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SEPARATION AXIOMS IN CUBIC TOPOLOGICAL SPACES

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Abstract

In this paper, the separation axioms cubic W-T_0 cubic W-T_1, cubic W-Hausdorff, cubic K-T_0, cubic K-T_1 and cubic K-Hausdorff are introduced and analysed.

I. Introduction

Decision making is one of the most complex issues that need scientific analysis of various factors both tangibles and intangibles like attitude, belief, taste and preferences of people involved. Real world decision making problems are very often uncertain or vague in a number of ways. In 1965, Zadeh [7] introduced the concept of fuzzy set theory to meet those problems. In 1975, Zadeh [8] made an extension of the concept of a fuzzy set by an interval valued fuzzy set with an interval valued membership function. Mondal and Samantha [6] defined the topology of interval valued fuzzy sets and studied some of its properties. In 2012, Jun, Kim and Yang [4] introduced cubic sets by combining an interval valued fuzzy set and a fuzzy set. Zeb et al. [1] introduced the topological structure of cubic sets. In Section II of this paper, preliminary definitions regarding cubic sets and cubic topological spaces are given. Several versions of separation axioms have been defined

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and studied in the case of fuzzy topological spaces by many famous topologists. In Section III of this paper the separation axioms of Gantner, Steinlage and Warren [3] and Katsaras [5] are extended to cubic topological spaces and it is proved that these axioms are hereditary and productive.

II. Preliminary Definitions

Definition 2.1[2]**.** Let X be a non empty set. A function $f : X \to I$ is called a fuzzy set in X. I^X denotes the collection of all fuzzy sets in X, where I is the closed unit interval [0, 1].

Definition 2.2[2]**.** For any two fuzzy sets $f, g \in I^X$

1. $f \leq g$ iff $f(x) \leq g(x)$ for all $x \in X$

2. $f = g$ iff $f(x) = g(x)$ for all $x \in X$

3. The **union** $f \vee g$ and the **intersection** $f \wedge g$ are defined respectively,

$(f \vee g)(x) = \max\{f(x), g(x)\}$ and

$(f \wedge g)(x) = \min\{f(x), g(x)\}$ for every $x \in X$

4. The **complement** of f denoted by f^c is defined by

$f^c(x) = 1 - f(x)$ for every $x \in X$

5. For a family $\{f_{\lambda / \lambda \in \Lambda}\}$ of fuzzy sets defined on a set X, the union $\vee_{\lambda \in \Lambda}(f_\lambda)$ and the intersection $\wedge_{\lambda \in \Lambda}(f_\lambda)$ are defined respectively,

$(\vee_{\lambda \in \Lambda} f_\lambda)(x) \vee_{\lambda \in \Lambda} (f_\lambda(x))$

$(\wedge_{\lambda \in \Lambda} f_\lambda)(x) \wedge_{\lambda \in \Lambda} (f_\lambda(x))$

6. The **constants** zero and one in fuzzy sets are denoted as **0** and **1** are defined respectively,

$\mathbf{0}(x) = 0$ for every $x \in X$ and

$\mathbf{1}(x) = 1$ for every $x \in X$.

Definition 2.3[4]**.** Let X be a non empty set. A function $\hat{\mu} : X \to [I]$ is called an **interval valued fuzzy** set in X, where $[I]$ is the set of all closed subintervals of $[0, 1]$.

$[I]^X$ denotes the collection of all interval valued fuzzy sets in X.

For every $\hat{\mu} \in [I]^X$ and $x \in X$, $\hat{\mu} : (x) = [\mu^-(x), \mu^+(x)]$ is called the degree of membership of an element x to $\hat{\mu}$, where $\mu^- : X \to I$ and $\mu^+ : X \to I$ are called the lower fuzzy set and upper fuzzy set in X respectively. For simplicity denote $\hat{\mu}$ as $\hat{\mu} = [\mu^-, \mu^+]$.

Definition 2.4[4]**.** For any two interval valued fuzzy sets $\hat{\mu}, \hat{\lambda}$ in $[I]^X$

1. $\hat{\mu} \subseteq \hat{\lambda}$ iff $\hat{\mu}^-(x) \leq \lambda^-(x)$ and $\hat{\mu}^+(x) \leq \lambda^+(x)$ for every $x \in X$

2. $\hat{\mu} = \hat{\lambda}$ iff $\hat{\mu} \subseteq \hat{\lambda}$ and $\hat{\lambda} \subseteq \hat{\mu}$

3. The **union** $\hat{\mu} \,\hat{\cup}\, \hat{\lambda}$ and **intersection** $\hat{\mu} \,\hat{\cap}\, \hat{\lambda}$ are defined respectively as

$\hat{\mu} \,\hat{\cup}\, \hat{\lambda} = [\mu^- \vee \lambda^-, \mu^+ \vee \lambda^+]$

$\hat{\mu} \,\hat{\cap}\, \hat{\lambda} = [\mu^- \wedge \lambda^-, \mu^+ \wedge \lambda^+]$

4. The **complement** of $\hat{\mu}$, denoted by $\hat{\mu}^c$ is defined as $\hat{\mu}^c = [\mathbf{1}\text{-}\,\mu^+, \mathbf{1}\text{-}\,\mu^-]$, where **1** is the constant fuzzy set one.

