Advances and Applications in Mathematical Sciences
Volume 21, Issue 8, June 2022, Pages 4239-4252
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TOTAL DOMINATION NUMBER OF CYCLES WITH PARALLEL P_k CHORDS

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Abstract

A subset S of vertices in a graph $G = (V, E)$ is called a total dominating set if every vertex v in V is adjacent to an element of S and minimum cardinality of S is total domination number of G. In this paper we have determined total domination number of a new family of graphs – Cycles with parallel P_k chords denoted by $C_n(P_k)$.

1. Introduction

First time dominating set and domination number are used by Ore [3]. $\gamma(G)$ was first used to represent the domination number of a Graph G in. The total domination set and total domination were introduced in 1980 by Cockayne et al. [4]. Henning [5] introduced the disjunctive total domination set and its underlying number. As a result of our study, we are interested in finding the total domination number of new family of graphs namely Cycles with parallel Pkchords denoted by $C_n(P_k)$ defined by Elumalai and

2020 Mathematics Subject Classification: 05C69.

Keywords: Total domination number; Cycles with Parallel Chords.
Received January 10, 2022; Accepted March 2, 2022

Sethuraman [8].

2. Preliminaries

Definition 2.1 Cycle with Parallel P_k Chords. A graph G is called a cycle with parallel P_k chords denoted by $C_n(P_k)$ if G is obtained from a cycle $C_n : v_0 v_1 v_2, \ldots, v_{n-1} v_0 (n \geq 6)$ by adding paths P_k's $(k \geq 3$ and k fixed) between pair of vertices

$$(v_1, v_{n-1}), (v_2, v_{n-2}), (v_3, v_{n-3}), \ldots, (v_\alpha, v_\beta) \text{ where } \alpha = \left\lfloor \frac{n}{2} \right\rfloor - 1$$

$$\beta = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor + 1 & \text{when } n \text{ is even} \\ \left\lfloor \frac{n}{2} \right\rfloor + 2 & \text{when } n \text{ is odd} \end{cases}$$

Definition 2.2 Domination number. A dominating set D for a graph $G = (V, E)$ is a subset of V such that every vertex not in $V \setminus D$ is adjacent to at least one member of D, minimum cardinality of this set is the domination number and is denoted by $\gamma(G)$.

Definition 2.3 Total domination number. A subset S of vertices in a graph $G = (V, E)$ is called a total dominating set if every vertex v in V is adjacent to some vertex of S and minimum cardinality of S is total domination number of G and is denoted by $\gamma_t(G)$.

3. Known Results

Theorem 3.1 [2]. *Domination number of a path P_n is* $\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil$.

Theorem 3.2 [3]. *Domination number of a Cycle C_n is* $\gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

Theorem 3.3 [6]**.** For $n > 2$ $\gamma_t(P_n) = \gamma_t(C_n) = \begin{cases} \dfrac{n}{2} & if\ n \equiv 0 (\bmod 4) \\ \dfrac{n+2}{2} & if\ n \equiv 2 (\bmod 4) \\ \dfrac{n+1}{2} & otherwise \end{cases}$

Theorem 3.4 [1]**.** For $n > 2$ $\gamma_t(P_n) = \gamma_t(C_n) = \left\lfloor \dfrac{n}{2} \right\rfloor + \left\lceil \dfrac{n}{4} \right\rceil - \left\lfloor \dfrac{n}{4} \right\rfloor.$

4. Our Findings

Based on the above study we were motivated to achieve the following results. Throughout this paper our study is concerned with finding total domination number of the graph of following type shown below.

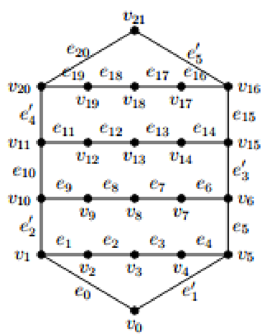

Figure 1.

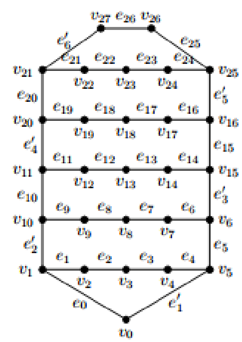

Figure 2.

