Advances and Applications in Mathematical Sciences
Volume 20, Issue 4, February 2021, Pages 649-661

MAJORITY DOMINATING AND CONNECTED MAJORITY DOMINATING SETS IN THE CORONA AND JOIN OF GRAPHS

J. JOSELINE MANORA and T. MUTHUKANI VAIRAVEL

P.G and Research Department of Mathematics
Tranquebar Bishop Manickam Lutheran College
(Affiliated to Bharathidasan University)
Porayar 609307, Tamilnadu, India

Department of Mathematics
Sir Issac Newton College
(Affiliated to Bharathidasan University)
Nagapattinam 611102, Tamilnadu, India
E-mail: joseline_manora@yahoo.com
muthukanivairavel@gmail.com

Abstract

In this research article, majority dominating set, connected majority dominating set, majority domination number $\gamma_M(G)$ and connected majority domination number $\gamma_{CM}(G)$ for corona graph $G' = G \circ H$ of two graphs G and H are determined. Then the relationship among the domination numbers $\gamma(G')$, $\gamma_M(G')$ and $\gamma_{CM}(G')$ are studied. Some results are also established for Join of two graphs $G^J = G + H$.

1. Introduction

Let G be a finite, simple, connected and undirected graph with vertex set $V(G)$ and edge set $E(G)$. Let $G(V, E)$ be a graph with $p = |V(G)|$ and $q = |E(G)|$, denote the number of vertices and edges of a graph G. Let $v \in V(G)$. The neighbourhood of v is the set $N_G(v) = N(v)$

2010 Mathematics Subject Classification: 05Cxx.
Keywords: majority domination number, connected majority domination number.
Received January 15, 2020; Accepted May 13, 2020

$= \{u \in V(G) : uv \in E(G)\}$. If $X \subseteq V(G)$, then the open neighbourhood of X is the set $N_G(X) = N(X) = \bigcup_{v \in X} N_G(v)$. The closed neighbourhood of X is $N_G[X] = N[X] = X \cup N(X)$. A subset S of $V(G)$ is a dominating set [2] for G if every vertex of G either belongs to S or is adjacent to a vertex of S. The minimum cardinality of a minimal dominating set for G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set S is said to be a connected dominating set [6] if the subgraph $\langle S \rangle$ induced by S is connected in G. The minimum cardinality of a minimal connected dominating set is called the connected domination number of G and is denoted by $\gamma_c(G)$.

A subset S of $V(G)$ is a majority dominating set (*MD*) [4] if at least half of the vertices of $V(G)$ are either belong to S or adjacent to the elements of S i.e., $|N[S]| \geq \left\lceil \frac{V(G)}{2} \right\rceil$. The minimum cardinality of a minimal majority dominating set for G is called majority domination number of G and is denoted by $\gamma_M(G)$. This parameter was introduced by and J. Joseline Manora and V. Swaminathan in [5]. Let G be any graph with p vertices and let $u \in V(G)$. Then u is said to be Majority Dominating (*MD*) vertex if $d(u) \geq \left\lceil \frac{p}{2} \right\rceil - 1$.

A subset S of $V(G)$ is a Connected Majority Dominating Set [3] (*CMD*) if (i) S is a majority dominating set and (ii) the subgraph $\langle S \rangle$ induced by S is connected in G. The minimum cardinality of minimal connected majority dominating set for G is called the Connected Majority Domination number of G, denoted by $\gamma_{CM}(G)$.

2. MD and CMD Sets in the Corona Graphs

Definition 2.1 [1]**.** The Corona $G \circ H$ of two graphs G and H is the graph obtained by taking one copy of G of order n and n copies of H, and then joining the i^{th} vertex of G to every vertex in the i^{th} copy of H. For every $v \in V(G)$, denote by H^v the copy of H whose vertices are attached one by one to the vertex v. Subsequently, denote by $v + H^v$ the subgraph of the corona $G \circ H$ corresponding to the join $\langle\{v\}\rangle + H^{v}$, $v \in V(G)$.