5. For a family $\{\hat{\mu}_\lambda / \lambda \in \wedge\}$ of interval valued fuzzy sets on a set X, the union $\hat{\bigcup}_{\lambda \in \Lambda} \hat{\mu}_\lambda$ and the intersection $\hat{\bigcap}_{\lambda \in \Lambda} \hat{\mu}_\lambda$ are defined respectively

$$\hat{\bigcup_{\lambda \in \Lambda}} \hat{\mu}_\lambda = [\vee_{\lambda \in \Lambda} (\mu_\lambda^-), \vee_{\lambda \in \Lambda} (\mu_\lambda^+)]$$

$$\hat{\bigcap_{\lambda \in \Lambda}} \hat{\mu}_\lambda = [\vee_{\lambda \in \Lambda} (\mu_\lambda^-), \vee_{\lambda \in \Lambda} (\mu_\lambda^+)]$$

6. The **constants** interval valued fuzzy sets zero and one are denoted as $\hat{0}$ and $\hat{1}$ which are defined respectively $\hat{0} = [\mathbf{0}, \mathbf{0}], \hat{1} = [\mathbf{1}, \mathbf{1}]$.

Definition 2.5[4]. Let X be a nonempty set. A cubic set on X denoted as $\boxed{A}$ is a structure $\langle \widehat{\mu}, f\rangle$ in which $\widehat{\mu}$ is an interval valued fuzzy set in X and is a fuzzy set in X. C^X denotes the collection of all cubic sets in X.

Definition 2.6[4]. For any two cubic sets $\boxed{A} = \langle \widehat{\mu}, f\rangle$ and $\boxed{B} = \langle \widehat{\lambda}, g\rangle$ in C^X.

1. $\boxed{A} \subseteq \boxed{B}$ iff $\widehat{\mu} \subseteq \widehat{\lambda}$ and $f \leq g$.

2. $\boxed{A} = \boxed{B}$ iff $\boxed{A} \subseteq \boxed{B}$ and $\boxed{B} \subseteq \boxed{A}$

3. The union $\boxed{A} \cup \boxed{B}$ and the intersection $\boxed{A} \cap \boxed{B}$ are defined respectively as

$$\boxed{A} \cup \boxed{B} = \left\langle \widehat{\mu} \,\widehat{\cup}\, \widehat{\lambda}, f \vee g \right\rangle$$

$$\boxed{A} \cap \boxed{B} = \left\langle \widehat{\mu} \,\widehat{\cap}\, \widehat{\lambda}, f \wedge g \right\rangle.$$

4. The **complement** $\boxed{A}^c$ is defined as $\boxed{A}^c = \langle \widehat{\mu}^c, f^c \rangle$

5. For a family $\{\boxed{A} / \lambda \in \wedge\}$ of cubic sets defined on a set X, the union $\bigcup_{\lambda \in \Lambda} \boxed{A_\lambda}$ and the intersection $\bigcap_{\lambda \in \Lambda} \boxed{A_\lambda}$ are defined respectively as

$$\bigcup_{\lambda \in \Lambda} \boxed{A_\lambda} = \langle \widehat{\bigcup_{\lambda \in \Lambda}} \widehat{\mu}_\lambda, \vee_{\lambda \in \Lambda} f_\lambda \rangle$$

$$\bigcap_{\lambda \in \Lambda} \boxed{A_\lambda} = \langle \widehat{\bigcap_{\lambda \in \Lambda}} \widehat{\mu}_\lambda, \wedge_{\lambda \in \Lambda} f_\lambda \rangle$$

6. The **constants** cubic sets zero and one are denoted as $\boxed{0}$ and $\boxed{1}$ are defined respectively as

$$\boxed{0} = \langle \widehat{0}, \mathbf{0} \rangle,\ \boxed{1} = 1\langle \widehat{1}, \mathbf{1} \rangle$$

Definition 2.7[1]. Let X be a nonempty set and be a family of cubic sets of X. The family is called a **cubic topology** on X iff $\mathcal{C}$ satisfies the following conditions

