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THE MAX-RADIAL NUMBER IN SOME SPECIAL CLASSES OF GRAPHS

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Abstract

For a graph $G(V, E)$, the S-radial set, $B_R(S)$, is defined for any set $S \subseteq V$, as the set of vertices $u \in V \setminus S$ which are at a distance of radius of G from some vertex $v \in S$. The Max-radial number of G is defined as $\max_S \{| B_R(S)| - |S|\}$. The study on this parameter faces the challenge of placing the maximum number of maximal length strings with certain conditions in any graph model. The Max-Radial subdivision number $sd_{\partial_R}(G)$ of a graph G is the minimum number of edges that must be subdivided (where an edge can be subdivided at most once) in order to increase the Max-Radial number. If there is no edges in G that can be subdivided in order to increase the Max-Radial number, then the Max-Radial subdivision number $sd_{\partial_R}(G) = \infty$. In this paper, we determine the value of these parameters for some special graphs.

1. Introduction

In this paper, the graphs considered are connected, undirected and finite graphs without loops and multiple edges. Other notation and terminology, can be referred in [3, 9].

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Let G be a graph of order n. A vertex $v \in V(G)$ is called a full degree vertex, if it is adjacent to all other vertices. The distance $d(x, y)$ between two vertices x and y in G is the length of a shortest path joining them. The eccentricity $e(v)$ of a vertex v in a connected graph G is the distance of a farthest vertex u in G. The radius $rad(G)$ is the minimum eccentricity. Whereas the diameter $d(G)$ is the maximum eccentricity of all vertices in G. A vertex of maximum eccentricity in a graph G is called peripheral and the set of all such vertices is the peripherian, denoted $PeriG$. For further reference on distance in graphs, one can refer [4].

Two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by $R(G)$, has the vertex set as in G and two vertices are adjacent in $R(G)$ if and only if they are radial in G. If G is disconnected, then two vertices are adjacent in $R(G)$ if they belong to different components of G. A graph G is called a radial graph if $R(G) = G$ for some graph H. For further reference on radial graphs, one can refer [1, 11]. The Corona product of two graphs G and H is the graph formed from one copy of G and $|V(G)|$ copies of H where the i^{th} vertex of G is adjacent to every vertex in the i^{th} copy of H. It is denoted by $G[H]$.

M. Mathan, M. Bhuvaneshwari and Selvam Avadayappan [12] discussed about an application of Max-Radial number of graphs in game theory. For a graph $G(V, E)$, the S-radial set, $B_R(S)$, is defined for any set $S \subseteq V$, as the set of vertices $u \in V \setminus S$ which are at a distance of radius of G from some vertex $v \in S$. The Max-radial number of G is defined as $\max_S \{|B_R(S)| - |S|\}$. The Max-radial subdivision number $sd_{\partial_R}(G)$ of a graph G is the minimum number of edges that must be subdivided (where an edge can be subdivided at most once) in order to increase the Max-radial number. If there is no edges in G, $sd_{\partial_R}(G) = \infty$.

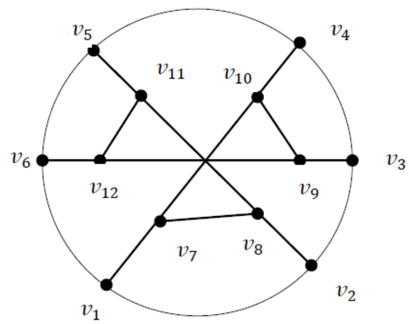
In this paper, determine the Max-radial number and the Max-Radial subdivision number for some special graphs.

2. Main Result

In this section, we start off with the Max-Radial number and The Max-Radial subdivision number $sd_{\partial_R}(G)$ of simple graph namely, bull graph.

Bull graph [5]

The bull graph is a planar undirected graph with 5 vertices and 5 edges, in the form of a triangle with two disjoint pendent edges. The bull graph is nothing but a pendent vertex removed from $K_3 \circ K_1$.



Franklin graph.

Figure 1.

For the bull graph G, $\partial_R(G) = 1$ and $sd_{\partial_R}(G) = 1$.