Theorem 4.1. *Every n cycle* $C_n (n \geq 6)$ *with parallel* $P_{4k} (k > 0)$ *chords denoted as* $C_n(P_{4k})$ *has the total domination number*

$$\gamma_t(C_n(P_{4k})) = \begin{cases} 2k\alpha & when\ n\ is\, even \\ 2k\alpha + 1 & when\ n\ is\, odd \end{cases} \quad where\ \alpha = \left\lfloor \frac{n}{2} \right\rfloor - 1.$$

Proof. Let $G = C_n(P_{4k})$

$$E(G) = \{e_i : v_i v_j,\ j - i = 1/0 \leq i < n^* - 2 \text{ or } \tilde{n} - 2\}$$

$$\cup \{e_i : v_i v_j,\ j - i > 1/0 \leq i < \alpha + 2\}$$

depending on even cycle or odd cycle and e_i are on the cycle

$$|E(G)| = n + \alpha(4k - 1)$$

When n is even $|V(G)| = 2 + \alpha(4k) = n^{*}(say)$

Total domination set of G determining total domination number given by

$S = \{\text{end points of edges}: e_3, e_7, e_{11}, \ldots, e_{n^{*}-3}\}$

$$\gamma_t(G) = 2\left\{\frac{n^{*} - 6}{4} + 1\right\} = 2\left\{\frac{\alpha(4k) - 6}{4} + 1\right\}$$

$$\gamma_t(G) = 2k\alpha$$

When n is odd $|V(G)| = 3 + \alpha(4k) = \tilde{n}(say)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges}: e_3, e_7, e_{11}, \ldots, e_{\tilde{n}-4}\} \cup \{v_{\tilde{n}-2}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^{*} - 7}{4} + 1\right\} + 1 = 2\left\{\frac{\alpha(4k) - 4}{4} + 1\right\} + 1$$

$$\gamma_t(G) = 2k\alpha + 1$$

Figure 3 below shows total dominating set of graph $C_{13}(P_8)$

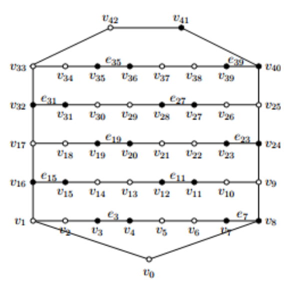

Figure 3.

Total domination number of above graph $C_{13}(P_8)$ is 21

Theorem 4.2. *Every n cycle* $C_n\,(n > \text{or} = 6)$ *with parallel* $P_{4k+1}(k > 0)$ *chords denoted as* $C_n(P_{4k+1})$ *has*

$$\gamma_t(C_n(P_{4k+1})=\begin{cases}2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+1)+1\right) & when\,\alpha\equiv 0(\bmod 4)\\ 2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+1)+k+1\right) & when\,\alpha\equiv 1(\bmod 4)\\ 2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+1)+2k+1\right) & when\,\alpha\equiv 2(\bmod 4)\\ 2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+1)+3k+1\right)+1 & when\,\alpha\equiv 3(\bmod 4)\ and\ n\,is\ even\\ 2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+1)+3k+2\right) & when\,\alpha\equiv 3(\bmod 4)\ and\ n\,is\ odd\end{cases}$$

where $\alpha = \left\lfloor\frac{n}{2}\right\rfloor - 1$

Proof. Let $G = C_n(P_{4k+1})$

$$E(G) = \{e_i : v_i v_j,\ j - i = 1/0 \leq i < n^* - 2 \text{ or } \tilde{n} - 2\}$$

$$\cup \{e_i : v_i v_j,\ j - i > 1/0 \leq i < \alpha + 2\}$$

depending on even cycle or odd cycle and e_i are on the cycle.

$$|E(G)| = n + \alpha(4k)$$

When n is even $|V(G)| = 2 + \alpha(4k+1) = n^*(say)$

Case 1. When $\alpha \equiv 0(\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_1,\ e_5,\ e_9,\ \ldots,\ e_{n^*-5} \text{ and } e_{n^*-2}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^*-6}{4} + 2\right\} = 2\left\{\frac{\alpha(4k+1)-4}{4} + 2\right\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+1)+1\right\}.$$

