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NEUTROSOPHIC GENERALISED REGULAR* CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract

Exploring a new type of neutrosophic set in neutrosophic topology is the major aim of our research. In this paper, the concepts “Neutrosophic generalised regular star closed sets” and “Neutrosophic generalised regular star open sets” are newly defined and their properties and some interesting theorems are introduced. We have analyzed the relationships between this newly introduced sets and the already existing neutrosophic sets.

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I. Introduction

In the past several decades, fuzzy set theory has played an important role in the research of mathematics. The research on the theory of fuzzy sets has been witnessing an exponential growth in mathematics. Zadeh established the fuzzy set as an extension of a classical notion of crisp set in 1965 [13]. K. Atanassov, extended the intuitionistic fuzzy set as a generalization of fuzzy set in 1983 [3]. Then Florentin Smarandache generalized the concept intuitionistic fuzzy sets as Neutrosophic sets in 1999 [5]. Later A. Salama and S. A. Alblowi studied the concept of neutrosophic topological spaces [9]. Wadei Al-Omeri and Saeid Jafari discovered the generalized closed sets and generalized pre-closed sets in neutrosophic topological space which belong to the important class of neutrosophic sets [12].

II. Preliminaries

Definition 2.1 [9]**.** Let $\mathcal{X}$ be a non-empty fixed set. A neutrosophic set (NS) G is an object having the form $\{\langle x, \mu_G(x), \sigma_G(x), \nu_G(x)\rangle : x \in \mathcal{X}\}$ where $\mu_G(x)$, $\sigma_G(x)$ and $\nu_G(x)$ represent the degree of membership, degree of indeterminacy and the degree of nonmembership respectively of each element $x \in \mathcal{X}$ to the set G. A Neutrosophic set $G = \{\langle x, \mu_G(x), \sigma_G(x), \nu_G(x)\rangle : x \in \mathcal{X}\}$ can be identified as an ordered triple $\langle \mu_G, \sigma_G, \nu_G\rangle$ in $]0, 1^+[$ on $\mathcal{X}$.

But in real life application, using Neutrosophic set with values from real standard or non-standard subset of $]0, 1^+[$ is very difficult. So that we used the Neutrosophic set which takes values from the subsets of $[0, 1]$. Here, the neutrosophic topological space is denoted by $(\mathcal{X}, \tau_N)$. Also the neutrosophic interior, neutrosophic closure of a neutrosophic set G are denoted by $N_{\text{int}}(G)$ and $N_{cl}(G)$. The complement of a neutrosophic set G is denoted by (G) and the empty and whole sets are denoted by 0_N and 1_N respectively.

Definition 2.2 [1]**.** We have used the following definitions throughout this paper.

1. $G \subseteq H \Leftrightarrow \mu_G(x) \leq \mu_H(x)$, $\sigma_G(x) \leq \sigma_H(x)$ and $\nu_G(x) \geq \nu_H(x) \forall x \in \mathcal{X}$

2. $G \cap H = \langle x, \mu_G(x) \wedge \mu_H(x), \sigma_G(x) \wedge \sigma_H(x), \nu_G(x) \vee \nu_H(x) \rangle$

3. $G \cup H = \langle x, \mu_G(x) \vee \mu_H(x), \sigma_G(x) \vee \sigma_H(x), \nu_G(x) \wedge \nu_H(x) \rangle$

4. $C(G) = \{\langle x, \nu_G(x), 1\text{-}\sigma_G(x), \mu_G(x) \rangle : x \in \mathcal{X}\}$

5. $0_N = \{\langle x, 0, 0, 1 \rangle : x \in \mathcal{X}\}$

6. $1_N = \{\langle x, 1, 1, 0 \rangle : x \in \mathcal{X}\}$.

Definition 2.3. [2]. A subset G of a neutrosophic topological space $(\mathcal{X}, \tau_N)$. is called,

(1) *a neutrosophic semi open* set (NSO set) if $G \subseteq N_{cl}(N_{\text{int}}(G))$ and a *neutrosophic semi closed* set (NSC set) if $N_{\text{int}}(N_{cl}(G)) \subseteq G$.

(2) *a neutrosophic preopen* set (NPO set) if $G \subseteq N_{\text{int}}(N_{cl}(G))$ and a *neutrosophic pre closed* set(NPC set) if $N_{cl}(N_{\text{int}}(G)) \subseteq G$.

(3) *a neutrosophic* α *open* set ($N\alpha O$ set) if $G \subseteq N_{\text{int}}(N_{cl}(N_{\text{int}}(G)))$ and *aneutrosophic* α *closed* ($N\alpha C$ set) set if $N_{cl}(N_{\text{int}}(N_{cl}(G))) \subseteq G$.

(4) *a neutrosophic semi pre open* set (NSPO set) if $G \subseteq N_{cl}(N_{\text{int}}(N_{cl}(G)))$ and *a neutrosophic semi pre closed* set (NSPC set) if $N_{\text{int}}(N_{cl}(N_{\text{int}}(G))) \subseteq G$.

