proof.** Let $A$ be a $NGR^*C$ set and $F$ is a NC set. If $U$ is an NO set with $A \cap F \subseteq U$, then $U$ is NGO and $A \subseteq U \cup (1_N \cdot F)$ So, $NR_{cl}(A) \subseteq U \cup (1_N \cdot F)$. Then $N_{cl}(A \cap F) = N_{cl}(A) \cap N_{cl}(F) \subseteq NR_{cl}(A) \cap N_{cl}(F) = NR_{cl}(A) \cap F \subseteq U$. So $A \cap F$ is a NGC set.

**Remark 3.4.** The intersection of a $NGR^*C$ set and a NRC set is a $NGR^*C$ set and the intersection of two NRC set is a $NGR^*C$.

**Theorem 3.9.** Let $A \subseteq B \subseteq NR_{cl}(A)$ and $A$ is a $NGR^*C$ subset of $(\chi, \tau_N)$, then $B$ is also a $NGR^*C$ subset of $(\chi, \tau_N)$.

**Proof.** Since $A$ is a $NGR^*C$ subset of $(\chi, \tau_N)$. So, $NR_{cl}(A) \subseteq U$, whenever $A \subseteq U$, $U$ being an NGO subset of $(\chi, \tau_N)$. Let $A \subseteq B$
\( \subseteq \text{NR}_{cl}(A) \), i.e. \( \text{NR}_{cl}(B) \subseteq \text{R}_{cl}(A) \). Let if possible, there exists an NO subset \( V \) of \((\chi, \tau_N)\), such that \( B \subseteq V \). So, \( A \subseteq V \) and \( B \) being \( \text{NGR}^* C \) subset of \((\chi, \tau_N)\), \( \text{NR}_{cl}(A) \subseteq V \), then \( \text{NR}_{cl}(B) \subseteq V \). Hence \( B \) is also \( \text{NGR}^* C \) subset of \((\chi, \tau_N)\).

**Theorem 3.10.** Let \( A \subseteq B \subseteq (\chi, \tau_N) \), where \( A \) is NGO in \((\chi, \tau_N)\). If \( A \) is \( \text{NGR}^* C \) then \( A \) is \( \text{NGR}^* C \) in \( B \).

**Proof.** Let \( A \subseteq B \), where \( U \) is NGO set of \((\chi, \tau_N)\). Let \( U = V \cap B \) for some NGO set \( V \) of \((\chi, \tau_N)\) and \( B \) is NGO in \((\chi, \tau_N)\). Using assumption \( A \) if \( \text{NGR}^* C \) in \((\chi, \tau_N)\), we have \( \text{NR}_{cl}(A) \subseteq U \) and so \( \text{NR}_{cl}(A) = N_{cl}(A) \cap B \subseteq U \cap B \subseteq U \). Hence \( A \) is \( \text{NGR}^* C \) in \( B \).

**Theorem 3.11.** Let \( A \subseteq B \subseteq (\chi, \tau_N) \), where \( B \) is NGO and \( \text{NGR}^* C \) in \((\chi, \tau_N)\). If \( A \) is \( \text{NGR}^* C \) in \( B \), then \( A \) is \( \text{NGR}^* C \) in \((\chi, \tau_N)\).

**Proof.** Let \( U \) be a NGO set of \((\chi, \tau_N)\) such that \( A \subseteq U \). Since \( A \subseteq U \cap B \), where \( U \cap B \) is NGO in \( B \) and \( A \) is \( \text{NGR}^* C \) in \( B \), \( \text{NR}_{cl}(A) \subseteq U \cap B \) holds, we have \( \text{NR}_{cl}(A) \cap B \subseteq U \cap B \). Since \( A \subseteq B \) we have \( \text{NR}_{cl}(A) \subseteq \text{NR}_{cl}(B) \). Since \( B \) is NGO and \( \text{NGR}^* C \) in \((\chi, \tau_N)\) and by Theorem 3.5, \( B \) is NRC. Therefore \( \text{NR}_{cl}(B) = B \). Thus \( \text{NR}_{cl}(A) \subseteq B \) implies \( \text{NR}_{cl}(A) = \text{NR}_{cl}(A) \cap B \subseteq U \cap B \subseteq U \). Hence \( A \) is \( \text{NGR}^* C \) in \((\chi, \tau_N)\).