Example 2.2. Let the graphs $G = C_4$ and $H = K_3$ and let $G' = G \circ H$. i.e., $G' = C_4 \circ K_3$.

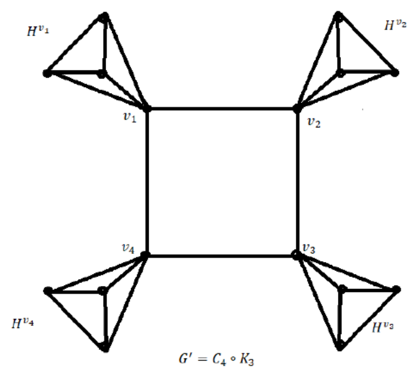

Figure 2.1

Consider the vertices of G' is $V(G') = \{v_1, H^{v_1}, v_2, H^{v_2}, v_3, H^{v_3}, v_4, H^{v_4}\}$ where H^{v_i} denotes the i^{th} copy of H joined to v_i of G. Let $m = O(G) = 4$. Let $S_1 = \{v_1, v_2, v_3, v_4\}$ be a Dominating Set of G'. Therefore, $\gamma(G') = |S_1| = 4$. Let $S_2 = \{v_1, v_2\}$ be a MD set and CMD set of G'. This implies that $\gamma_M(G') = |S_2| = 2$. Also, $\gamma_{CM}(G') = 2$. Hence $\gamma_M(G') = \gamma_{CM}(G') < \gamma(G')$.

Example 2.3. Let $G = P_7$ and $H = K_1$ and let $G' = G \circ H$. Consider the vertex sets $V(G) = \{v_1, \ldots, v_7\}$ and $V(G') = \{v_1, \ldots, v_7, v_1', \ldots, v_7'\}$. Here $S_1 = V(G)$ is a Dominating Set of G'. Hence, $\gamma(G') = |S_1| = 7$. Let $S_2 = \{v_2, v_5\}$ is a MD set of G'. Thus, $\gamma_M(G') = |S_2| = 2$. Let $S_3 = \{v_2, v_3, v_4\}$ is a CMD set of G'. Hence, $\gamma_{CM}(G') = 3$. Hence $\gamma_M(G') < \gamma_{CM}(G') < \gamma(G')$.

Example 2.4. Let $G = C_5$ and $H = K_5$ and let $G' = G \circ H$. Consider the vertices of G' is $V(G') = \{v_1, H^{v_1}, v_2, H^{v_2}, v_3, H^{v_3}, v_4, H^{v_4}, v_5, H^{v_5}\}$ and $|V(G')| = 30$. Let $m = O(G) = 5$. Let $S_1 = \{v_1, v_2, v_3, v_4, v_5\}$ be a Dominating Set of G'. Therefore, $\gamma(G') = |S_1| = 5$. Let $S_2 = \{v_1, v_4\}$ be a

MD set of G'. This implies that $\gamma_M(G') = |S_2| = 2$. Let $S_3 = \{v_2, v_3, v_4\}$ be a CMD set of G'. Hence, $\gamma_{CM}(G') = 3$. Hence $\gamma_M(G') < \gamma_{CM}(G') < \gamma(G')$.

Observations 2.5.

1. For any graph G and H, the corona graph $G' = G \circ H$, $\gamma_M(G') \leq \left\lceil \frac{\gamma(G')}{2} \right\rceil$ and $\gamma_{CM}(G') \leq \left\lceil \frac{\gamma(G')}{2} \right\rceil$.

2. For any corona graph G', (i) $\gamma_M(G') \leq \gamma_{CM}(G')$ (ii) $\gamma_M(G') < \gamma(G')$ (iii) $\gamma_M(G') < \gamma_{CM}(G') < \gamma(G')$.

3. For any graphs G and H with $O(G) = m$ and $O(H) = n$, if the corona $G' = G \circ H$ with $|V(G')| = p$ where $p = mn + m$ then $\gamma_M(G') \leq \left\lceil \frac{m}{2} \right\rceil$ and $\gamma_{CM}(G') \leq \left\lceil \frac{m}{2} \right\rceil$.