Case 2. When $\alpha \equiv 1(\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_1,\ e_5,\ e_9,\ \ldots,\ e_{n^*-2}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^* - 3}{4} + 1\right\} = 2\left\{\frac{\alpha(4k+1) - 2}{4} + 1\right\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+1) + k + 1\right\}.$$

Case 3. When $\alpha \equiv 2 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_1, e_5, e_9, \ldots, e_{n^*-3}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^* - 4}{4} + 1\right\} = 2\left\{\frac{\alpha(4k+1) - 2}{4} + 1\right\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+1) + 2k + 1\right\}.$$

Case 4. When $\alpha \equiv 3 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_1, e_5, e_9, \ldots, e_{n^*-4}\} \cup (v_{n^*2})$$

$$\gamma_t(G) = 2\left\{\frac{n^* - 5}{4} + 1\right\} + 1 = 2\left\{\frac{\alpha(4k+1) - 3}{4} + 1\right\} + 1$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+1) + 3k + 1\right\} + 1$$

When n is odd $|V(G)| = 3 + \alpha(4k+1) = \tilde{n}(say)$.

Case 1. When $\alpha \equiv 0 (\bmod 4)$

$$S = \{\text{endpointsof edges} : e_1, e_5, e_9, \ldots, e_{\tilde{n}-6} \text{ and } e_{\tilde{n}-3}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+1) + 1\right\}.$$

Case 2. When $\alpha \equiv 1 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges}: e_1, e_5, e_9, \ldots, e_{\tilde{n}-3}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+1) + k + 1\right\}.$$

Case 3. When $\alpha \equiv 2 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges}: e_1, e_5, e_9, \ldots, e_{\tilde{n}-4}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+1) + 2k + 1\right\}.$$

Case 4. When $\alpha \equiv 3 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges}: e_1, e_5, e_9, \ldots, e_{\tilde{n}-5} \text{ and } e_{\tilde{n}-3}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+1) + 3k + 2\right\}$$

Figure 4 below shows total dominating set of graph $C_{15}(P_9)$

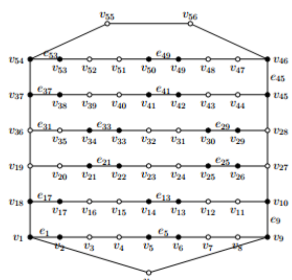

Figure 4.

Total domination number of above graph $C_{15}(P_9)$ is 28.

Theorem 4.3. *Every n cycle* $C_n (n > \text{or } 6)$ *with parallel* $P_{4k+2} (k > 0)$ *chords denoted as* $C_n(P_{4k+2})$ *has*

$$\gamma_t(C_n(P_{4k+1}) = \begin{cases} 2\left(\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2)\right) & when\, \alpha \equiv 0 (\mathrm{mod} 4) and n is even \\ 2\left(\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2)\right) + 1 & when\, \alpha \equiv 0 (\mathrm{mod} 4)\ and\ n is\ odd \\ 2\left(\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2) + k + 1\right) & when\, \alpha \equiv 1 (\mathrm{mod} 4) \\ 2\left(\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2) + k + 1\right) & when \alpha \equiv 2 (\mathrm{mod} 4)\ and\ n is\ even \\ \left(\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2) + k + 1\right) & when\, \alpha \equiv 2 (\mathrm{mod} 4)\ and\ n is\ odd \\ 2\left(\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2) + 3k + 2\right) & when\, \alpha \equiv 3 (\mathrm{mod} 4) \end{cases}$$

where $\alpha = \left\lfloor \frac{n}{2} \right\rfloor - 1.$

Proof. Let $G = C_n(P_{nk+2})$

$$E(G) = \{e_i : v_i v_j,\ j - i = 1/0 \leq i < n^* - 2 \text{ or } \widetilde{n} - 2\}$$

$$\cup \{e_i : v_i v_j,\ i > 1/0 < i < \alpha + 2\}$$

depending on even cycle or odd cycle and e_i are on the cycle.