Let $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$. Now we claim that $\partial_R(G) = 1$. Let $S = \{v_1\} \subseteq V(G)$. Then $B_R(S) = \{v_4, v_5\}$. Therefore, the Max-Radial number of the set S is $\partial_R(S) = |B_R(S)| - |S| = 1$. We can easily verify that $\partial_R(G) = 1$ as any other set S' has $B_R(S)$ with at most $|S'| + 1$ vertices. We subdivide an edge v_2v_3, then we get Max-Radial number of resultant graph as 2. Hence $sd_{\partial_R}(G) = 1$.

Franklin graph [6]

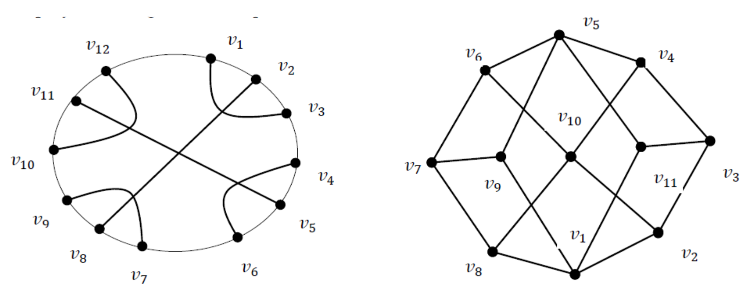
The Franklin graph is a 3-regular graph with 12 vertices and 18 edges as shown in the Figure 1.

For the Franklin graph G, $\partial_R(G) = 5$ and $sd_{\partial_R}(G) = 1$. Let

$S = \{v_1, v_2, v_6\}$. Then $B_R(S) = \{v_3, v_4, v_5, v_8, v_9, v_{10}, v_{11}, v_{12}\}$. Therefore, the Max-Radial number of the set S is $\partial_R(S) = |B_R(S)| - |S| = 5$. Thus $\partial_R(G) \geq 5$. It is enough to prove $\partial_R(G) = 5$. For a singleton subset $S' \subseteq V$, $B_R(S')$ contains 3 vertices. Therefore, $\partial_R(S') = 2$. Suppose a subset $S' \subseteq V(G)$ contains exactly two vertices. Then $B_R(S')$ contains at most 6 vertices. Therefore, $\partial_R(S') = 4$. If a subset $S'' \subseteq V(G)$ contains at least four vertices. Then $B_R(S'')$ contains at most 8 vertices. Therefore, $\partial_R(S'') \leq 4$. Also if S contains any three vertices of $V(G)$, then also $B_R(S)$ contains at most 8 vertices. Therefore, $\partial_R(S) = 5$. We subdivide an edge v_4v_5, then the Max-Radial number becomes 6 with ∂_R-set $\{v_1, v_2, v_6\}$. Hence $sd_{\partial_R}(G) = 1$.

The Truncated Tetrahedron [5]

The Truncated Tetrahedron G is an Archimedean Solid. It has 4-regular hexagonal faces, 4-regular triangular face, 12 vertices and 18 edges as shown in Figure 2. Archimedean Solid means one of 13 possible solids whose faces are all regular polygons whose faces are all regular whose polyhedral angles are all equal.



The Truncated Tetrahedron The Herschel graph

Figure 2.

For The Truncated Tetrahedron $\partial_R(G) = 6$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_1, v_8, v_{10}\} \subseteq V(G)$. Then $B_R(X) = \{v_5, v_6, v_7, v_9, v_2, v_3, v_4, v_{11}, v_{12}\}$. Therefore, $\partial_R(X) = |B_R(X)| - |X| = 9 - 3 = 6$. Thus $\partial_R(G) \geq 6$. It is enough we prove $\partial_R(G) \leq 6$. For any singleton set S of $V(G)$, we have $\partial_R(S) < 6$. Also for any set S' contains exactly two vertices, $\partial_R(S') < 6$.

Therefore, $X = \{v_1, v_8, v_{10}\}$ is the max-radial set of G. Hence $\partial_R(G) = 6$. We subdivide an edge $v_{10}v_{11}$, then the Max-Radial number becomes 7 with ∂_R-set $\{v_1, v_8, v_{10}\}$. Hence $sd_{\partial_R}(G) = 1$.