Theorem 2.6. *Let G be a connected graph and H be any graph with order m and n respectively. Let* $G' = G \circ H$ *and the set* $S \subseteq V(G)$ *is a MD set of* G' *if and only if* $[V(u + H^u) \cap S]$ *is a MD set of* $(u + H^u)$ *such that* $|N[S]| \geq \left\lceil \frac{p}{2} \right\rceil$, *for at least one vertex* $u \in V(G)$.

Proof. Let $V(G') = \{u_1, H^{u_1}, u_2, H^{u_2}, \ldots, u_m, H^{u_m}\}$. Let $S = \{u_1\}$ be a MD set of G'. Then $|N_{G'}[S]| \geq \left\lceil \frac{p}{2} \right\rceil$. Let $|V(G')| = p = mn + m$. If $u_1 \in V(G)$ then $\{u_1\}$ is a MD set of (u_1, H^{u_1}). Since every dominating set of G is a MD set of G, $[V(u_1 + H^{u_1}) \cap S]$ is a MD set of $(u_1 + H^{u_1})$. If $|N[u_1]| \geq \left\lceil \frac{p}{2} \right\rceil$ then $[V(u_1 + H^{u_1}) \cap S]$ is a MD set of G', atleast one vertex $u_1 \in V(G)$.

If not, take $S = \{u_1, u_2\}$ is a MD set of G'. Then $[V(u_1 + H^{u_1}) \cap S]$ is a MD set of $(u_1 + H^{u_1})$ and $[V(u_2 + H^{u_2}) \cap S]$ is a MD set of $(u_2 + H^{u_2})$ in which u_1 dominates at most $(m + n)$ vertices and at least $(n + 2)$ vertices and u_2 dominates only n vertices. For two vertices $u_1, u_2 \in V(G)$, $|N[u_1] + N[u_2]| \geq 2n + 2$ such that $|N[S]| \geq \left\lceil \frac{p}{2} \right\rceil$. If not, continue this argument till we obtain a set S with at least one vertex $u \in V(G)$ such that $|N[S]| \geq \left\lceil \frac{p}{2} \right\rceil$ and $[V(u + H^u) \cap S]$ is a MD set of $(u + H^u)$. Conversely, for at most one vertex $u \in V(G)$, $[V(u + H^u) \cap S]$ is a MD set of $(u + H^u)$ such that $|N[u]| \geq \left\lceil \frac{p}{2} \right\rceil$. It implies that $S = \{u\}$ is a MD set of G'. Suppose for two vertices $u_1, u_2 \in V(G)$, $[V(u_1 + H^{u_1}) \cap S]$ and $[V(u_2 + H^{u_2}) \cap S]$ are the MD sets of the subgraphs $(u_1 + H^{u_1})$ and $(u_2 + H^{u_2})$ respectively such that $|N[u_1] \cup N[u_2]| \geq \left\lceil \frac{p}{2} \right\rceil$. It implies that $S = \{u_1, u_2\}$ is a MD set of G'. Hence the set $S \subseteq V(G)$ is a MD set of the corona graph G'.

Corollary 2.7. *Let G be a connected graph and H be any graph with order m and n respectively. Let* $G' = G \circ H$ *and the set* $S \subseteq V(G)$ *is a CMD set of* G' *if and only if* $[V(u + H^u) \cap S]$ *is a CMD set of* $(u + H^u)$ *such that* $|N[S]| \geq \left\lceil \frac{p}{2} \right\rceil$. *and the induced subgraph* $\langle S \rangle$ *is connected for at least one vertex* $u \in V(G)$.

Corollary 2.8. *Let G be any connected graph and H be any graph with m and n vertices respectively. Then* $\gamma_M(G') \leq \left\lceil \frac{m}{2} \right\rceil$ *and* $\gamma_{CM}(G') \leq \left\lceil \frac{m}{2} \right\rceil$.