**Theorem 3.12.** A subset \( A \) of \((\chi, \tau_N)\) is \( \text{NGR}^* C \) set iff \( \text{NR}_{cl}(A) \cap C(A) \) does not contain the non-zero NC set in \((\chi, \tau_N)\).

**Proof.** Let \( A \) be a \( \text{NGR}^* C \) subset of \((\chi, \tau_N)\). Also if possible, let \( M \) be a NC subset of \((\chi, \tau_N)\) such that \( \text{NR}_{cl}(M) \cup C(M) \) i.e., \( \text{M} \subseteq \text{NR}_{cl}(A) \cap C(A) \) and \( \text{M} \subseteq \text{C(A)} \). Since \( M \) is a NC subset of \((\chi, \tau_N)\), \( \text{C(M)} \) is a NO subset of \((\chi, \tau_N)\) contained in \( A \). \( A \) being \( \text{NGR}^* O \) subset of \((\chi, \tau_N)\), \( \text{NR}_{cl}(A) \subseteq \text{C(M)} \). But \( \text{M} \subseteq \text{NR}_{cl}(A) \). So, we get a contradiction, which leads to the conclusion that \( \text{M} = 0 \). So the condition is true. Conversely, Let \( A \subseteq N \), \( N \) being an
NO subset of \((\mathcal{X}, \tau_N)\). Then \(C(N) \subseteq C(A)\), \(C(N)\) is a NC subset of \((\mathcal{X}, \tau_N)\). Let it possible \(NR_{cl}(A) \subseteq N\), then \(NR_{cl}(A) \cap C(N)\) is a non-zero NC subset of \(NR_{cl}(A) \cap C(A)\), which is a contradiction. Hence \(A\) is a \(NGR^*O\) subset of \((\mathcal{X}, \tau_N)\).

**Theorem 3.13.** A subset \(A\) of \((\mathcal{X}, \tau_N)\) is \(NGR^*O\) set in \((\mathcal{X}, \tau_N)\) iff \(NR_{cl}(A) \cap C(N)\) contains no non-zero NGC set in \((\mathcal{X}, \tau_N)\).

**Proof.** Suppose that \(F\) is a non-zero NGC subset of \(NR_{cl}(A)\). Now \(F \subseteq NR_{cl}(A)\). Then \(F \subseteq NR_{cl}(A) \cap C(A)\). Therefore \(F \subseteq NR_{cl}(A)\) and \(F \subseteq C(A)\). Since \(C(F)\) is NGO set and \(A\) is \(NGR^*C\), \(NR_{cl}(A) \subseteq C(F)\). That is \(F \subseteq C(NR_{cl}(A))\). Hence \(F \subseteq NR_{cl}(A) \cap C(NR_{cl}(A)) = 0_N\). i.e. \(F = 0_N\). Thus \(NR_{cl}(A)\) contains no non-zero NGC set. Conversely, assume that \(NR_{cl}(A)\) contains no non-zero NGC set. Let \(A \subseteq U\), \(U\) is NGO. Suppose that \(NR_{cl}(A)\) is not contained in \(U\). Then \(NR_{cl}(A) \cap C(U)\) is a non-zero NGC set and contained in \(NR_{cl}(A)\) which is a contradiction. Therefore \(NR_{cl}(A) \subseteq U\) and hence \(A\) is \(NGR^*C\) set.

