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FINITE ELEMENT MODEL TO STUDY THE THERMAL EFFECT OF CYST AND MALIGNANT TUMOR IN WOMEN'S BREAST DURING MENSTRUAL CYCLE UNDER COLD ENVIRONMENT

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Abstract

Most of the thermal studies in human organs are reported on malignant disorders. In this paper a finite element model is proposed to study the thermal changes in women's breast due to malignant tumor and two benign disorders namely cyst and menstrual cycle. The variations in physical and physiological parameters like blood mass flow rate, metabolic heat generation and thermal conductivity due to benign and malignant changes in the breast have been incorporated in the model. The physical and physiological conditions have been used to frame the appropriate boundary conditions. The modelling of physical and physiological processes in women's breast leads to boundary value problem involving partial differential equations for a two-dimensional study state case. A finite element solution has been obtained and used to study the thermal effect of malignant tumor and cyst in women's breast during various phases of menstrual cycle under cold environmental conditions. Such models can be developed to generate thermal information which can be useful to improve the accuracy and specificity of thermo graphic techniques in detecting malignant and benign disorders in women's breast.

Introduction

The various benign changes and disorders take place in human body organs during the life of human subjects. These changes may take place due

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to aging, hormonal changes in the structures, shape and size of human organs which lead to thermal stress affecting the performance of human subjects. One of the notable example is of woman's breast. The various physical and physiological changes can take place in woman's breast due to aging, hormonal changes, menstrual cycle and they lead to various benign and malignant disorders. The cyst is a common benign disorder among 57% women. The menstrual cycle is another cause of benign disorder in women's breast. Changes in temperature affects menstrual cycles by changing the metabolic rate which leads again to hormonal imbalance. When a person geographically relocates to a place where the climate is warmer/colder the body does not adjust automatically. Women who are used to cold will burn fat faster in a hot climate. This sudden change in temperature may change a woman's menstrual cycle. Cold weather or any weather changes does affect menstrual cycle. The metabolic rate constantly adjusts itself in response to change in weather conditions. Change in metabolic rate in turn affects the hormonal balance resulting in disturbances or changes in menstrual cycle. In women specifically, there is a monthly variation that is affected by the menstrual cycle. Thus, the study of thermal changes in woman's breast due to cyst or a malignant tumor in presence and absence of various phases of menstrual cycle can be useful to generate the thermal information in cold environment, which in turn can be useful in development of clinical thermography.

Patterson [26] performed the experimental investigations to study temperature distribution in the human peripheral region. The theoretical investigations are also reported in the literature for study of temperature distribution in peripheral tissues of flat regions of human body like trunk [9,10, 11, 12, 13, 33, 34, 35], the cylindrical shaped human organs like limbs [1, 2, 4, 5, 6, 13, 32,36,37] and spherical shaped organs like human head [15, 16, 17,18, 19 & 20] and woman's breast [22,25] under normal, physiological and environmental conditions. Also, theoretical studies on thermal effect of malignant tumors in various regions of human body like flat, cylindrical and spherical shaped organs [3, 21, 23, 24, 26, 27, 28, 29, 31, 42] are reported in the literature. Acharya et al. [7, 8] studied temperature distribution in dermal layers of male versus female subjects under normal, environmental and physiological conditions for one and two-dimensional cases. Acharya et al. [9] Investigated this temperature distribution in peripheral region of female subject during menstrual cycle.

From the literature survey, it is evident that very little attention is paid to study the effect of benign disorders on temperature distribution in human body organs. The different types of benign changes can occur in human body organs especially in woman's breast simultaneously. No investigation is reported for study of thermal changes in women's breast due to simultaneously occurring multiple benign disorders. Further no model is reported in the past for study of thermal changes due to a cyst or a malignant tumor in woman's breast during different phases of menstrual cycle in the cold environment. In this paper, a finite element model is proposed to study thermal effect of cyst and malignant tumor in women's breast during different phases of menstrual cycle under cold environment. The mathematical model and method are presented in the next section.