3. $\gamma_M(G')$ and $\gamma_{CM}(G')$ for Some Classes of Graphs

In this section, it is worth noting that if G and H are connected and non-trivial graph then $\gamma_M(G') \geq 1$ and $\gamma_{CM}(G') \geq 1$.

Proposition 3.1. *Let* $G = K_m$ *be a complete graph of m vertices and* $= K_1$. *Then* $\gamma_M(G') = \gamma_{CM}(G') = 1$, *where* $G' = G \circ H$.

Proposition 3.2. *Let* $G = K_4$ *and H be any complete graph with* $n \geq 3$. *Then* $\gamma_M(G \circ H) = \gamma_{CM}(G \circ H) = 2$ *and* $\gamma_M(G') = \gamma_{CM}(G') = \frac{\gamma(G')}{2}$.

Corollary 3.3. *When* $n = 1, 2, \gamma_M(G') = \gamma_{CM}(G') = 1$.

Corollary 3.4. *For* $G' = K_4 \circ K_n, n \geq 3, \gamma_M(G') = \gamma_{CM}(G') = \frac{\gamma(G')}{2}$.

Proof. Since $\gamma(G') = 4$, by the above theorem, $\gamma_M(G') = \gamma_{CM}(G') = 2$.

Proposition 3.5. *Let* $G = K_5$ *and* $H = K_n$ *where* $G' = K_5 \circ K_n$. *Then*

(i) $\gamma_M(G') = \gamma_{CM}(G') = 2$, *if* $2 \leq n \leq 5$.

(ii) $\gamma_M(G') = \gamma_{CM}(G') = 3$, *if* $n \geq 6$.

Proposition 3.6. *Let* $G = S(K_{1,t})$ *be a subdivision of a star and* $H = K_1$. *Then* $\gamma_M(G \circ H) = \gamma_{CM}(G \circ H) = \left\lceil \frac{t-1}{2} \right\rceil + 1$.

Corollary 3.7. *Let* $G = K_{1,m}$ *be a star with* $(m + 1)$ *vertices. Then* $\gamma_M(G \circ K_1) = \gamma_{CM}(G \circ K_1) = 1$.

Proposition 3.8. *Let* $G = D_{r,s}, r \leq s$ *be a double star with* $m = (r + s + 2)$ *vertices and* $H = K_1$. *Then* $\gamma_M(G \circ H) = \gamma_{CM}(G \circ H) = 2$.

Proof. Let $O(G) = m = (r + s + 2)$ and $O(H) = 1$ with $p = 2(r + s + 2)$. Let the corona $G' = (G \circ K_1) = (D_{r,s} \circ K_1)$. There are two vertices u_1 and u_2 with *r* and *s* pendants respectively in *G*.

Case (i): If $r = s = 1$ pendant at each vertices u_1 and u_2 with $m = 4$ and $p = 8$ then u_1 dominates $(r + 3) = 4$ vertices. This implies that $|N[u_1]| = 4 = \frac{p}{2}$. Therefore $\gamma_M(G') = \gamma_{CM}(G') = 1$.

Case (ii): When $r, s \geq 2$ and $r = s$. In G', the vertex u_1 dominates $(r + 3)$ and u_2 dominates $(s + 1)$ vertices.

Choose $S = \{u_1, u_2\}$. Then $|N[S]| = r + s + 4 > \frac{p}{2}$. Since u_1 and u_2 are adjacent, *S* is a MD set for G'. Therefore $\gamma_M(G') = \gamma_{CM}(G') = |S| = 2$.

Case (iii): When $r, s \geq 2$ and $r < s$ and $s \geq r + 1$. In this case, the vertex u_2 dominates $(s + 3)$ and u_1 dominates $(r + 1)$ vertices. Therefore $|N[u_2]| = s + 3 < \frac{p}{2}$. Then choose $S = \{u_2, u_2\}$ with $|N[S]| = (s + 3) + (r + 1) = r + s + 4 > \frac{p}{2}$. This implies that S is a MD and CMD set of G'. Hence $\gamma_M(G') = \gamma_{CM}(G') = 2$.