(5) *a neutrosophic regular open* (NRO)set if $G = N_{\text{int}}(N_{cl}(G))$ and *a neutrosophic regular closed* (NRC)set if $G = N_{cl}(N_{\text{int}}(G))$.

Definition 2.4. [4]. A subset G of a neutrosophic topological space $(\mathcal{X}, \tau_N)$ is called *a neutrosophic generalized closed* set (NGC set) if $N_{cl}(G) \subseteq H$ whenever $G \subseteq H$ and H is NO in $(\mathcal{X}, \tau_N)$. The complement of a NGC set is called a NGO set.

Definition 2.5. [10]. A subset G of a neutrosophic topological space $(\mathcal{X}, \tau_N)$ is called *a neutrosophic generalized semi closed* set (briefly NGSC) if $NS_{cl}(G) \subseteq H$ whenever $G \subseteq H$ and H is NSO in $(\mathcal{X}, \tau_N)$. The complement of a NGSC set is called a NGSO set.

Definition 2.6. [11]. A subset G of a neutrosophic topological space

$(\mathcal{X}, \tau_N)$ is called *a neutrosophic generalized* α *closed* set (briefly NGαC) if $N\alpha_{cl}(G) \subseteq H$ whenever $G \subseteq H$ and H is $N\alpha O$ in $(\mathcal{X}, \tau_N)$. The complement of a NGαC set is called a NGαO set.

Definition 2.7 [7]. A subset G of a neutrosophic topological space $(\mathcal{X}, \tau_N)$ is called *a neutrosophic* α *generalized closed* set (briefly NαGC) if $N\alpha_{cl}(G)G \subseteq H$ whenever $G \subseteq H$ and G is NO in $(\mathcal{X}, \tau_N)$. The complement of a NαGC set is called a NαGO set.

Definition 2.8 [6]. A subset G of a neutrosophic topological space $(\mathcal{X}, \tau_N)$ is called *a neutrosophic regular generalized closed* set (briefly NRGC) if $N_{cl}(G) \subseteq H$ whenever $G \subseteq H$ and H is NRO in $(\mathcal{X}, \tau_N)$. The complement of a NRGC set is called a NRGO set.

Definition 2.9 [8]. A subset G of a neutrosophic topological space $(\mathcal{X}, \tau_N)$ is called *a neutrosophic generalized pre regular closed* set (briefly NGPRC) if $NP_{cl}(G) \subseteq H$ whenever $G \subseteq H$ and H is NRO in $(\mathcal{X}, \tau_N)$. The complement of a NGPRC set is called a NGPRO set.

III. Neutrosophic Generalized Regular* Closed Set

Definition 3.1. A subset G of $(\mathcal{X}, \tau_N)$ is called *neutrosophic generalized semi-preclosed* (briefly NGSPC) if $NSP_{cl}(G) \subseteq U$ whenever $G \subseteq U$ and U is NRO in $(\mathcal{X}, \tau_N)$. The complement of a NGSPC set is called a NGSPO set.

Example 3.1. Let $\mathcal{X} = \{a, b\}, \tau_N = \{0_N, A, 1_N\}$ where $A = \{\langle a, 0.4, 0.3, 0.7\rangle, \langle b, 0.6, 0.4, 0.9\rangle\}$. Then τ_N is a NT and consider $G = \{\langle a, 0.5, 0.5, 0.6\rangle, \langle b, 0.7, 0.8, 8\rangle\}$. Let U be any NRO such that $G \subseteq U$. Then $NSP_{cl}(G) \subseteq U$. Hence G is NGSPC.

Definition 3.2. A subset G of $(\mathcal{X}, \tau_N)$ is called *neutrosophic semi-generalized closed* (NSGC) if $NS_{cl}(G) \subseteq U$ whenever $G \subseteq U$ and U is NSO in $(\mathcal{X}, \tau_N)$. The complement of NSGC is called NSGO.

Example 3.2. Let $\mathcal{X} = \{a, b\}, \tau_N = \{0_N, A, 1_N\}$ where $A = \{\langle a, 0.3, 0.3, 0.6\rangle, \langle b, 0.5, 0.5, 0.7\rangle\}$. Then τ_N is a NT and consider

$G = \{\langle a, 0.6, 0.4, 0.7\rangle, \langle b, 0.6, 0.5, 0.8\rangle\}$. Let U be any NSO such that $G \subseteq U$. Then $NS_{cl}(G) \subseteq U$. Hence G is NSGC.

Definition 3.3. A subset G of $(\mathcal{X}, \tau_N)$ is called *neutrosophic generalized regular closed* (NGRC) if $NR_{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is NO in $(\mathcal{X}, \tau_N)$. The complement of a NGRC set is called a NGRO set.