2. Mathematical Model

The equation for temperature distribution in living tissues for a two - dimensional steady state case in spherical coordinates is given by [10, 25]:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(Kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(K\sin\theta\frac{\partial T}{\partial\theta}\right)+M(T_A-T_v)+S=0. \tag{1}$$

Where $M = m_b c_b$ and $m_b = \rho_b\omega_b$. Here, we consider K = thermal conductivity of tissue, ω_b = blood perfusion rate, c_b = specific heat of blood, ρ_b = density of blood; T = tissue temperature at position (r, θ), c = specific heat of tissues at time t, T_v = Venous blood temperature, S = rate of metabolic heat generation, ρ = tissue density. T_A = Arterial blood temperature. m_b = Blood mass flow rate. The woman's breast is assumed to be of hemi spherical shape consisting of various tissue layers.

The outer surface of the region is exposed to the environment and heat loss at this surface due to convection, radiation, and evaporation [18]. Is expressed as following the boundary condition.

$$-K\frac{\partial T}{\partial r}=h(T-T_a)+LE\,at\,r=r_n,\ \theta\in(0,\pi). \tag{2}$$

Where h is the heat transfer coefficient, T_a is the atmospheric temperature, L and E are, latent heat and rate of sweat evaporation, respectively. r_n is the radius of the outer surface of the breast. Therefore, the following condition is imposed at [25]:

$$T(r, \theta) = F(\theta) \text{ at } r = r_0, \text{ Where } F(\theta) = b_1 + b_2\theta + b_3\theta^2. \tag{3}$$

And $T(r_0, \theta) = \alpha_1$ at $\theta = 0$, $T(r_0, \theta) = \alpha_2$ at $\theta = \pi/2$, $T(r_0, \theta) = \alpha_3$ at $\theta = \pi$.

Here α_1 and α_3 are the temperatures of the portion of core of the breast near the trunk and therefore taken to be equal to the body core temperature T_b. Here α_2 is the temperature of the extreme part of the breast core $(\theta = \pi/2)$ at a radial distance r_0 from the trunk and is generally lower than α_1 and α_3, at low atmospheric temperature. The values of the constants b_1, b_2 and b_3 are determined by using conditions (3). The normal tissues of the breast have self-controlled metabolic activity which is expressed as [23]:

$S = \bar{S}[1 + Q_d(T_b - T)]$, where $Q_d = \dfrac{2}{T_a + T_b}$ and $\bar{S}$ is the normal rate of metabolic heat generation. In order to make the model non-dimensional the following scaling of parameters is used:

$$U = \frac{T_b - T}{T_b},\ T = T_b(1 - U),\ U_A = \frac{T_b - T_A}{T_b},$$

$$T_A = T_b(1 - U_A),\ U_v = \frac{T_b - T_v}{T_b},\ T_v = T_b(1 - U_v)$$

$$T_a = T_b(1 - U_a),\ r = r'r_n \Rightarrow r' = \frac{r}{r_n},\ \theta = \theta'\pi \Rightarrow \theta' = \frac{\theta}{\pi},$$

$$M = \rho_b \omega_b c_b r_n^2 \pi^2,\ S^* = \bar{S}[1 + Q_d U T_b] r_n^2 \pi^2.$$

$S'' = \eta S$ and $Q_d = 0$ in malignant tissues, η is parameter which represents ratio of metabolic activity in tumor and normal tissues and the metabolic activity in tumors is found to vary between 1 to 7 times of that in normal tissues [21].

3. Solution of the Problem

Women's breast is divided into 16 layers (see Figure 1) surrounding the inner core with first four layers above the core as muscles, five layers of glands above the muscles and four layers of fat above the glands. The three layers of skin and sub dermal tissues cover the fat layers. A spherical shaped lump initially assumed to be cyst and alternatively assumed as malignant tumor is assumed to be present in the fat layers. The whole region is divided into 640 coaxial circular sector elements and 697 nodes.