$$|\, E(G)\,| = n + \alpha(4k+1)$$

When n is even $|\, V(G)\,| = 2 + \alpha(4k+2) = n^*(say).$

Case 1. When $\alpha \equiv 0 (\mathrm{mod}\, 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_2,\ e_6,\ e_{10},\ \ldots,\ e_{n^*-4}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^* - 6}{4} + 1\right\} = 2\left\{\frac{\alpha(4k+2) - 4}{4} + 1\right\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2)\right\}.$$

Case 2. When $\alpha \equiv 1 (\mathrm{mod}\, 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_2, e_6, e_{10}, \ldots, e_{n^*-2}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^* - 4}{4} + 1\right\} = 2\left\{\frac{\alpha(4k+2) - 2}{4} + 2\right\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2) + k + 1\right\}.$$

Case 3. When $\alpha \equiv 2 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_2, e_6, e_{10}, \ldots, e_{n^*-4}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^* - 6}{4} + 1\right\} = 2\left\{\frac{\alpha(4k+2) - 4}{4} + 1\right\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2) + 2k + 1\right\}.$$

Case 4. When $\alpha \equiv 3 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_2, e_6, e_{10}, \ldots, e_{n^*-2}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^* - 4}{4} + 1\right\} + 1 = 2\left\{\frac{\alpha(4k+2) - 2}{4} + 1\right\} + 1$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2) + 3k + 2\right\}$$

When n is odd $|V(G)| = 3 + \alpha(4k+1) = \tilde{n}(say)$.

Case 1. When $\alpha \equiv 0 (\bmod 4)$

$$S = \{\text{end points of edges}: e_2, e_6, e_{10}, \ldots, e_{\tilde{n}-5}\} \cup \{v_{\tilde{n}-3}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2)\right\} + 1.$$

Case 2. When $\alpha \equiv 1 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges}: e_2, e_6, e_{10}, \ldots, e_{\tilde{n}-3}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2) + k + 1\right\}.$$

Case 3. When $\alpha \equiv 2 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges}: e_2, e_6, e_{10}, \ldots, e_{\tilde{n}-5}\} \cup \{v_{\tilde{n}-3}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2) + 2k + 1\right\} + 1.$$

Case 4. When $\alpha \equiv 3 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges}: e_2, e_6, e_{10}, \ldots, e_{\tilde{n}-3}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+2) + 3k + 2\right\}.$$

Figure 5 below shows total dominating set of graph $C_{15}(P_{10})$

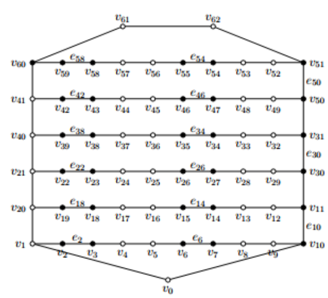

Figure 5.

Total domination number of above graph $C_{15}(P_{10})$ is 31.

Theorem 4.4. Every n cycle $C_n(n > \text{or } 6)$ with parallel $P_{4k+3}(k > 0)$ chords denoted as $C_n(P_{4k+3})$ has

$$\gamma_t(C_n(P_{4k+3}))=\begin{cases}2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+3)\right) & when\,\alpha\equiv 0(\mathrm{mod}4) and n is even\\ 2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+3)\right)+1 & when\,\alpha\equiv 0(\mathrm{mod}4)\text{ and } n \text{ is odd}\\ 2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+3)+k+1\right) & when\,\alpha\equiv 1(\mathrm{mod}4) and n is even\\ 2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+3)+k+1\right)+1 & when\,\alpha\equiv 1(\mathrm{mod}4)\text{ and } n \text{ is odd}\\ 2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+3)+2k+1\right)+1 & when\,\alpha\equiv 2(\mathrm{mod}4)\\ 2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+3)+3k+2\right)+1 & when\,\alpha\equiv 3(\mathrm{mod}4) and n is even\\ 2\left(\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+3)+3k+3\right) & when\,\alpha\equiv 3(\mathrm{mod}4)\text{ and } n \text{ is odd}\end{cases}$$

where $\alpha = \left\lfloor\frac{n}{2}\right\rfloor - 1.$

Proof. Let $G = C_n(P_{4k+3})$

$$E(G) = \{e_i : v_i v_j,\ j-i = 1/0 \le i < n^* - 2 \text{ or } \tilde{n} - 2\}$$

$$\cup \{e_i : v_i v_j,\ j-i > 1/0 < i < \alpha + 2\}$$

depending on even cycle or odd cycle and e_i are on the cycle.