Example 3.3. Let $\mathcal{X} = \{a, b\}$, $\tau_N = \{0_N, A, B, 1_N\}$ where $A = \{\langle a, 0.2, 0.3, 0.7\rangle, \langle b, 0.4, 0.4, 0.8\rangle\}$, $B = \{\langle a, 0.6, 0.5, 0.5\rangle, \langle b, 0.7, 0.6, 0.5\rangle\}$. Then τ_N is a NT and consider $G = \{\langle a, 0.4, 0.5, 0.6\rangle, \langle b, 0.6, 0.5, 0.7\rangle\}$. Let U be any NO such that $G \subseteq U$. Then $NR_{cl}(G) = G \cup C(B) = C(B) \subseteq U$. Hence G is NGRC.

Definition 3.4. A subset G of a neutrosophic topological space $(\mathcal{X}, \tau_N)$ is called *neutrosophic generalized*semi closed* (NG^*SO) if $NS_{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is NGO in $(\mathcal{X}, \tau_N)$. The complement of a NG^*SC set is called a NG^*SO set.

Example 3.4. Let $\mathcal{X} = \{a, b\}$, $\tau_N = \{0_N, A, B, 1_N\}$ where $A = \{\langle a, 0.6, 0.7, 0.3\rangle, \langle b, 0.7, 0.9, 0.2\rangle\}$, $B = \{\langle a, 0.5, 0.6, 0.4\rangle, \langle b, 0.6, 0.8, 0.3\rangle\}$. Then τ_N is a NT and consider $G = \{\langle a, 0.2, 0.3, 0.9\rangle, \langle b, 0.2, 0.1, 0.9\rangle\}$. Let U be any NGO such that $G \subseteq U$. Then $NS_{cl}(G) = N_{\text{int}}(C(A)) \cup G = O_N \cup G \subseteq U$. Hence G is NGR^*C.

Definition 3.5. A subset G is called *neutrosophic generalized regular star closed* (NGR^*C) if $NR_{cl}(G) \subseteq U$ whenever $G \subseteq U$ and U is NGO in $(\mathcal{X}, \tau_N)$.

Example 3.5. Let $\mathcal{X} = \{a, b\}$, $\tau_N = \{0_N, A, 1_N\}$ where $A = \{\langle a, 0.4, 0.4, 0.4\rangle, \langle b, 0.4, 0.5, 0.7\rangle\}$. Then τ_N is a NT and consider $G = \{\langle a, 0.5, 0.5, 0.4\rangle, \langle b, 0.6, 0.8, 0.3\rangle\}$. Let U be any NGO such that $G \subseteq U$. Then $NR_{cl}(G) \subseteq U$. Hence G is NGR^*C.

Definition 3.6. A neutrosophic space $(\mathcal{X}, \tau_N)$ is called a neutrosophic partition space, if every NO set is NC and every NC set is NO.

Remark 3.1. 0_N and 1_N are NGR^*C subset of $(\mathcal{X}, \tau_N)$.

Theorem 3.1. *Every NC set in* $(\mathcal{X}, \tau_N)$ *is* NGR^*C *in* $(\mathcal{X}, \tau_N)$.

Proof. Let A be NC in $(\mathcal{X}, \tau_N)$. Let U be any NGO such that $A \subseteq U$. Since A is NC, we get $N_{cl}(A) = A$. Therefore, $A \subseteq U \Rightarrow N_{cl}(A) \subseteq U$. Therefore $NR_{cl}(A) \subseteq U$, since $N_{cl}(A) \subseteq NR_{cl}(A)$. Hence A is NGR^*C set.

Theorem 3.2. *Every NRC set in* $(\mathcal{X}, \tau_N)$ *is* NGR^*C *in* $(\mathcal{X}, \tau_N)$.

Proof. Let A be NRC in $(\mathcal{X}, \tau_N)$. Let U be NGO such that $A \subseteq U$. Since A is NRC, we have $NR_{cl}(A) = A \subseteq U$. Hence A is NGR^*C set in $(\mathcal{X}, \tau_N)$.

Example 3.6. The converse of the above theorem need not be true. For, let $\mathcal{X} = \{a, b\}$, $\tau_N = \{0_N, A, 1_N\}$ where $A = \{\langle a, 0.4, 0.4, 0.4\rangle, \langle b, 0.4, 0.5, 0.7\rangle\}$. Then τ_N is a NT. Consider $G = \{\langle a, 0.5, 0.5, 0.4\rangle, \langle b, 0.6, 0.8, 0.3\rangle\}$. Let U be any NGO such that $G \subseteq U$. Then $NR_{cl}(G) \subseteq U$. Hence G is NGR^*C. But G is not NC. Hence G is not NRC.

Theorem 3.3. *For a neutrosophic topological space* $(\mathcal{X}, \tau_N)$, *the following conditions are hold.*

Every NGR^*C *set is NGSC set.*

Every NGR^*C *set is NSGC set.*

Every NGR^*C *set is NGPC set.*

Every NGR^*C *set is NGSPC set.*

Every NGR^*C *set is NRGC set.*

Every NGR^*C *set is NGPRC set.*

Every NGR^*C *set is N*α*GC set.*

Every NGR^*C *set is NGC set.*

Proof.