The equation (1) along with the boundary conditions (2) and (3) in the variational form is written as given below

$$I^{(e)} = \frac{1}{2}\iint\left[K^{(e)}\pi^2 r'^2\left(\frac{\partial U^{(e)}}{\partial r'}\right)^2 + K^{(e)}\left(\frac{\partial U^{(e)}}{\partial(\theta'\pi)}\right)^2\right.$$

$$\left. + \{M^*(U_v^e - U_A^{(e)}) + S^{*(e)}\}U^{(e)}r'^2 dr' d(\theta'\pi)\right]$$

$$+ \frac{\lambda^{(e)}}{2}\int_{\theta_j}^{\theta_1'}\{h^*(U^{(e)} - U_a)^2 + 2L^* E^* U^{(e)}\}r'^2 d(\theta'\pi) \text{ for } e = 1(1)640. \tag{4}$$

Where r_i' and r_j' are boundaries of e^{th} element. $K^{(e)}, M^{(e)}, S^{*(e)}, U_A^{(e)}, U_V^{(e)}$ and U denote the values of K, M^*, S^*, U_A, U_V and U respectively in e^{th} layer [24]. $\lambda^{(e)} = 1$ for elements along the surface and $\lambda^{(e)} = 0$ for all elements which are not along the outer surface. Here we assume non dimensionlized arterial blood temperature equal to the average of non dimensionlized nodal temperatures of the previous element along radial direction as the arterial blood will enter the eth element from its previous element below it and the blood will have almost average temperature of its previous element because blood gives heat to the element from which it passes and cools down to almost same temperature of that element. In the same way the venous blood temperature is also taken as average of nodal temperatures of element from which it is coming in to next element.

$$U_A^{(e)} = \left(\frac{U_i + U_j + U_k + U_l}{4}\right)^{(e-1)} \text{ and } U_V^{(e)} = \left(\frac{U_i + U_j + U_k + U_l}{4}\right)^{(e+1)}.$$

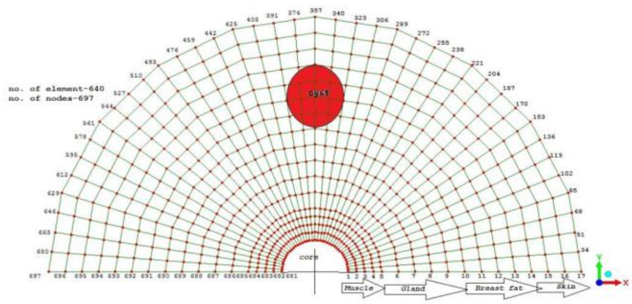

Figure 1. Element wise discretization of woman's Breast Involving Cyst or a malignant tumor.

The following bilinear shape function for variation of temperature within each element has been employed in the present study:

$$U^{(e)} = c_1^{(e)} + c_2^{(e)} r' + c_3^{(e)} \theta' + c_4^{(e)} r' \theta'. \tag{5}$$

Where $c_1^{(e)}, c_2^{(e)}, c_3^{(e)}$ and $c_4^{(e)}$ are constants for the e^{th} element. This can be obtained by the nodal conditions given below:

$$U^{(e)} = U_p,\ P = i,\ j,\ k,\ l. \tag{6}$$

The Integral $I^{(e)}$ in expression (4) are evaluated for each element using expression (6) and assembled as given below

$$I^{(e)} = \sum_{e=1}^{N} I^{(e)}. \tag{7}$$

The integral I is extremized with respect to each nodal temperature U_i as shown below:

$$\frac{dI}{d\overline{U}} = 0. \tag{8}$$

From (8) we get a system of linear algebraic equations given below;

$$[X]_{697\times 697}[\overline{U_{697\times 1}}] = [Y]_{697\times 1}. \tag{9}$$

Here X is the system matrix of order 697×697, Y is system vector of order 697×1 and $\overline{U} = [U_1\ U_2\ U_3\ ----\ U_{697}]$. The Gauss elimination method has been used to obtain the solution of (9).

4. Numerical Results and Discussion

The numerical results are obtained by [15] using the values of physical and physiological constants given in Table: 1 and Table: 2

Table 1. The values of physical and physiological constants [15, 26 and 27] without menstrual Cycle.