Proposition 3.9. *Let* $G = W_m$ *and* $H = K_1$ *with* $O(G) = m$. *Then* $\gamma_M(G \circ H) = \gamma_{CM}(G \circ H) = 1$.

Proposition 3.10. *Let G be a totally disconnected graph of m vertices and* $H = K_1$. *Then* $\gamma_M(G \circ H) = \left\lceil \frac{p}{4} \right\rceil$ *and* $\gamma_{CM}(G \circ H)$ *does not exist.*

Proposition 3.11. *Let* $G = rK_2, r \geq 1$ *and* $H = K_1$. *Then* $\gamma_M(G \circ H) = \left\lceil \frac{p}{6} \right\rceil$ *and* $\gamma_{CM}(G \circ H)$ *does not exist.*

Proposition 3.12 *Let* $G = C_4$ *and* $= K_n, n \geq 2$. *Then* $\gamma_M(G \circ H) = \gamma_{CM}(G \circ H) = 2$ *and* $\gamma_M(G \circ H) = \gamma_{CM}(G \circ H) = \frac{\gamma(G \circ H)}{2}$.

Proposition 3.13. *Let* $G = C_m$ *be a cycle of m vertices,* $m \geq 3$ *and* $H = K_1$. *If* $|V(G \circ H)| = p$, *then*

(i) $\gamma_M(G') = \left\lceil \frac{p}{8} \right\rceil$ *and*

(ii) $\gamma_{CM}(G') = \begin{cases} \left\lfloor \frac{p}{4} \right\rfloor, & \text{if } p \equiv 2 \pmod 4 \\ \left\lfloor \frac{p-1}{4} \right\rfloor, & \text{if } p \equiv 0 \pmod 4 \end{cases}$.

Proof. Let $G = C_m, m \geq 3$ and $H = K_1$. Then $G' = G \circ H$. Let $V(G') = \{v_1, \ldots, v_m, v'_1, \ldots, v'_m\}$ where v_i be the inner vertices and v'_i be the pendants in G'.

Case (i): Let $S = \{v_1, \ldots, v_{\lceil \frac{p}{8} \rceil}\}$ with $|S| = \lceil \frac{p}{8} \rceil = t$. Then $|N[S]| = \sum_{i=1}^{t} d(v_i) + t = 3t + t = 4t = 4\lceil \frac{p}{8} \rceil \geq \lceil \frac{p}{2} \rceil$. Therefore S is a MD set of G'. Hence, $\gamma_M(G') \leq |S| = \lceil \frac{p}{8} \rceil$.

Suppose $|S'| = |S| - 1 = t - 1$. Then $|N[S']| = \sum_{i=1}^{t-1} d(v_i) + (t-1)$ $= 4t - 4 = 4(t-1) = 4\left(\lceil \frac{p}{8} \rceil - 1\right) < \frac{p}{2}$. (Since p is even). Therefore S' is not a MD set of G'.

Hence, $\gamma_M(G') \geq |S| = \lceil \frac{p}{8} \rceil$. It implies that $\gamma_M(G') = \lceil \frac{p}{8} \rceil$.

Case (ii): When $p \equiv 2 \pmod 4$.

Subcase (a). Let $S = \{v_1, v_{\lfloor \frac{p}{4} \rfloor}\}$ with $|S| = \lfloor \frac{p}{4} \rfloor = t$. Then $|N[S]| = \sum_{i=1}^{t-1} d(v_i) - 1 = 3t - 1 = \frac{p}{2} + 1 > \frac{p}{2}$, $p \equiv 2 \pmod 4$. Since the induced subgraph $\langle S \rangle$ is connected, S a CMD set of G'. Hence,

$$\gamma_{CM}(G') \leq |S| = \left\lfloor \frac{p}{4} \right\rfloor. \qquad (1).$$

Suppose $|S'| = |S| - 1 = t - 1$. Then $|N[S']| = \sum_{i=1}^{t-1} d(v_i) - 1 =$ $3t - 4 = 3\lfloor \frac{p}{4} \rfloor - 4 < \frac{p}{2}$. (Since p is even). Therefore S' is not a CMD set of G'. Hence,

$$\gamma_{CM}(G') \geq |S| = \left\lfloor \frac{p}{4} \right\rfloor \qquad (2).$$

From (1) and (2), $\gamma_{CM}(G') = \lfloor \frac{p}{4} \rfloor$.