(1) Let A be a NGR^*C set in $(\mathcal{X}, \tau_N)$. Let U be NO such that $A \subseteq U$. Since every NO is NGO and A is NGR^*C, we have $NS_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore A is NGSC set in $(\mathcal{X}, \tau_N)$.

(2) Let A be NGR^*C, in $(\mathcal{X}, \tau_N)$. Let U be any NSO such that $A \subseteq U$. Since every NSO is NGO and A is NGR^*C, we have $NS_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore A is NSGC set in $(\mathcal{X}, \tau_N)$.

(3) Let A be NGR^*C, in $(\mathcal{X}, \tau_N)$. Let U be any NO such that $A \subseteq U$. Since every NO set is NGO and A is NGR^*C, we have $NP_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore A is NGPC in $(\mathcal{X}, \tau_N)$.

(4) Let A be a NGR^*C, set in $(\mathcal{X}, \tau_N)$. Let U be any NO set such that $A \subseteq U$. Since every NO set is NGO and A is NGR^*C, we have $NSP_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore A is NGSPC set in $(\mathcal{X}, \tau_N)$.

(5) Let A be NGR^*C set in $(\mathcal{X}, \tau_N)$. Let U be a NRO set such that $A \subseteq U$. Since every NRO is NGO and A is NGR^*C, We have $N_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore A is NRGC set in $(\mathcal{X}, \tau_N)$.

(6) Let A be NGR^*C set in $(\mathcal{X}, \tau_N)$. Let U be a NRO set such that $A \subseteq U$. Since every NRO set is NSO and A is NGR^*C, We have $NP_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore A is NGPRC set in $(\mathcal{X}, \tau_N)$.

(7) Let A be NGR^*C set in $(\mathcal{X}, \tau_N)$. Let U be a NO set such that $A \subseteq U$. Since every NO set is NGO and A is NGR^*C, We have $N\alpha_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Therefore A is NαGC in $(\mathcal{X}, \tau_N)$.

It is obvious.

The converse of the above need not be true.

Example 3.7. Let $\mathcal{X} = \{a, b\}, \tau_N = \{0_N, A, 1_N\}$ where

$A = \{\langle a, 0.3, 0.4, 0.7\rangle, \langle b, 0.5, 0.4, 0.8\rangle\}$. Then τ_N is a NT. Consider $G = \{\langle a, 0.5, 0.5, 0.7\rangle, \langle b, 0.6, 0.8, 0.8\rangle\}$. and let $V = \{\langle a, 0.6, 0.6, 0.5\rangle, \langle b, 0.7, 0.8, 0.4\rangle\}$, then V is NGO. Now $G \subseteq V$, but $NR_{cl}(G) = C(A) \nsubseteq V$. Hence G is not a NGR^*C set.

(1) For any NO set U such that $G \subseteq U$. then $N_{cl}(G) = C(A) \subseteq U$. Hence G is NGC.

(2) For any NO set U such that $G \subseteq U$, $NS_{cl}(G) = C(A) \subseteq U$, Hence G is NGSC.

(3) For any NO set U such that $G \subseteq U$, $NP_{cl}(G) = C(A) \subseteq U$, Since 1_N is the only open set containing G. Hence G is NGPC.

(4) For any NO set U such that $G \subseteq U$, $NSP_{cl}(G) = A \subseteq U$. Hence G is NGSPC.

(5) Let U be any NRO set such that $G \subseteq U$. Then U is a NO set. Hence $N_{cl}(G) = C(A) \subseteq U$. Hence G is NRGC.

(6) Let U be any NRO set such that $G \subseteq U$. Then U is a NO set. Hence $NP_{cl}(G) \subseteq U$. Hence G is NGPRC.

(7) Let U be any NSO set such that $G \subseteq U$. Then $NS_{cl}(G) \subseteq U$. Hence G is NSGC.

(8) Let U be any NO set such that $N\alpha_{cl}(G) \subseteq U$. Hence G is NαGC.

Theorem 3.4. *Every NGR^*C set is NGRC set but not conversely.*

Proof. Let A be NGR^*C set in $(\mathcal{X}, \tau_N)$. Let U be a NO set such that $A \subseteq U$. Since every NO set is NGO and A is NGR^*C We have $NR_{cl}(A) \subseteq U$. Therefore A is NGRC set in $(\mathcal{X}, \tau_N)$.

Example 3.8. Let $\mathcal{X} = \{a, b\}, \tau_N = \{0_N, A, 1_N\}$ where $A = \{\langle a, 0.2, 0.4, 0.7\rangle, \langle b, 0.1, 0.5, 0.6\rangle\}$. Then τ_N is a NT. Consider G $G = \{\langle a, 0.5, 0.5, 0.3\rangle, \langle b, 0.4, 0.8, 0.2\rangle\}$. Let U be any NO such that $G \subseteq U$. Then $NR_{cl}(G) = C(A) \subseteq U$. Hence G is NGRC. Consider

$V = \{\langle a, 0.6, 0.6, 0.2\rangle, \langle b, 0.6, 0.7, 0.2\rangle\}$. Then V is NGO and $G \subseteq U$. but $NR_{cl}(G) \not\subseteq V$. Hence G is not $NGR^{*}C$. Hence NGRC set need not be a $NGR^{*}C$. set.