(i) $\boxed{0}, \boxed{1} \in \mathcal{C}$

(ii) $\boxed{A}, \boxed{B} \in \mathcal{C}$ implies $\boxed{A} \cap \boxed{B} \in \mathcal{C}$

(iii) $\boxed{A_\lambda} \in \mathcal{C}$ for each $\lambda \in \Lambda$ implies $(\bigcup_{\lambda \in \Lambda} \boxed{A_\lambda} \in \mathcal{C})$

Definition 2.8. Let $\boxed{A} = \langle \hat{\mu}, f \rangle$ and $\boxed{B} = \langle \hat{\lambda}, f \rangle$ be two cubic sets on X and Y respectively. The **Cartesian product** of $\boxed{A}$ and $\boxed{B}$ is a cubic set on $X \times Y$ denoted $\boxed{A} * \boxed{B}$ as and is defined as $(\boxed{A} * \boxed{B}) = \langle (\hat{\mu} * \hat{\lambda})(x, y), (f * g)(x, y) \rangle$.

Where $(\hat{\mu} * \hat{\lambda})(x, y) = \{\min(\mu^-(x), \lambda^-(y)), \min(\mu^+(x), \lambda^+(y))\}$ and $(f * g)(x, y) = \min(f(x), g(y))$

Definition 2.9. Let $(X, \mathcal{C}_1)$ and $(Y, \mathcal{C}_2)$ be two cubic topological spaces. Then the **cubic product topology** $\mathcal{C}_1 \times \mathcal{C}_2$ on $X \times Y$ is the cubic topology having the collection $\{(\boxed{A} * \boxed{B}) / \boxed{A} \in \mathcal{C}_1, \boxed{B} \in \mathcal{C}_2\}$ as a basis.

Definition 2.10. Let $\{(X_\lambda\ \mathcal{C}_\lambda)/\lambda \in \Lambda\}$ be a family of cubic topological spaces and $X = \prod_{\lambda \in \wedge} X_\lambda$. The cubic product topology on X is the one with basic cubic open sets of the form $\prod_{\lambda \in \wedge} \boxed{A_\lambda}$, where $\boxed{A_\lambda} \in \mathcal{C}_\lambda$ and $\boxed{A_\lambda} = \boxed{1_\lambda}$ except for finitely many λ's.

Definition 2.11. Let $(X, \mathcal{C})$ be a cubic topological space. Let $Y \subseteq X$ and $\boxed{A} \in \mathcal{C}$. Define $\boxed{A}/Y$ as follows: $(\boxed{A}/Y)(z) = \boxed{A}(z)$ if $z \in Y$ Define $(\mathcal{C}/Y) = \{(\boxed{A}/Y)/\boxed{A} \in \mathcal{C})\}$. Then $(\mathcal{C}/Y)$ is called a **cubic subspace topology** of Y and $(Y, \mathcal{C}/Y)$ is called a cubic subspace of $(X, \mathcal{C})$.

III. Separations Axioms

Definition 3.1. A cubic topological space $(X, \mathcal{C})$ is said to be **cubic** W-T_0, if for any two distinct points $x, y \in X$, there exist a cubic open set $\boxed{A} = \langle \hat{\mu}, f \rangle \in \mathcal{C}$ such that $\boxed{A}(x) = \boxed{1}, \boxed{A}(y) = \boxed{0}$ (or) $\boxed{A}(x) = \boxed{0}, \boxed{A}(y) = \boxed{1}$.

Definition 3.2. *A* cubic topological space $(X, \mathcal{C})$ is said to be **cubic** W-T_1, if for any two distinct points $x, y \in X$, there exist two cubic open sets $\boxed{A} = \langle \hat{\mu}, f \rangle, \boxed{B} = \langle \hat{\lambda}, f \rangle \in \mathcal{C}$ such that $\boxed{A}(x) = \boxed{1}, \boxed{A}(y) = \boxed{0}, \boxed{B}(x) = \boxed{0}, \boxed{B}(y) = \boxed{1}$.

Definition 3.3. A cubic topological space $(X, \mathcal{C})$ is said to be **cubic W-Hausdorff** or cubic W-T_2, if for all pair of disjoint points $x, y \in X$, there exist two cubic open sets $\boxed{A} = \langle \hat{\mu}, f \rangle \in \mathcal{C}$ and $\boxed{B} = \langle \hat{\lambda}, g \rangle \in \mathcal{C}$ such that $\boxed{A}(x) = \boxed{1}$, $\boxed{B}(y) = \boxed{1}$ and $\boxed{A} \cap \boxed{B} = \boxed{0}$.