Subcase (b) When $p \equiv 0 \pmod 4$.

Proceeding the same discussion as in subcase (a), we conclude that $\gamma_{CM}(G') = \left\lfloor \frac{p-1}{4} \right\rfloor$, if $p \equiv o \pmod 4$.

Corollary 3.14. *Let* $G = P_m$ *be a path of m vertices,* $m \geq 3$ *and* $H = K_1$. *If* $|V(G \circ H)| = p$, *then* (i) $\gamma_M(G') = \left\lceil \frac{p}{8} \right\rceil$ *and* (ii)

$$\gamma_{CM}(G') = \begin{cases} \left\lfloor \frac{p}{4} \right\rfloor, & if \ p \equiv 2 \pmod 4 \\ \left\lfloor \frac{p-1}{4} \right\rfloor, & if \ p \equiv 0 \pmod 4 \end{cases}.$$

4. Relationships among $\gamma(G \circ H)$, $\gamma_M(G \circ H)$ and $\gamma_{CM}(G \circ H)$

Theorem 4.1. *If a connected graph G has at least one full degree vertex u with* $O(G) = m$ *and* $O(H) = 1$ *if and only if* $\gamma_M(G \circ H) = \gamma_{CM}(G \circ H) = 1$.

Proof. Let $O(G) = m$ and $H = K_1$. Then $|V(G')| = 2m$. If G has only one full degree vertex u then $|N[u]| = m$ and u dominates $(m+1)$ vertices of G'. This implies that $S = \{u\}$ is a MD and CMD set of G'. Suppose G has two full degree vertex u_1 and u_2. Then $|N[u_1]| = m+1$ and $|N[u_2]| = m+1$. But u_1 dominates $(m+1)$ vertices and u_2 dominates $(m+1)$ vertices of G'. This implies $S = \{u_1\}$ or $S = \{u_2\}$ is a MD and CMD set of G'. If the graph G has more than two full degree vertices then each vertex of G dominates $(m+1)$ vertices of $G' = G \circ K_1$, $S = \{u\}$, for any vertex $u \in V(G)$ is a MD and CMD set of G'. This implies that $\gamma_M(G \circ H) = \gamma_{CM}(G \circ H) = 1$. The converse is obvious.

Proposition 4.2. *If a graph G has exactly two MD vertices and others are pendants such that* $O(G) = m$, $m \geq 6$, *then* $\gamma_M(G \circ H) = \gamma_{CM}(G \circ H) = 2$, *where* $H = K_1$.

Theorem 4.3. *If* $O(G) < O(H)$, G *and* H *are complete then* $\gamma_M(G \circ H) = \gamma_{CM}(G \circ H) = \left\lceil \frac{\gamma(G \circ H)}{2} \right\rceil$.

Proof. Let $O(G) = m$ and $O(H) = n$ and $G' = G \circ H$. Let $V(G) = \{u_1, \ldots, u_m\}$ and $V(H) = \{v_1, \ldots, v_n\}$. Consider $p = mn + m = m(n+1)$ and $\left\lceil \frac{p}{2} \right\rceil = \left\lceil \frac{m(n+1)}{2} \right\rceil = \frac{mn}{2} + \frac{m}{2}$. In G', a vertex u_1 dominates $(m+n)$ vertices i.e., $|N[u_1]| = m + n$ and $u_2, u_3, \ldots$ dominates n vertices only since they are adjacent. Let $S = \{u_1, u_{\lceil \frac{m}{2} \rceil}\} \subseteq V(G)$. Then $|N[S]| = n\left(\left\lceil \frac{m}{2} \right\rceil - 1\right) + n + m = \frac{mn}{2} + m > \frac{mn}{2} + \frac{m}{2} = \left\lceil \frac{p}{2} \right\rceil$. It implies that S is a MD set of G'. Hence, $\gamma_M(G') \leq |S| = \left\lceil \frac{m}{2} \right\rceil = \left\lceil \frac{\gamma(G')}{2} \right\rceil$. Since all vertices are adjacent in S and S is a CMD set of G'. It implies that $\gamma_{CM}(G') = \left\lceil \frac{\gamma(G')}{2} \right\rceil$.