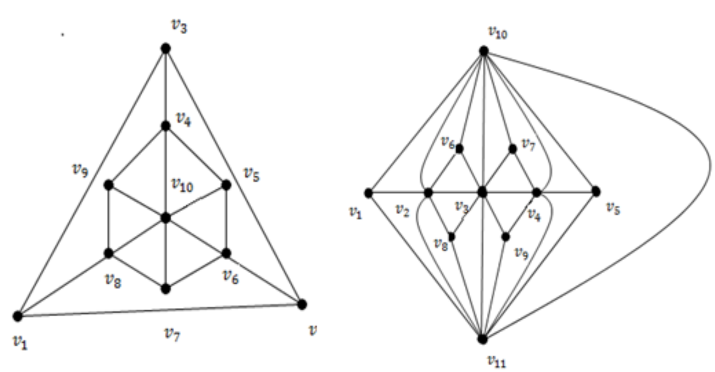
The Herschel graph [6]

The Herschel graph is a bipartite undirected graph with 11 vertices and 18 edges as shown in Figure 2, the smallest non Hamiltonian polyhedral graph. It is named after British astronomer Alexander Stewart Herschel.

For the Herschel graph G, $\partial_R(G) = 4$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_3, v_7\}$ where v_3 and v_7 are peripheral vertices. Then $B_R(X) = \{v_2, v_4, v_6, v_8, v_9, v_{11}\}$. Therefore, $\partial_R(X) = |B_R(X)| - |X| = 4$. Take any singleton set S of $V(G)$, then we have $\partial_R(S) < 4$. Also for any subset S of $V(G)$ contains more than two vertices. Then $B_R(S)$ contains less than or equal to 6 vertices. Therefore, we have $\partial_R(S) < 4$. Thus, for any subset S of $V(G)$, $\partial_R(S) \leq 4$. Hence $\partial_R(G) = 4$. We subdivide an edge v_6v_{10}, then the Max-Radial number becomes 5 with ∂_R-set $\{v_3, v_7\}$. Hence $sd_{\partial_R}(G) = 1$.

The Golomb graph [6]

The Golomb graph is a unit distance graph discovered around 1960-1965 by Golomb with 10 vertices and 18 edges as shown in the Figure 3.



The Golomb graph The Goldner-Harary graph

Figure 3.

For the Golomb graph G, $\partial_R(G) = 6$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_1, v_8\} \subseteq V(G)$. Then $B_R(X) = \{v_2, v_3, v_4, v_5, v_6, v_7, v_9, v_{10}\}$. Therefore, $\partial_R(X) = | B_R(X) | - | X | = 6$. Suppose there exists a Max-Radial set $S \subseteq V(G)$ containing more than two vertices, then $B_R(S)$ must contain less than 6 vertices. Therefore, $\partial_R(S) < 6$. Also for any singleton set $S \subseteq V(G)$ with $\partial_R(S) < 6$. Therefore, $\partial_R(G) = 6$. We subdivide an edge v_4v_9, then the Max-Radial number becomes 7 with ∂_R-set $\{v_1, v_8\}$. Hence $sd_{\partial_R}(G) = 1$.

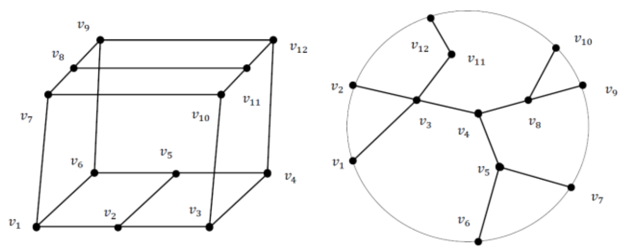
The Goldner - Harary graph [6]

The Goldner - Hararygraph is a simple undirected graph with 11 vertices and 27 edges as shown in the Figure 3. It is named after *A* Goldner and Frank Harary, who proved in 1975 that it was the smallest non Hamiltonian maximal planer graph.