Remark 3.2. $NGR^{*}C$ sets and NGαC sets are independent of each other. It is shown by the following example.

Example 3.9. Let $\mathcal{X} = \{a, b\}, \tau_N = \{0_N, A, 1_N\}$ where $A = \{\langle a, 0.7, 0.3, 0.5\rangle, \langle b, 0.8, 0.5, 0.6\rangle\}$. Then τ_N is a NT. Consider G $G = \{\langle a, 0.6, 0.3, 0.5\rangle, \langle b, 0.7, 0.5, 0.6\rangle\}$. Let U be any NGO such that $G \subseteq U$. Then $NR_{cl}(G) = C(A) \cup G = G \subseteq U$. Hence G is $NGR^{*}C$. Now $G \subseteq A$ and A is NαO, but $N\alpha_{cl}(G) = G \cup 1_N = 1_N \not\subseteq A$. Hence G is not NGαC. Therefore $NGR^{*}C$ set need not be a NGαC set.

Example 3.10. Consider the Example 3.7. In this example, 0_N A and 1_N are the only NαO sets and G is not $NGR^{*}C$. Since $G \not\subseteq A$, we consider 1_N as the only NαO set such that $G \subseteq 1_N \Rightarrow N\alpha_{cl}(G) = G \cup N_{cl}(N_{\text{int}}(N_{cl}(G)))$ $= A^{c} \subseteq 1_N$, Hence G is NGαC. Hence NGαC set need not be a $NGR^{*}C$ set.

Remark 3.3. $NGR^{*}C$ sets and $NG^{*}SC$ sets are independent of each other. It is shown by following example.

Example 3.11. Let $\mathcal{X} = \{a, b\}, \tau_N = \{0_N, A, B, 1_N\}$ where $A = \{\langle a, 0.4, 0.5, 0.8\rangle, \langle b, 0.4, 0.4, 0.9\rangle\}$. and $B = \{\langle a, 0.6, 0.5, 0.8\rangle,$ $\langle b, 0.6, 0.7, 0.9\rangle\}$. Then τ_N is a NT. Consider $G = \{\langle a, 0.5, 0.5, 0.7\rangle,$ $\langle b, 0.4, 0.5, 0.8\rangle\}$. Then NGO $(\mathcal{X}, \tau_N) = \{U/U \subseteq \{\langle a, 0.6, 0.5, 0.8\rangle,$ $\langle b, 0.6, 0.7, 0.9\rangle$ and $U = 1_N\}$. Let be any NGO such that $G \subseteq U$. Then $NS_{cl}(G) = G \cup N_{\text{int}}(N_{cl}(G)) = G \cup A = G \subseteq U$. Hence G is $NG^{*}SC$. For $G \subseteq B$, $NR_{cl}(G) = A^{c} \not\subseteq B$ and B is NGO. Hence G is not $NGR^{*}C$. Hence $NG^{*}SC$ set need not be a $NGR^{*}C$ set.

Example 3.12. Let $\mathcal{X} = \{a, b\}, \tau_N = \{0_N, A, 1_N\}$ where $A = \{\langle a, 0.7, 0.4, 0.5\rangle, \langle b, 0.9, 0.4, 0.5\rangle\}$. Then τ_N is a NT. Consider

$G = \{\langle a, 0.6, 0.2, 0.7\rangle, \langle b, 0.6, 0.1, 0.6\rangle\}$. Let U be any NGO such that $G \subseteq U$. Then $NR_{cl}(G) = G \cup 0_N \subseteq U$. Hence G is NGR^*C. Now $G \subseteq A$ and A is NGO. But $NS_{cl}(G) = G \cup 1_N \subseteq U$. Hence G is not NG^*SC. Hence NGR^*C set need not be a NG^*SC set.

Theorem 3.5. *Let A be a NGO. Then A is NRC if A is* NGR^*C*.*

Proof. It is obvious.

Theorem 3.6. *The finite union of the* NGR^*C *sets is* NGR^*C*.*

Proof. Let A and B be NGR^*C sets in $(\mathcal{X}, \tau_N)$. Let U be a NGO in $(\mathcal{X}, \tau_N)$ such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are NGR^*C sets, $NR_{cl}(A) \subseteq U$ and $NR_{cl}(B) \subseteq U$. Hence $NR_{cl}(A \cup B) = NR_{cl}(A) \cup NR_{cl}(B) \subseteq U$. Therefore $A \cup B$ is NGR^*C.

Theorem 3.7. *The finite intersection of two* NGR^*C *sets is* NGR^*C*.*

Proof. The proof is obvious.