Corollary 4.4. *If* $O(G) \geq O(H)$, G *and* H *are complete then* $\gamma_M(G') = \gamma_{CM}(G') \leq \left\lceil \frac{\gamma(G')}{2} \right\rceil$, *where* $G' = G \circ H$.

Result 4.5. There exists a graph $G' = G \circ H$ with $O(G) = m$ and $O(H) = 1$ for which (i) $\gamma(G') = m$ (ii) $\gamma_M(G') = \left\lceil \frac{m}{4} \right\rceil$, (iii)

$$\gamma_{CM}(G') = \begin{cases} \left\lfloor \frac{m}{2} \right\rfloor, & if\ m \equiv 1 \pmod 2 \\ \frac{m}{2} - 1, & if\ m \equiv 0 \pmod 2 \end{cases}.$$

Proof. Let $O(G) = m$ and $G' = G \circ K_1$. Then $|V(G')| = p = 2m$. There is a graph $G = P_m$ and the corona graph $G' = P_m \circ K_1$. It implies that $\gamma(G') = m$.

Case (i): Since each middle vertex dominates 4 vertices of G' with the distance $d(v_i, v_j) \geq 3$, $i \neq j$, $v_i, v_j \in V(G)$, $\left\lceil \frac{m}{4} \right\rceil$ vertices needed to dominate $\left\lceil \frac{p}{4} \right\rceil$ vertices of G'. Hence $\gamma_M(G') = \left\lceil \frac{m}{4} \right\rceil$.

Case (ii): Choose the middle vertices are adjacently in G'. Then $\left\lfloor \frac{m}{2} \right\rfloor$ vertices needed to dominate $\left\lceil \frac{p}{2} \right\rceil$ vertices of G'. Hence, $\gamma_{CM}(G') = \left\lfloor \frac{m}{2} \right\rfloor$ if $m \equiv 1 \pmod 2$. Again applying the same argument, we get, $\gamma_{CM}(G') = \frac{m}{2} - 1$, if $m \equiv 0 \pmod 2$.

Hence, we obtain an inequality $\gamma_M(G') \leq \gamma_{CM}(G') < \gamma(G')$.

Result 4.6. There exists a graph $G' = G \circ H$ with $O(G) = m + 1$ and $O(H) = 1$ for which $\gamma(G') - \gamma_M(G') = m$ and $\gamma(G') - \gamma_{CM}(G') = m$.

Proof Let $G' = G \circ H$ with $O(G) = m + 1$ and $O(H) = 1$. Then $|V(G')| = p = 2m + 2$. There exists a graph $G = K_{1,m}$ and $H = K_1$ with $p = 2m + 2$. Then the corona $G' = (K_{1,m} \circ K_1)$. By the known result in [1], $\gamma(G') = m + 1(1)$

Let u be a center vertex of G with $d(u) = m + 1$ and $|N[u]| = m + 2 > \frac{p}{2}$. This implies that $S = \{u\}$ is a MD and CMD set of G'. Hence $\gamma_M(G') = 1 = \gamma_{CM}(G')(2)$

From (1) and (2), $\gamma(G') - \gamma_M(G') = m$ and $\gamma(G') - \gamma_{CM}(G') = m$.

5. MD and CMD Sets in Join of Two Graphs

Definition 5.1 [1]. The join $G + H$ of two graphs G and H is the graph with vertex set $V(G + H) = V(G) \cup V(H)$ and edge set $E(G + H) = E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}$. It is denoted by $G^J = (G + H)$.