For the Goldner - Harary graph G, $\partial_R(G) = 6$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_1\} \subseteq V(G)$, then $B_R(X) = \{v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$. Therefore, $\partial_R(X) = | B_R(X) | - | X | = 6$. For any singleton set S of $V(G)$, we have $\partial_R(S) \leq 6$. Suppose for any subset S'' contains more than one vertices. Then $B_R(S'')$ contains less than 7 vertices. Therefore, $\partial_R(S'') < 6$. Hence $\partial_R(G) = 6$. We subdivide an edge v_3v_{10}, then the Max-Radial number becomes 7 with ∂_R-set $\{v_1\}$. Hence $sd_{\partial_R}(G) = 1$.

The Bidiakis cube [6]

The Bidiakis cube is a 3-regular graph with 12 vertices and 18 edges as shown in Figure 4.



The Bidiakis cube The Frucht graph

Figure 4.

For the Bidiakis cube graph $G, \partial_R(G) = 6$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_1, v_3\} \subseteq V(G)$. Then $B_R(X) = \{v_4, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$. Therefore, $\partial_R(X) = |B_R(X)| - |X| = 6$. Suppose for any singleton set S, we have $\partial_R(S) < 6$. Also for any subset S'' of $V(G)$ contains more than two vertices, $B_R(S'')$ contains less than 8 vertices. Therefore, $\partial_R(S'') = |B_R(S'')| - |S''| < 6$. Therefore, for any subset S of $V(G), \partial_R(S) \leq 6$. Hence $\partial_R(G) = 6$. We subdivide an edge v_8v_{11}, then the Max-Radial number becomes 7 with ∂_R-set $\{v_1, v_3\}$. Hence $sd_{\partial_R}(G) = 1$.

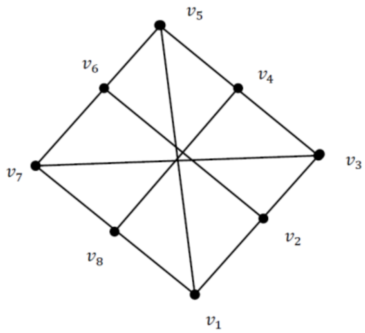
The Frucht graph [6]

The Frucht graph is a 3-regular graph with 12 vertices, 18 edges and no non-trivial symmetries as shown in Figure 4.

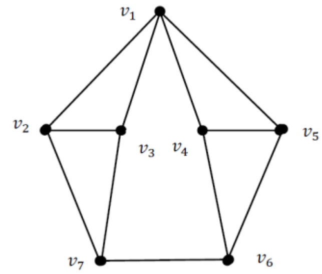
For The Frucht graph $G, \partial_R(G) = 5$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_3, v_8\} \subseteq V(G)$. Then $B_R(X) = \{v_1, v_2, v_6, v_7, v_9, v_{10}, v_{11}\}$. Therefore, $\partial_R(X) = |B_R(X)| - |X| = 7 - 2 = 5$. Suppose for any singleton set X' of $V(G), B_R(X')$ contains less than 5 vertices. Therefore, $\partial_R(X') < 5$. Also, for any subset X'' of $V(G)$ contain more than two vertices. Then $B_R(X'')$ contains less than 7 vertices. Therefore, $\partial_R(X'') = |B_R(X'')| - |X''| < 5$. For any subset S of $V(G), \partial_R(X) \leq 5$. Thus $\partial_R(G) = 5$. Here $X = \{v_3, v_8\}$ is a minimum Max-Radial set of G. We subdivide an edge v_5v_6, then the Max-Radial number becomes 6 with ∂_R-set $\{v_3, v_8\}$. Hence $sd_{\partial_R}(G) = 1$.

The Wagner graph [6]

The Wagner graph is a 3-regular graph with 8 vertices and 12 edges as shown in Figure 5 named after Klaus Wagner. It is the 8 vertex Mobius ladder graph. Mobius ladder is a cubic circular graph with an even number 'n' vertices, formed from an n-cycle by adding edges connecting opposite pairs of vertices in the cycle.



The Wagner graph



Moser Spindle

Figure 5.