S.No.	Tissues	$K(Cal/cm\min^{\circ}C)$	$c_b(cal/g^{\circ}C)$	$S(cal/cm^3\min)$	$m_b(cal/cm^3\min^{\circ}C)$
1	Bone	0.075	17	0	0
2	Muscle	0.042	3768	0.0684	0.001046
3	Fat	0.016	23	0.058	0.0001880
4	Skin	0.047	368	0.0357	0.003
5	cysts	0.056	0	0	0

Table 2. The values of physical and physiological constants [5] with menstrual Cycle.

S.No.	Tissues	$m_b follicular$ $(cal/cm^3\min^0 C)$	$m_{bluteal}$ $(cal/cm^3\min^0 C)$	$S_{follicular}$ $(cal/cm^3\min)$	S_{luteal} $(cal/cm^3\min)$
1	Bone	0	0	0	0
2	Muscle	0.0005	0.0007	0.133	0.1415
3	Fat	0.00009	0.00012	0.0985	0.1135
4	Skin	0.01538	0.02197	0.0665	0.0707
5	cysts	0	0	0	0

The constants $r_i(i=1(1)17)$ can be assigned any value depending upon particular sample of tissues layers under study. The thickness of layers for a particular sample of breast is taken as: $r_1 = 4$cm; $r_2 = 5.5$cm; $r_3 = 6$cm; $r_4 = 6.4$cm; $r_5 = 6.5$cm; $r_6 = 7.0$cm; $r_7 = 7.1$cm$r_8 = 7.2$cm; $r_9 = 7.3$cm; $r_{10} = 7.8$cm; $r_{11} = 8.1$cm; $r_{12} = 8.25$cm; $r_{13} = 8.5$cm; $r_{14} = 8.6$; $r_{15} = 8.8$cm; $r_{16} = 8.9$cm; $r_{17} = 9.0$cm.

The cyst is taken between $\theta = 75^0$ and $\theta = 105^0$ in the fat layers of breast. For variable boundary condition the following sets of temperatures have been assumed at inner boundary. $\alpha = 37^0 C, \beta = 36^0 C, \gamma = 37^0 C.$ The numerical results have been computed and graphs are plotted for the low atmospheric temperature $T_a = 15^0 C,\ E = 0.$

The simulation was performed for $N = 640$ elements initially. Then again simulation was performed for $N = 1280, 2560$ and 5120 elements. We get a temperature of 30.3666 at node number 352 in cyst during luteal phase of menstrual cycle for model with $N = 640$ elements and temperature 30.3443 at node number 692 in cyst for model with $N = 1120$ elements. The error is [(30.3443-30.3433)/30.3443] $*100$ which works out to be $29.39 \times 10^{-6}\%$ only. For better clarity, we take only the ratio of four decimal points (0.3433/0.3443) $*$ 100 = 99.70% and call this term as confidence level. A confidence level of 100 implies that a saturation point has reached. In this work, we have used $N = 640$ elements as the number of elements in our model as we can safely say that results are mesh insensitive at this number.

The numerical results have been computed and graphs are plotted for temperature distribution in women's breast involving cyst and malignant tumor in presence and absence of menstrual cycle at $T_a = 15^0 C,\ E = 0.$ Figures 2, 3 and 4 represent the temperature distribution in women's breast in absence and presence of different phases of menstrual cycle. The temperature distribution in women's breast along radial and angular direction in presence and absence of menstrual cycle with cyst and malignant tumor is shown in Figures 5 to 9.

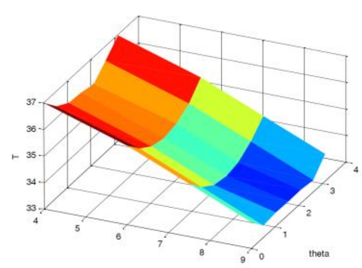

Figure 2. Temperature distribution along r and θ direction in woman's breast with variable boundary condition for $T_a = 15^0 C,\ \ E = 0$ in absence of menstrual cycle.