Theorem 3.8. *The intersection of a* NGR^*C *set and a NC set is an NGC set.*

Proof. Let A be a NGR^*C set and F is a NC set. If U is an NO set with $A \cap F \subseteq U$, then U is NGO and $A \subseteq U \cup (1_N\text{-}F)$ So, $NR_{cl}(A) \subseteq U \cup (1_N\text{-}F)$. Then $N_{cl}(A \cap F) = N_{cl}(A) \cap N_{cl}(F) \subseteq NR_{cl}(A) \cap N_{cl}(F) = NR_{cl}(A) \cap F \subseteq U$. So $A \cap F$ is a NGC set.

Remark 3.4. The intersection of a NGR^*C set and a NRC set is a NGR^*C set and the intersection of two NRC set is a NGR^*C.

Theorem 3.9. *Let* $A \subseteq B \subseteq NR_{cl}(A)$ *and* A *is a* NGR^*C *subset of* $(\mathcal{X}, \tau_N)$*, then B is also a* NGR^*C *subset of* $(\mathcal{X}, \tau_N)$*.*

Proof. Since A is a NGR^*C subset of $(\mathcal{X}, \tau_N)$. So, $NR_{cl}(A) \subseteq U$, whenever $A \subseteq U, U$ being an NGO subset of $(\mathcal{X}, \tau_N)$. Let $A \subseteq B$

$\subseteq NR_{cl}(A)$. i.e. $NR_{cl}(B) \subseteq R_{cl}(A)$. Let if possible, there exists an NO subset V of $(\mathcal{X}, \tau_N)$. such that $B \subseteq V$. So, $A \subseteq V$ and B being NGR^*C subset of $(\mathcal{X}, \tau_N)$, $NR_{cl}(A) \subseteq V$, then $NR_{cl}(B) \subseteq V$, Hence B is also a NGR^*C subset of $(\mathcal{X}, \tau_N)$.

Theorem 3.10. *Let* $A \subseteq B \subseteq (\mathcal{X}, \tau_N)$, *where A is NGO in* $(\mathcal{X}, \tau_N)$. *If A is* NGR^*C *then A is* NGR^*C *in B.*

Proof. Let $A \subseteq B$, where U is NGO set of $(\mathcal{X}, \tau_N)$. Let $U = V \cap B$ for some NGO set V of $(\mathcal{X}, \tau_N)$ and B is NGO in $(\mathcal{X}, \tau_N)$. Using assumption A if NGR^*C in $(\mathcal{X}, \tau_N)$, we have $NR_{cl}(A) \subseteq U$ and so $NR_{cl}(A) = N_{cl}(A)$ $\cap B \subseteq U \cap B \subseteq U$. Hence A is NGR^*C in B.

Theorem 3.11. *Let* $A \subseteq B \subseteq (\mathcal{X}, \tau_N)$, *where B is NGO and* NGR^*C *in* $(\mathcal{X}, \tau_N)$ *If A is* NGR^*C *in B, then A is* NGR^*C *in* $(\mathcal{X}, \tau_N)$.

Proof. Let U be a NGO set of $(\mathcal{X}, \tau_N)$ such that $A \subseteq U$. Since $A \subseteq U \cap B$, where $U \cup B$ is NGO in B and A is NGR^*C in B, $NR_{cl}(A) \subseteq U \cap B$ holds, we have $NR_{cl}(A) \cap B \subseteq U \cap B$. Since $A \subseteq B$ we have $NR_{cl}(A) \subseteq NR_{cl}(B)$. Since B is NGO and NGR^*C in $(\mathcal{X}, \tau_N)$ and by Theorem 3.5, B is NRC. Therefore $NR_{cl}(B) = B$. Thus $NR_{cl}(A) \subseteq B$ implies $NR_{cl}(A) = NR_{cl}(A) \cap B \subseteq U \cap B \subseteq U$. Hence A is NGR^*C in $(\mathcal{X}, \tau_N)$.

Theorem 3.12. *A subset A of* $(\mathcal{X}, \tau_N)$ *is* NGR^*C *set iff* $NR_{cl}(A) \cap C(A)$ *does not contain the non-zero NC set in* $(\mathcal{X}, \tau_N)$.

Proof. Let A be a NGR^*C subset of $(\mathcal{X}, \tau_N)$. Also if possible, let M be a NC subset of $(\mathcal{X}, \tau_N)$ such that $M \subseteq NR_{cl}(A) \cap C(A)$ i.e., $M \subseteq NR_{cl}(A)$ and $M \subseteq C(A)$. Since M is a NC subset of $(\mathcal{X}, \tau_N)$, $C(M)$ is a NO subset of $(\mathcal{X}, \tau_N)$ contained in A. A being NGR^*O subset of $(\mathcal{X}, \tau_N)$, $NR_{cl}(A) \subseteq C(M)$. But $M \subseteq NR_{cl}(A)$. So, we get a contradiction, which leads to the conclusion that $M = 0_N$. So the condition is true. Conversely, Let $A \subseteq N$, N being an

NO subset of $(\mathcal{X}, \tau_N)$. Then $C(N) \subseteq C(A)$, $C(N)$ is a NC subset of $(\mathcal{X}, \tau_N)$. Let if possible $NR_{cl}(A) \subseteq N$, Then $NR_{cl}(A) \cap C(N)$ is a non-zero NC subset of $NR_{cl}(A) \cap C(A)$, which is a contradiction. Hence A is a $NGR^{*}O$ subset of $(\mathcal{X}, \tau_N)$.