For The Wagner graph G, $\partial_R(G) = 4$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_3, v_8\} \subseteq V(G)$, $B_R(X) = \{v_2, v_3, v_4, v_6, v_7, v_8\}$. Therefore, $\partial_R(X) = |B_R(X)| - |X| = 4$. Suppose every singleton set X' of $V(G)$, then $|B_R(X')| = 4$. Therefore, $\partial_R(X') = 3$. Also, for any subset X'' contains two vertices. Then $B_R(X'')$ contains 6 vertices. Therefore, $\partial_R(X'') = 4$. Suppose any subset X''' of $V(G)$ contains more than two vertices. Then $B_R(X''')$ contains less than 6 vertices. Therefore, $\partial_R(X''') < 4$. Thus for any subset S of $V(G)$, $\partial_R(S) \leq 4$. Hence $\partial_R(G) = 4$. We subdivide an edge v_4v_5, then the Max-Radial number becomes 5 with ∂_R-set $\{v_1, v_5\}$. Hence $sd_{\partial_R}(G) = 1$.

Moser Spindle [6]

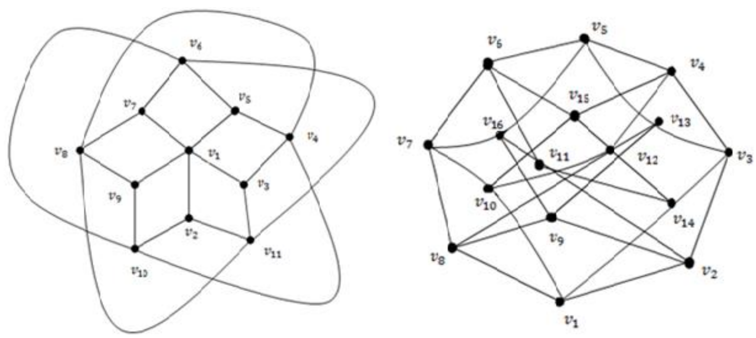
Moser Spindle (also called the Moser's Spindle or Moser graph) is an undirected graph, named after mathematicians Leo Moser and his brother Willian with seven vertices and eleven edges as shown in Figure 5.

For the Moser Spindle graph G, $\partial_R(G) = 3$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_6, v_7\} \subseteq V(G)$, $B_R(X) = \{v_1, v_2, v_3, v_4, v_5\}$. Therefore, $\partial_R(X) = |B_R(X)| - |X| = 3$. For any singleton set S' of $V(G)$, $\partial_R(S') < 3$. Also, every subset S'' of $V(G)$ contains more than one vertices, $\partial_R(S'') \leq 3$. Therefore, $\partial_R(G) = 3$ and $X = \{v_6, v_7\}$ is a minimum Max-Radial set of G. We subdivide an edge v_5v_6, then the Max-Radial number becomes 4 with

∂_R-set $\{v_6, v_7\}$. Hence $sd_{\partial_R}(G) = 1$.

The Grotzch graph [6]

The Grotzch graph is a triangle free graph with 11 vertices, 20 edges, chromatic number 4 and crossing number 5 as shown in Figure 6. It is named after German mathematician Herbert Grotzsch.



The Grotzch graph The Hoffman graph

Figure 6.

For the Grotzch graph G, $\partial_R(G) = 7$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_1, v_2\} \subseteq V(G)$, then $B_R(X) = \{v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}\}$. Therefore, $\partial_R(X) = 9 - 2 = 7$. Suppose for any singleton set S of $V(G)$. Then $B_R(S)$ contains at most 6 vertices. Therefore, $\partial_R(S) \leq 5$. Also, for any subset S' contain more than three vertices. Then $B_R(S')$ contains at most eight vertices. Therefore, $\partial_R(S') \leq 5$. Therefore, for any subset S of $V(G)$, $\partial_R(S) \leq 7$. Hence $X = \{v_1, v_2\}$ is one of the minimum Max-Radial set of $V(G)$. Thus, $\partial_R(G) = 7$. We subdivide an edge v_5v_6, then the Max-Radial number becomes 8 with ∂_R-set $\{v_1, v_2\}$. Hence $sd_{\partial_R}(G) = 1$.