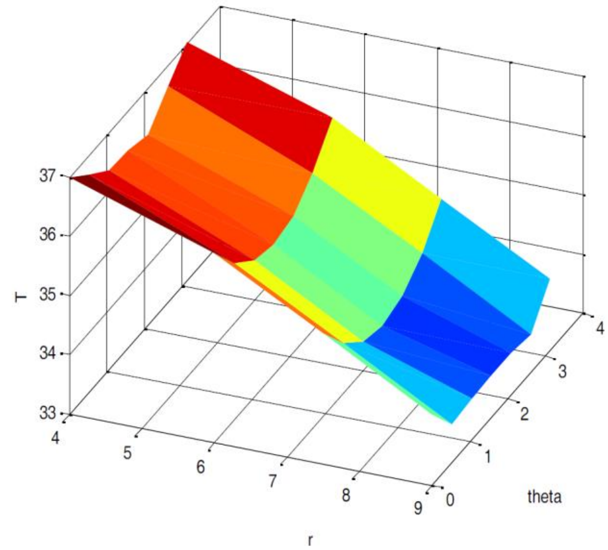

Figure 3. Temperature distribution along r and θ direction in woman's breast during follicular phase of menstrual cycle with variable boundary condition for $T_a = 15^0 C$, $E = 0$.

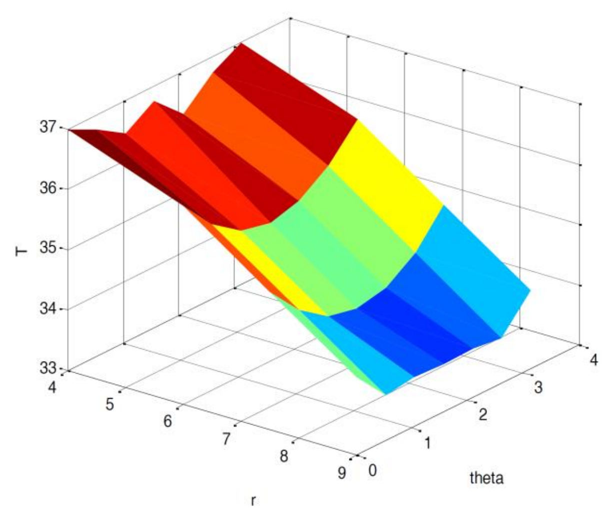

Figure 4. Temperature distribution along r and θ direction in woman's breast during luteal phase of menstrual cycle with variable boundary condition for $T_a = 15^0 C$, $E = 0$.

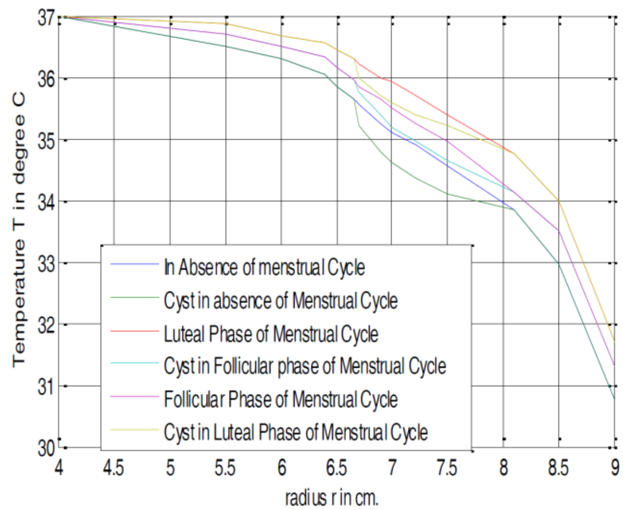

Figure 5. Radial temperature distribution in women's breast due to cyst in absence and presence of menstrual cycle. For $T_a = 15^0 C,\ E = 0\text{gm/cm}^2\,\text{min}$. Normal case means in absence of menstrual cycle.