Theorem 3.13. *A subset A of* $(\mathcal{X}, \tau_N)$ *is* $NGR^{*}O$ *set in* $(\mathcal{X}, \tau_N)$ *iff* $NR_{cl}(A)$-(A) *contains no non-zero NGC set in* $(\mathcal{X}, \tau_N)$.

Proof. Suppose that F is a non-zero NGC subset of $NR_{cl}(A)$-A. Now $F \subseteq NR_{cl}(A)$-A. Then $F \subseteq NR_{cl}(A) \cap C(A)$. Therefore $F \subseteq NR_{cl}(A)$ and $F \subseteq C(A)$. Since C(F) is NGO set and A is $NGR^{*}C$, $NR_{cl}(A) \subseteq C(F)$. That is $F \subseteq C(NR_{cl}(A))$. Hence $F \subseteq NR_{cl}(A) \cap C(NR_{cl}(A)) = 0_N$. i.e. $F = 0_N$. Thus $NR_{cl}(A)$-A contains no non-zero NGC set. Conversely, assume that $NR_{cl}(A)$-A. contains no non- zero NGC set. Let $A \subseteq U$, U is NGO. Suppose that $NR_{cl}(A)$ is not contained in U. Then $NR_{cl}(A) \cap C(U)$ is a non- zero NGC set and contained in $NR_{cl}(A)$-A which is a contradiction. Therefore $NR_{cl}(A) \subseteq U$ and hence A is $NGR^{*}C$ set.

Theorem 3.14. *For each* $A \in (\mathcal{X}, \tau_N)$. *either A is NGC or* $C(A)$ *is* $NGR^{*}C$ *in* $(\mathcal{X}, \tau_N)$.

Proof. If A is not NGC, then the only NGO set containing $C(A)$ is 1_N. Thus $R_{cl}C(A)$ is contained in 1_N and hence $C(A)$ is $NGR^{*}C$ in $(\mathcal{X}, \tau_N)$.

Theorem 3.15. *In a neutrosophic partition space* (*Definition* 3.5), *every* $NGR^{*}C$ *is NGC set*.

Proof. Let A be a $NGR^{*}C$ and $A \subseteq U$, where U is NO. Since every NO set is a NGO set, U is NGO. By hypothesis A is $NGR^{*}C$ set. Hence we have $NR_{cl}(A) \subseteq U$. In neutrosophic partition space every NC set is NO.

Hence the class of NRC sets coincides with the class of NC sets. Therefore we have $N_{cl}(A) \subseteq NR_{cl}(A) \subseteq U$. Thus we have A is NGC.

Theorem 3.16. *In a neutrosophic partition space, every* $NGR^{*}C$ *is NRGC set.*

Proof. Let A be a $NGR^{*}C$ and $A \subseteq U$, where U is NRO. In neutrosophic partition space, the class of NRC sets coincides with the class of NC sets (NO sets) and the class of NRO sets also coincides with the class of NC sets (NO sets). Therefore we have $(\mathcal{X}, \tau_N) = NRO(\mathcal{X}, \tau_N) = NRC(\mathcal{X}, \tau_N)$. Hence we also get in a neutrosophic partition space every NRO set is a NGO set. So we have U is a NGO set with $A \subseteq U$. By hypothesis A is $NGR^{*}C$. Hence we have $NR_{cl}(A) \subseteq U$. Thus we have A is NRGC.

The relationship of $NGR^{*}C$. sets with some other sets discussed in this section is showed by the Figure (a).

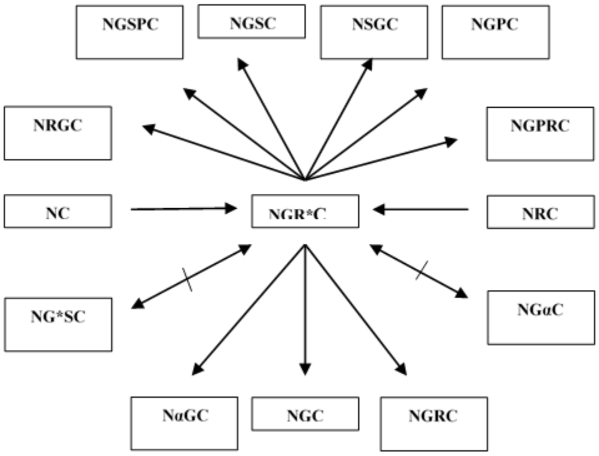

Figure (a)

where A$\longrightarrow$B (resp. A$\not\longleftrightarrow$B) represents A implies B (resp. *A* and *B* are independent).

IV. Neutrosophic Generalized Regular* Open Set

Definition 4.1. A subset A of a neutrosophic topological space $(\mathcal{X}, \tau_N)$ is called $NGR^{*}O$ set if the complement of A $(C(A))$ is $NGR^{*}C$.