The Hoffman graph [6]

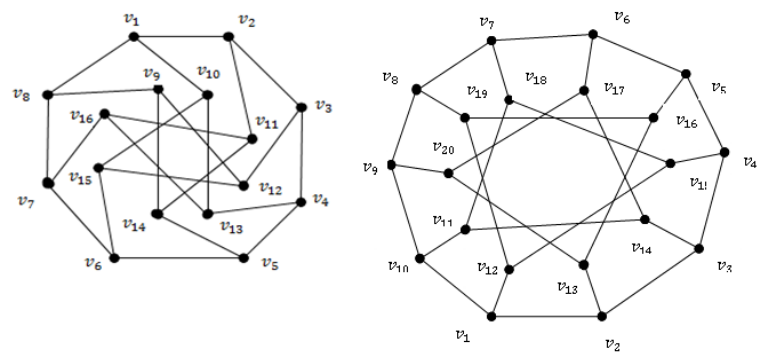
The Hoffman graph is a 4-regular graph with 16 vertices and 32 edges as shown in Figure 6 discovered by Alan Hoffman.

For the Hoffman graph G, $\partial_R(G) = 7$ and $sd_{\partial_R}(G) = 1$. Let

$X = \{v_1, v_2, v_5\} \subseteq V(G)$. Then $B_R(X) = \{v_4, v_6, v_7, v_8, v_{10}, v_{16}, v_{15}, v_{13}, v_{12}, v_{14}\}$. Therefore, $\partial_R(X) = |B_R(X)| - |X| = 7$. For any subset S of $V(G)$, $\partial_R(S) \leq 7$. Therefore, $\partial_R(G) = 7$. We subdivide an edge v_3v_4, then the Max-Radial number becomes 8 with ∂_R-set $\{v_1, v_2, v_5\}$. Hence $sd_{\partial_R}(G) = 1$.

The Mobius - Kantor graph [6]

The Mobius - Kantor graph is a symmetric bipartite cubic graph with 16 vertices and 24 edges as shown in Figure 7, named after August Ferdinand Mobius and Seligmann Kantor.



The Mobius - Kantor graph The Desargues graph

Figure 7.

For the Mobius - Kantor graph G, $\partial_R(G) = 0$ and $sd_{\partial_R}(G) = 1$. Let $R(G) \cong 8K_2$ be the radial graph of G. We claim that $\partial_R(G) = 0$. Now $\partial_R(G) = \partial(R(G)) = \partial(8K_2) = \partial(K_2 + K_2 + \ldots + K_2) = \partial(K_2) + \partial(K_2) + \ldots + \partial(K_2)$ (8 times) $= 0$. Hence $\partial_R(G) = 0$. We subdivide an edge v_1v_2, then the Max-Radial number becomes 1 with ∂_R-set $\{v_1\}$. Hence $sd_{\partial_R}(G) = 1$.

The Desargues graph [6]

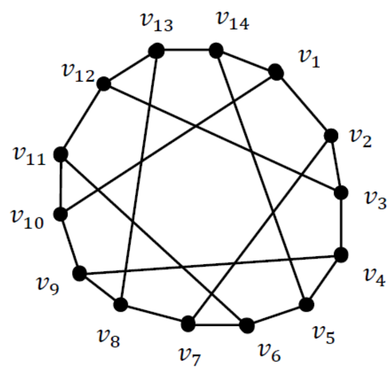
The Desargues graph is a distance transitive graph with 20 vertices and 30 edges as shown in Figure 7. It is named after Gerard Desargues.

For the Desargues graph G, $\partial_R(G) = 0$ and $sd_{\partial_R}(G) = 1$. Let $R(G) \cong 10K_2$ be the radial graph of G. We claim that $\partial_R(G) = 0$. Now

$\partial_R(G) = \partial(R(G)) = \partial(10K_2) = \partial(K_2 + K_2 + \ldots + K_2) = \partial(K_2) + \partial(K_2) + \ldots + \partial(K_2)$ (10 times) $= 0$. Hence $\partial_R(G) = 0$. We subdivide an edge v_5v_6, then the Max-Radial number becomes 1 with ∂_R-set $\{v_1\}$. Hence $sd_{\partial_R}(G) = 1$.

Heawood graph [6]

The Heawood graph is an undirected graph with 14 vertices and 21 edges named after Percy John Heawood as shown in Figure 8.



Heawood graph.

Figure 8.