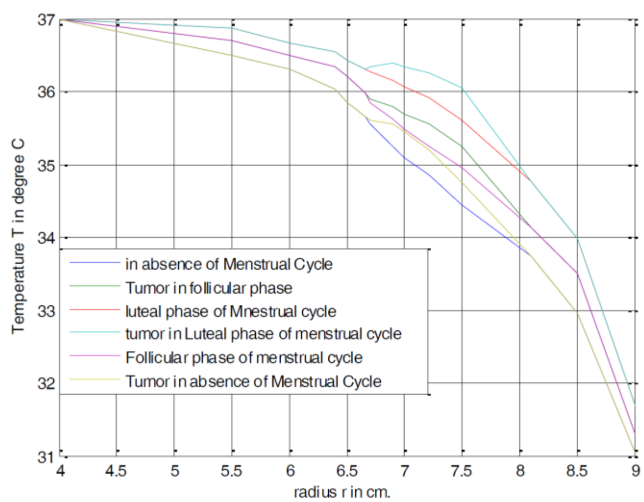

Figure 6. Radial temperature distribution in women's breast due to tumor in absence and presence of menstrual cycle. For $T_a = 15^0 C,\ E = 0\text{gm/cm}^2\,\text{min}$. Normal case means in absence of menstrual cycle.

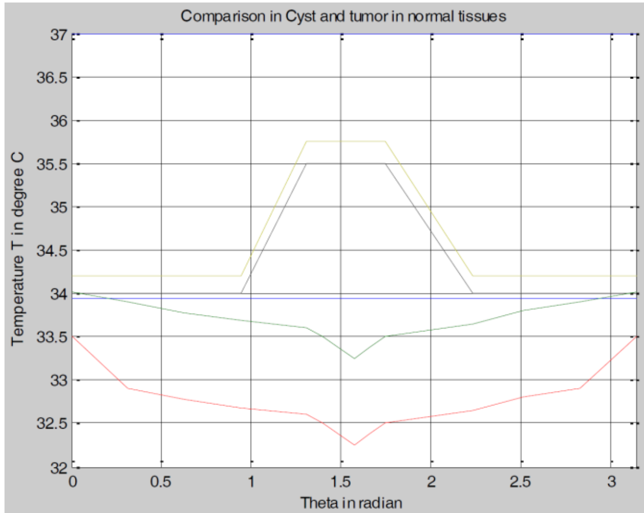

Figure 7. Angular temperature distribution in women's breast due to lump absence of menstrual cycle for $T_a = 15^0 C$, $E = 0\text{gm/cm}^2 \min$. Lump is cyst or malignant tumor. Normal case means absence of menstrual cycle.

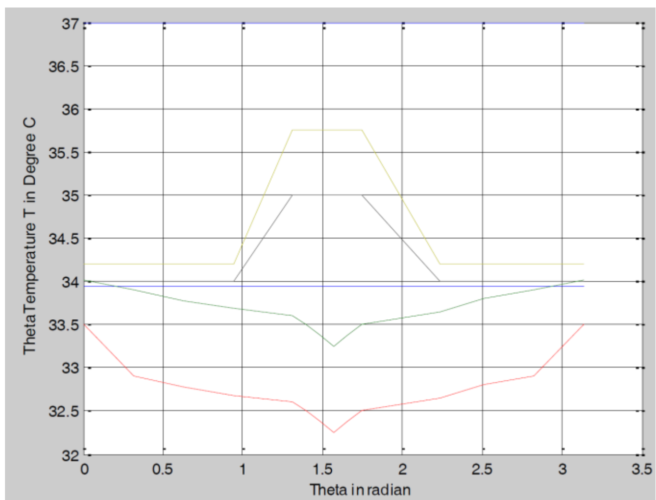

Figure 8. Angular temperature distribution in women's breast due to lump during follicular phase of menstrual cycle for $T_a = 15^0 C$, $E = 0\text{gm/cm}^2 \min$. Lump is cyst or malignant tumor.