Remark 3.4. From the above three definitions it follows that

(1) Cubic W-$T_2 \Rightarrow$ Cubic W-$T_1 \Rightarrow$ Cubic W-T_0

(2) Subspace of a cubic W-T_0 space is a cubic W-T_0 space

(3) Subspace of a cubic W-T_1 space is a cubic W-T_1 space.

Definition 3.5. A cubic topological space $(X, \mathcal{C})$ is said to be **cubic** K-T_0 if for any two distinct points $x, y \in X$, there exist cubic open sets $\boxed{A} = \langle \hat{\mu}, f \rangle \in \mathcal{C}$ such that $\boxed{A}(x) > \boxed{0}$, $\boxed{A}(y) = \boxed{0}$ (or) $\boxed{A}(x) = \boxed{0}$, $\boxed{A}(y) > \boxed{0}$.

Definition 3.6. A cubic topological space $(X, \mathcal{C})$ is said to be **cubic** K-T_1, if for any two distinct points $x, y \in X$, there exist two cubic open sets $\boxed{A} = \langle \hat{\mu}, f \rangle$, $\boxed{B} = \langle \hat{\lambda}, g \rangle \in \mathcal{C}$ such that $\boxed{A}(x) > \boxed{0}$, $\boxed{A}(y) = \boxed{0}$, $\boxed{B}(x) = \boxed{0}$, $\boxed{B}(y) > \boxed{0}$.

Definition 3.7. A cubic topological space $(X, \mathcal{C})$ is said to be **cubic K-Hausdorff** or **cubic** K-T_2, if for all pair of disjoint points $x, y \in X$, there exist two cubic open sets $\boxed{A} = \langle \hat{\mu}, f \rangle$, $\boxed{B} = \langle \hat{\lambda}, g \rangle \in \mathcal{C}$ such that $\boxed{A}(x) > \boxed{0}$, $\boxed{B}(y) > \boxed{0}$ and $\boxed{A} \cap \boxed{B} = \boxed{0}$.

Remark 3.8. From the above three definitions it follows that

(1) Cubic K-$T_2 \Rightarrow$ Cubic K-$T_1 \Rightarrow$ Cubic K-T_0

(2) Subspace of a cubic K-T_0 space is a cubic K-T_0 space

(3) Subspace of a cubic K-T_1 space is a cubic K-T_1 space.

Theorem 3.9. *Subspace of a cubic* W-*Hausdorff space is a cubic* W-*Hausdorff space.*

Proof. Let $(X, \mathcal{C})$ be a cubic W-Hausdorff space. Let Y be a subspace of

X. That is $(Y, \mathcal{C}/Y)$ is a cubic of $(X, \mathcal{C})$ where $\mathcal{C}/Y = \{(\boxed{A}/Y : \boxed{A} \in \mathcal{C})\}$.

To Prove: $(Y, \mathcal{C}/Y)$ is a cubic W-Hausdorff space.

Consider $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Then $y_1, y_2 \in X$. Since $(X, \mathcal{C})$ is a cubic W-Hausdorff space, there exist $\boxed{A} = \langle \hat{\mu}, f \rangle \in \mathcal{C}, \boxed{B} = \langle \hat{\lambda}, g \rangle \in \mathcal{C}$ such that $\boxed{A}(y_1) = \boxed{1}, \boxed{B}(y_2) = \boxed{1}$ and $\boxed{A} \cap \boxed{B} = \boxed{0}$.

Therefore $(\boxed{A}/Y), (\boxed{B}/Y) \in \mathcal{C}/Y$

Also

$$(\boxed{A}/Y)(y_1) = \boxed{A}(y_1) \text{ if } y_1 \in Y$$

$$= \boxed{1}$$

$$(\boxed{B}/Y)(y_2) = \boxed{B}(y_2) \text{ if } y_2 \in Y$$

$$= \boxed{1}$$

$$((\boxed{A}/Y) \cap (\boxed{B}/Y(y_2))y = ((\boxed{A}/Y)(y) \cap (\boxed{B}/Y))(y) \text{ for every } y \in Y \subseteq X$$

$$= (\boxed{A}/Y)(y) \cap (\boxed{B}/Y)(y) \text{ for every } y \in Y \subseteq X$$

$$= \boxed{A}(y) \cap \boxed{B}(y) \text{ for every } y \in Y \subseteq X$$

$$= (\boxed{A} \cap \boxed{B})(y) \text{ for every } y \in Y \subseteq X$$

$$= \boxed{0}(y) \text{ for every } y \in Y \subseteq X$$

$$(\boxed{A}/Y) \cap (\boxed{B}/Y) = \boxed{0}.$$

Hence $(Y, \mathcal{C}/Y)$ is a cubic W-Hausdorff space.