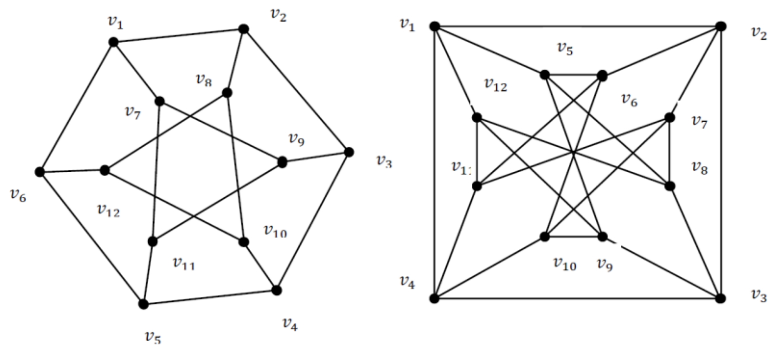
For the Heawood graph $G, \partial_R(G) = 7$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_1, v_2, v_3\} \subseteq V(G)$, then $B_R(X) = \{v_4, v_5, v_6, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}\}$. Therefore, the Max-Radial number of a set X is $\partial_R(X) = |B_R(X)| - |X| = 7$. Suppose a singleton set X' of $V(G)$, the S-radial set of X', $B_R(X')$ contains at most 4 vertices. Then the Max-Radial number $\partial_R(X') = |B_R(X')| - |X'| \leq 3$. Also, for a set X'' of $V(G)$ contains at most two vertices. Then $B_R(X'')$ contain at most 6 vertices.

Therefore, the Max-Radial number $\partial_R(X'') \leq 4$. Suppose a set X''' of $V(G)$ contains at least 4 vertices. Then $B_R(X''')$ contain at most 10 vertices. Therefore, $\partial_R(X''') = |B_R(X''')| - |X'''| \leq 6$. For any subset X of $V(G)$, $\partial_R(X) \leq 7$. Thus, $\partial_R(G) = 7$. We subdivide an edge $v_{12}v_{13}$, then the

Max-Radial number becomes 8 with ∂_R-set $\{v_1, v_2, v_3\}$. Hence $sd_{\partial_R}(G) = 1$.

Durer graph [6]

The Durer graph is an undirected graph with 12 vertices and 18 edges as shown in Figure 9.



Durer graph Chvatal graph

Figure 9

For the Durer graph $G, \partial_R(G) = 4$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_7, v_{10}\} \subseteq V(G)$. Then $B_R(X) = \{v_1, v_4, v_8, v_{11}, v_{12}\}$. Therefore, the Max-Radial number of a set $X, \partial_R(X) = |B_R(X)| - |X| = 4$. Suppose a singleton set X' of $V(G)$, then $B_R(X')$ contains at most 3 vertices. Therefore, $\partial_R(X') \leq 2$. Also, for any subset X'' contains more than two vertices. Then $B_R(X'')$ contains at most 6 vertices. $\partial_R(X'') \leq 4$. For any subset X''' of $V(G), \partial_R(X''') \leq 4$. Thus, $\partial_R(G) = 4$. We subdivide an edge v_3v_4, then the Max-Radial number becomes 5 with ∂_R-set $\{v_7, v_{10}\}$. Hence $sd_{\partial_R}(G) = 1$.

Chvatal graph [6]

The Chvatal graph is an undirected graph with 12 vertices and 24 edges, discovered by Vaclav Chvatal (1970) as shown in Figure 9.

For the Chvatal graph $G, \partial_R(G) = 8$ and $sd_{\partial_R}(G) = 1$. Let $X = \{v_1, v_4\} \subseteq V(G)$. Then $B_R(X) = \{v_2, v_3, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{10}, v_{11}, v_{12}\}$. Therefore, the Max-Radial number of a set $X, \partial_R(X) = |B_R(X)| - |X| = 10 - 2 = 8$.

For any subset $D \subseteq V(G), \partial_R(D) \leq 8$. Thus $\partial_R(G) = 8$. We subdivide an edge v_2v_3, then the Max-Radial number becomes 9 with ∂_R-set $\{v_1, v_4\}$. Hence $sd_{\partial_R}(G) = 1$.

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