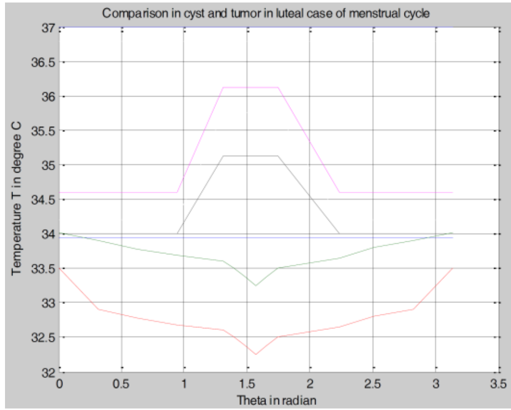

Figure 9. Angular temperature distribution in women's breast due to lump during luteal phase of menstrual cycle for $T_a = 15^0 C$, $E = 0 \text{gm/cm}^2 \text{min}$. Lump is cyst or malignant tumor.

In Figures 2, 3 and 4 it is observed that the temperature falls down gradually from inner core $r = 4.0\text{cm}$ of breast towards the outer surface of the breast. This is due to the heat loss from the outer surface of the breast to the environment. The temperature is found to be higher throughout the tissues of breast in luteal phase as compared to follicular phase and the temperature in follicular phase is higher than that in normal case i.e. in absence of menstrual cycle. Further we observe the change in slope of the curve at the junctions of each layer which is due to the different properties of each layer. Also, we observe the sharp change in slope of the curve and fall in temperature at the junctions of normal tissues and cyst and temperature is lowest at the central part of the cyst. The change in slope of the curve at the junctions of normal tissues and cyst indicates boundary and location of the cyst.

In Figures 4, 5 and 6 we observe the sharp changes in slope of the curve at the junctions of the lump and normal tissues. When this lump is cyst we observe the deviation of temperature profiles downwards from the junction of normal tissues and the cyst and this fall in temperature is maximum at the central part of the cyst and then elevation in temperature occurs from the center of cyst up to the junction of cyst and normal tissues. When this lump is a malignant tumor we observe the elevation in temperature profiles at the junction of normal tissues and tumor and this elevation of temperature continuous up to central part of tumor where it is maximum. From central part of the tumor a fall in temperature profile is observed up to the junction of tumor and normal tissues. Comparing the Figures 4, 5 and 6 it can be seen that the temperature in the tissues in absence of menstrual cycle (normal case), is lower than that for follicular phase which is again lower than that for luteal phase. In the same way the temperature in the cyst and tumor in normal case (absence of menstrual cycle) is lower than that in cyst and tumor respectively in follicular phase which is further lower than that in luteal phase. In Figures 7, 8 and 9 we also observe the change in slope of the curve along angular direction at the junctions of lump and normal tissues. We observe that when the lump is malignant tumor, there is elevation of temperature from the junction of normal and tumor tissues till the central part of the tumor and then fall in temperature is observed from central part of tumor up to normal tissues. Similarly, we observe the fall in temperature profiles along angular direction from the junction of normal tissues and cyst at different radial positions and this fall is highest at the central part of cyst and then this fall in temperature profiles decreases as we move away from central part of cyst up to the normal tissues. This fall in temperature is maximum in the tissues at radial position near the cyst and this fall in temperature in tissues is lower at radial positions away from the cyst under cold environment.

5. Conclusion

A two-dimensional finite element model is proposed and successfully employed to study the thermal patterns generated due to the cyst and malignant tumor in the women's breast in presence and absence of menstrual cycle in cold environment. From the results it is concluded that the malignant tumor acts as a local heat source in the breast and cyst acts as a local sink in the breast. These thermal properties of tumor and cyst give us the clear picture for distinguishing between the cyst and malignant tumor by thermography. Further the sharp changes in slope of curves at the junctions of lumps and normal tissues give as clear idea of location and size of the lump in the breast. The thermographic detection of malignant tumor or a cyst will be more effective during luteal phase as compared to that in follicular phase and in absence of menstrual cycle. Such models can be developed further to study the thermal effect of other malignant and benign disorders in women's breast which can be useful in generation of the thermal information for development of protocols for diagnosis of malignant and benign disorders in women's breast by thermography.

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