$$|E(G)| = n + \alpha(4k+3)$$

When n is even $|V(G)| = 2 + \alpha(4k+3) = n^*(say)$.

Case 1. When $\alpha \equiv 1(\mathrm{mod}\, 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_2,\ e_6,\ e_{10},\ \ldots,\ e_{n^*-4}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^*-6}{4}+1\right\} = 2\left\{\frac{\alpha(4k+3)-4}{4}+1\right\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor\frac{\alpha}{4}\right\rfloor(4k+3)\right\}.$$

Case 2. When $\alpha \equiv 1(\mathrm{mod}\, 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_2, e_6, e_{10}, \ldots, e_{n^*-3}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^* - 5}{4} + 1\right\} = 2\left\{\frac{\alpha(4k+3) - 3}{4} + 1\right\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+3) + k + 1\right\}.$$

Case 3. When $\alpha \equiv 2(\text{mod}\, 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_2, e_6, e_{10}, \ldots, e_{n^*-2}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^* - 4}{4} + 1\right\} = 2\left\{\frac{\alpha(4k+3) - 2}{4} + 1\right\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+3) + 2k + 2\right\}.$$

Case 4. When $\alpha \equiv 3(\text{mod}\, 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_2, e_6, e_{10}, \ldots, e_{n^*-5}\} \cup \{v_{n^*-3}\}$$

$$\gamma_t(G) = 2\left\{\frac{n^* - 7}{4} + 1\right\} + 1 = 2\left\{\frac{\alpha(4k+3) - 5}{4} + 1\right\} + 1$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+3) + 3k + 2\right\} + 1$$

When n is odd $|V(G)| = 2 + \alpha(4k+3) = \tilde{n}\,(say)$.

Case 1. When $\alpha \equiv 0(\text{mod}\, 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges} : e_2, e_6, e_{10}, \ldots, e_{\tilde{n}-5}\} \cup \{v_{\tilde{n}-3}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+3)\right\} + 1.$$

Case 2. When $\alpha \equiv 1 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges}: e_2, e_6, e_{10}, \ldots, e_{\tilde{n}-4}\} \cup \{v_{\tilde{n}-2}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+3) + k + 1\right\} + 1.$$

Case 3. When $\alpha \equiv 2 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges}: e_2, e_6, e_{10}, \ldots, e_{\tilde{n}-3}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+3) + 2k + 2\right\}.$$

Case 4. When $\alpha \equiv 3 (\bmod 4)$

Total domination set of G determining total domination number given by

$$S = \{\text{end points of edges}: e_2, e_6, e_{10}, \ldots, e_{\tilde{n}-6}\} \cup \{e_{\tilde{n}-2}\}$$

$$\gamma_t(G) = 2\left\{\left\lfloor \frac{\alpha}{4} \right\rfloor (4k+3) + 3k + 3\right\}$$

Figure 6 below shows total dominating set of graph $C_{14}(P_{11})$

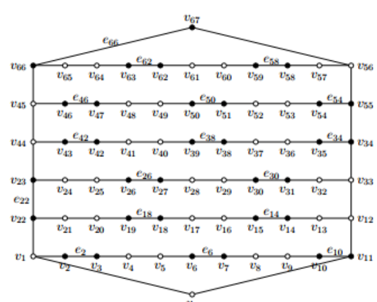

Figure 6.

Total domination number of above graph $C_{14}(P_{11})$ is 34.

5. Conclusion

In this paper, for the family of graph with parallel P_k Chords in Cycle we have found the total domination number. This study will be extended to find disjunctive total domination of this family of graphs as well as other new families of graphs.

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