**Theorem 3.14.** For each \(A \in (\mathcal{X}, \tau_N)\), either \(A\) is NGC or \(C(A)\) is \(NGR^*C\) in \((\mathcal{X}, \tau_N)\).

**Proof.** If \(A\) is not NGC, then the only NGO set containing \(C(A)\) is \(1_N\). Thus \(R_{cl}C(A)\) is contained in \(1_N\) and hence \(C(A)\) is \(NGR^*C\) in \((\mathcal{X}, \tau_N)\).

**Theorem 3.15.** In a neutrosophic partition space (Definition 3.5), every \(NGR^*C\) is NGC set.

**Proof.** Let \(A\) be a \(NGR^*C\) and \(A \subseteq U\), where \(U\) is NO. Since every NO set is a NGO set, \(U\) is NGO. By hypothesis \(A\) is \(NGR^*C\) set. Hence we have \(NR_{cl}(A) \subseteq U\). In neutrosophic partition space every NC set is NO.

Hence the class of NRC sets coincides with the class of NC sets. Therefore we have \(N_{cl}(A) \subseteq NR_{cl}(A) \subseteq U\). Thus we have \(A\) is NGC.
**Theorem 3.16.** In a neutrosophic partition space, every $NGR^C$ is NRGC set.

**Proof.** Let $A$ be a $NGR^C$ and $A \subseteq U$, where $U$ is NRO. In neutrosophic partition space, the class of NRC sets coincides with the class of NC sets (NO sets) and the class of NRO sets also coincides with the class of NC sets (NO sets). Therefore we have $(X, \tau_N) = NRO(X, \tau_N) = NRC(X, \tau_N)$. Hence we also get in a neutrosophic partition space every NRO set is a NGO set. So we have $U$ is a NGO set with $A \subseteq U$. By hypothesis $A$ is $NGR^C$. Hence we have $NR_{cl}(A) \subseteq U$. Thus we have $A$ is NRGC.

The relationship of $NGR^C$ sets with some other sets discussed in this section is showed by the Figure (a).

**IV. Neutrosophic Generalized Regular* Open Set**

**Definition 4.1.** A subset $A$ of a neutrosophic topological space $(X, \tau_N)$ is called $NGR^O$ set if the complement of $A$ ($C(A)$) is $NGR^C$.

**Example 4.1.** Let $X = \{a, b\}$, $\tau_N = \{0_N, A, 1_N\}$ where
A = \{\langle a, 0.4, 0.5, 0.4 \rangle, \langle b, 0.4, 0.3, 0.7 \rangle\}. Then \( \tau_N \) is a NT. Consider
\( G = \{\langle a, 0.4, 0.5, 0.5 \rangle, \langle b, 0.3, 0.2, 0.6 \rangle\}. Then \( C(G) = \{\langle a, 0.5, 0.5, 0.4 \rangle, \langle b, 0.6, 0.8, 0.3 \rangle\}. \) Let \( U \) be any NGO such that \( C(G) \subseteq U \). Then
\( NR_{\text{cl}}(C(G)) = C(A) \subseteq U \). Hence \( C(G) \) is \( NGR^*C \). Hence \( G \) is a \( NGR^*O \) set.

**Theorem 4.1.** A subset \( A \) of a neutrosophic topological space \((X, \tau_N)\) is
\( NGR^*O \) if and only if \( B \subseteq NR_{\text{int}}(A) \) where \( B \) is NGC in \((X, \tau_N)\) and
\( B \subseteq A \).

**Proof.** **Necessity:** Suppose \( B \subseteq NR_{\text{int}}(A) \) where \( B \) is NGC in \((X, \tau_N)\) and
\( B \subseteq A \). Let \( C(A) \subseteq M \), where \( M \) is NGO. Hence \( C(M) \subseteq A \), where
\( C(M) \) is NGC. Hence by assumption \( C(M) \subseteq NR_{\text{int}}(A) \), which implies
\( C(NR_{\text{int}}(A)) \subseteq M \). Therefore \( NR_{\text{cl}}(C(A)) \subseteq M \). Thus \( C(A) \) is \( NGR^*C \), implies \( A \) is \( NGR^*O \).