Example 4.1. Let $\mathcal{X} = \{a, b\}$, $\tau_N = \{0_N, A, 1_N\}$ where

$A = \{\langle a, 0.4, 0.5, 0.4\rangle, \langle b, 0.4, 0.3, 0.7\rangle\}$. Then τ_N is a NT. Consider $G = \{\langle a, 0.4, 0.5, 0.5\rangle, \langle b, 0.3, 0.2, 0.6\rangle\}$. Then $C(G) = \{\langle a, 0.5, 0.5, 0.4\rangle, \langle b, 0.6, 0.8, 0.3\rangle\}$. Let U be any NGO such that $C(G) \subseteq U$. Then $NR_{cl}(C(G)) = C(A) \subseteq U$. Hence $C(G)$ is NGR^*C. Hence G is a NGR^*O set.

Theorem 4.1. *A subset A of a neutrosophic topological space* $(\mathcal{X}, \tau_N)$ *is* NGR^*O *if and only if* $B \subseteq NR_{\text{int}}(A)$ *where B is NGC in* $(\mathcal{X}, \tau_N)$ *and* $B \subseteq A$.

Proof. Necessity: Suppose $B \subseteq NR_{\text{int}}(A)$ where B is NGC in $(\mathcal{X}, \tau_N)$ and $B \subseteq A$. Let $C(A) \subseteq M$, where M is NGO. Hence $C(M) \subseteq A$, where $C(M)$ is NGC. Hence by assumption $C(M) \subseteq NR_{\text{int}}(A)$, which implies $C(NR_{\text{int}}(A)) \subseteq M$. Therefore $NR_{cl}(C(A)) \subseteq M$. Thus $C(A)$ is NGR^*C, implies A is NGR^*O.

Sufficiency. Let A is NGR^*O in $(\mathcal{X}, \tau_N)$ with $N \subseteq A$, where N is NGC. We have $C(A)$ is NGR^*C with $C(A) \subseteq C(N)$ where $C(N)$ is NGO. Then we have $NR_{cl}(C(A)) \subseteq C(N)$ implies $N \subseteq 1_N\text{-}R_{cl}(C(A)) = NR_{\text{int}}(1_N - (C(A)) = NR_{\text{int}}(A))$ Hence proved.

Theorem 4.2. *Every NRO set is* NGR^*O *set.*

Proof. Let A be a NRO set. Then $1_N\text{-}A$ is NRC. By Theorem 3.2, $1_N\text{-}A$ is NGR^*C. Hence A is NGR^*O set.

Theorem 4.3. *If* $NR_{\text{int}}(A) \subseteq B \subseteq A$ *and A is a* NGR^*O *subset of* $(\mathcal{X}, \tau_N)$, *then B is also a* NGR^*O *subset of* $(\mathcal{X}, \tau_N)$.

Proof. $NR_{\text{int}}(A) \subseteq B \subseteq A$ implies $C(A) \subseteq C(B) \subseteq NR_{cl}(C(A))$. Given $C(A)$ is NGR^*C. By Theorem 3.09, $C(B)$ is NGR^*C. Therefore B is NGR^*O.

Theorem 4.4. *If a subset A of a neutrosophic topological space* $(\mathcal{X}, \tau_N)$ *is*

NGR^*O in $(\mathcal{X}, \tau_N)$, then $F = 1_N$, whenever F is NGO and $NR_{\text{int}}(A) \subseteq C(A) \subseteq F$.

Proof. Let A be a NGR^*O and F be NGO. Then $NR_{\text{int}}(A) \cup C(A) \subseteq F$. This gives $C(F) \subseteq (1_N\text{-}NR_{\text{int}}(A)) \cap A = NR_{cl}(C(A)) \cap A = NR_{cl}(C(A))\text{-}C(A)$. Since $C(F)$ is NGC and $C(A)$ is NGR^*O by Theorem 3.13, we have $C(F) = 0_N$. Thus $F = 1_N$.

Theorem 4.5. *If a subset A of a topological space $(\mathcal{X}, \tau_N)$, is NGR^*C, then $NR_{cl}(A) - A$ is NGR^*O.*

Proof. Let $A \subseteq (\mathcal{X}, \tau_N)$ be a NGR^*C and let F be NGC such that $F \subseteq NR_{cl}(A) - \text{A}$. Then by Theorem $F = 0_N$. So, $0_N = F \subseteq NR_{\text{int}}(NR_{cl}(A) - \text{A})$. This shows that A is NGR^*O set.

Conclusion

In this paper, we defined some new classes of neutrosophic generalized closed sets ($NGRC$, NG^*SC, NGR^*C). We studied some characteristics of neutrosophic generalized regular star closed sets (NGR^*C) in neutrosophic topological space and obtained some of their basic properties. We have analyzed the relationship between the NGR^*C sets and some other generalized closed sets which already defined by many authors. Next, we introduced neutrosophic generalized regular star open sets (NGR^*O) and analyzed some of their properties.

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