Proposition 5.2. *If G and H both are any two connected graphs then* $\gamma_M(G^J) = \gamma_{CM}(G^J) = 1$.

Proof. Let $V(G) = \{u_1, \ldots, u_m\}$ and $V(H) = \{v_1, \ldots, v_n\}$. Then $d^J(u_1) = n + d(u_1)$ and $d^J(v_1) = m + d(v_1)$. A vertex $u_1 \in V(G)$ dominates

$(n + 1 + d(u_1))$ vertices of G^J and $v_1 \in V(H)$ dominates $(m + 1 + d(v_1))$ vertices of G^J.

Case (i): When $m = n$. Then $V(G^J) = p = 2m$. Certainly, there exists at least one MD vertex in G^J. It implies that $S = \{u_1\}$ is a MD (or CMD) set of G^J. Hence $\gamma_M(G^J) = \gamma_{CM}(G^J) = 1$.

Case (ii): When $m < n$, where $n = m + r, r \geq 1$. Then $p = m + n = 2m + r, r \geq 1$. In this case, a vertex $u_1 \in V(G)$ is adjacent with n vertices adding with its degree $(d(u_1) + 1)$. Therefore, $|N[u_1]| = n + 1 + d(u_1) = m + r + 1 + d(u_1), r \geq 1$. Since G and H are connected $d(u_1) \geq 2$. It implies that $|N[u_1]| \geq \left\lceil \frac{p}{2} \right\rceil$ and each vertex $u_1 \in V(G)$ is a MD(or CMD) set of G^J. Hence $\gamma_M(G^J) = \gamma_M(G^J) = 1$.

Case (iii): When $m > n$, applying the same argument, each vertex $v_1 \in V(H)$ is adjacent to m vertices plus $(d(v) + 1)$ vertices. It implies that v_1 is a MD vertex of G^J. Hence $\gamma_M(G^J) = \gamma_{CM}(G^J) = 1$.

Corollary 5.3. *If the graphs G and H both are complete then* $\gamma_M(G + H) = \gamma_M(G + H) = 1$.

Corollary 5.4. *If G is any connected graph and H is complete then* $\gamma_M(G^J) = \gamma_M(G^J) = 1$.

Corollary 5.5. *Let* $G = \overline{K_m}$ *and* $H = \overline{K_n}$. *Then* $\gamma_M(G^J) = \gamma_M(G^J) = 1$.

Proof. Since $G^J = G + H = K_{m,n}$ a complete bipartite graph, each vertex of $V(G)$ (or $V(H)$) is a MD vertex of G^J.

6. Conclusion

In this article, the researcher thus discussed Majority Domination and Connected Majority Domination parameter of a graph *G*. Also, Majority

Domination number $\gamma_M(G)$ and Connected Majority Domination number $\gamma_{CM}(G)$ determined for some classes of Corona and Join of two graphs. Then bounds of $\gamma_M(G)$ and $\gamma_{CM}(G)$ are established for corona of complete graph.

References

[1] E. Go. Carmelito and R. Sergio Jr. Canoy, Domination in the corona and join of graphs, International Forum 6(16) (2011), 763-771.

[2] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of Domination in Graphs, Marces Dekker. Inc, New York, (1998).

[3] J. Joseline Manora and T. Muthukani Vairavel, Connected majority dominating set of a graph, Global Journal of Pure and Applied Mathematics 13(2) (2017), 534-543.

[4] J. Joseline Manora and T. Muthukani Vairavel, Connected majority domatic number of a graph, Malaya Journal of Matematik 1 (2019), 52-56.

[5] J. Joseline Manora and V. Swaminathan, Majority dominating sets in graphs I, Jamal Academic Research Journal 3(2) (2006), 75-82.

[6] J. Joseline Manora and V. Swaminathan, Results on majority dominating sets, Scientia Magna, Dept. of Mathematics, Northwest University, X'tian, P.R. China 7(3) (2011), 53-58.

[7] E. Sampathkumar and H. B. Walikar, The connected domination of a graph, Jour. Math. Phy. Sci. 13(6), (1979).