**Sufficiency.** Let \( A \) is \( NGR^*O \) in \((X, \tau_N)\) with \( N \subseteq A \), where \( N \) is NGC.
We have \( C(A) \) is \( NGR^*C \) with \( C(A) \subseteq C(N) \) where \( C(N) \) is NGO. Then we have
\( NR_{\text{cl}}(C(A)) \subseteq C(N) \) implies \( N \subseteq 1_{N^*} \cdot R_{\text{cl}}(C(A)) \)
= \( NR_{\text{int}}(1_{N} \cdot C(A)) = NR_{\text{int}}(A) \) Hence proved.

**Theorem 4.2.** Every NRO set is \( NGR^*O \) set.

**Proof.** Let \( A \) be a NRO set. Then \( 1_{N^*} \cdot A \) is NRC. By Theorem 3.2, \( 1_{N^*} \cdot A \)
is \( NGR^*C \). Hence \( A \) is \( NGR^*O \) set.

**Theorem 4.3.** If \( NR_{\text{int}}(A) \subseteq B \subseteq A \) and \( A \) is a \( NGR^*O \) subset of
\((X, \tau_N)\), then \( B \) is also a \( NGR^*O \) subset of \((X, \tau_N)\).

**Proof.** \( NR_{\text{int}}(A) \subseteq B \subseteq A \) implies \( C(A) \subseteq C(B) \subseteq NR_{\text{cl}}(C(A)) \). Given
\( C(A) \) is \( NGR^*C \). By Theorem 3.09, \( C(B) \) is \( NGR^*C \). Therefore \( B \) is
\( NGR^*O \).

**Theorem 4.4.** If a subset \( A \) of a neutrosophic topological space \((X, \tau_N)\) is
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NGR*O in \((\mathcal{X}, \tau_N)\), then \(F = 1_N\), whenever \(F\) is NGO and \(NR_{\text{int}}(A) \subseteq C(A) \subseteq F\).

**Proof.** Let \(A\) be a \(NGR^*O\) and \(F\) be NGO. Then \(NR_{\text{int}}(A) \cup C(A) \subseteq F\). This gives \(C(F) \subseteq (1_N \cdot NR_{\text{int}}(A)) \cap A = NR_{cl}(C(A)) \cap A = NR_{cl}(C(A)) \cdot C(A)\). Since \(C(F)\) is NGC and \(C(A)\) is \(NGR^*O\) by Theorem 3.13, we have \(C(F) = 0_N\). Thus \(F = 1_N\).

**Theorem 4.5.** If a subset \(A\) of a topological space \((\mathcal{X}, \tau_N)\), is \(NGR^*C\), then \(NR_{cl}(A) \cdot A\) is \(NGR^*O\).

**Proof.** Let \(A \subseteq (\mathcal{X}, \tau_N)\) be a \(NGR^*C\) and let \(F\) be NGC such that \(F \subseteq NR_{cl}(A) \cdot A\). Then by Theorem \(F = 0_N\). So, \(0_N = F \subseteq NR_{\text{int}}(NR_{cl}(A) \cdot A)\). This shows that \(A\) is \(NGR^*O\) set.

**Conclusion**

In this paper, we defined some new classes of neutrosophic generalized closed sets \((NGRC, NG^*SC, NGR^*C)\). We studied some characteristics of neutrosophic generalized regular star closed sets \((NGR^*C)\) in neutrosophic topological space and obtained some of their basic properties. We have analyzed the relationship between the \(NGR^*C\) sets and some other generalized closed sets which already defined by many authors. Next, we introduced neutrosophic generalized regular star open sets \((NGR^*O)\) and analyzed some of their properties.

**References**