Theorem 3.10. *Product of two cubic* W-*Hausdorff spaces is a cubic* W-*Hausdorff space in the product topology*.

Proof. Let $(X, \mathcal{C}_1)$ and $(Y, \mathcal{C}_2)$ be two cubic W-Hausdorff spaces.

To Prove: $(X \times Y, \mathcal{C}_1 \times \mathcal{C}_2)$ is a cubic W-Hausdorff space.

Consider two distinct points $(x_1, y_1), (x_2, y_2) \in X \times Y$. Either $x_1 \neq x_2$ or

$y_1 \neq y_2$. Assume $x_1 \neq x_2$. Therefore there exists two cubic open sets $\boxed{A} = \langle \hat{\mu}, f \rangle \in \mathcal{C}, \boxed{B} = \langle \hat{\lambda}, g \rangle \in \mathcal{C}$ such that $\boxed{A}(x_1) = \boxed{1}, \boxed{B}(x_2) = \boxed{1}$ and $\boxed{A} \cap \boxed{B} = \boxed{0}$. Therefore $\boxed{A} * \boxed{1} \in \mathcal{C}_1 \times \mathcal{C}_2$, since $\boxed{A} \in \mathcal{C}_1, \boxed{1} \in \mathcal{C}_2$ and $\boxed{B} * \boxed{1} \in \mathcal{C}_1 \times \mathcal{C}_2$, since $\boxed{B} \in \mathcal{C}_1, \boxed{1} \in \mathcal{C}_2$, where $\boxed{A} * \boxed{1} = \langle (\hat{\mu} * \hat{1}), (f * 1) \rangle$ and $\boxed{B} * \boxed{1} = \langle (\hat{\lambda} * \hat{1}), (g * 1) \rangle$.

Consider

$(\boxed{A} * \boxed{1})(x_1, y_1) = \langle (\hat{\mu} * \hat{1}), (f * \mathbf{1}) > (x_1, y_1) \rangle$ for all $(x_1, y_1) \in X \times Y$

$= (\hat{\mu} * \hat{1}), (x_1, y_1), (f * \mathbf{1})(x_1, y_1)$ for all $(x_1, y_1) \in X \times Y$

$[(\mu^- * 1^-)(x_1, y_1), (\mu^+ * 1^+)(x_1, y_1)], (f * \mathbf{1})(x_1, y_1)$ for all $(x_1, y_1) \in X \times Y$

$= [\min(\mu^-(x_1,)1^-(y_1)), \min(\mu^+(x_1,), 1^+(y_1))], \min(f(x_1), \mathbf{1}(y_1))$ for all $(x_1, y_1) \in X \times Y$

$= \langle [\min(1, 1), \min(1, 1)], \min(1, 1) \rangle$

$= \langle [\mathbf{1}, \mathbf{1}], \mathbf{1} \rangle = \langle \hat{1}, \mathbf{1} \rangle = \boxed{1}$ and

$(\boxed{B} * \boxed{1})(x_2, y_2) = \langle (\hat{\lambda} * \hat{1}), (g * 1) \rangle (x_2, y_2)$ for all $(x_2, y_2) \in X \times Y$

$= [(\lambda^- * 1^-)(x_2, y_2), (\lambda^+ * 1^+)(x_2, y_2)], (g * \mathbf{1})(x_2, y_2)$ for all $(x_2, y_2) \in X \times Y$

$= [\min(\lambda^-(x_2), 1^-(y_2)), \min(\lambda^+(x_2), 1^+(y_2))], \min(g(x_2), \mathbf{1}(y_2))$ for all $(x_2, y_2) \in X \times Y$

$= \langle [\min(1, 1), \min(1, 1)], \min(1, 1) \rangle$

$= \langle [\mathbf{1}, \mathbf{1}], \mathbf{1} \rangle = \langle \hat{1}, \mathbf{1} \rangle = \boxed{1}$

Also

$\boxed{A} \cap \boxed{B} = \boxed{0}. \Rightarrow \langle \hat{\mu}, f \rangle \cap \langle \hat{\lambda}, g \rangle = \langle \hat{0}, 0 \rangle$

$\Rightarrow \langle \hat{\mu} \hat{\cap} \hat{\lambda}, f \wedge g \rangle = \langle \hat{0}, \mathbf{0} \rangle$

$\Rightarrow (\mu^- \wedge \lambda^-, \mu^+ \wedge \lambda^+) = [0, 0], (f \wedge g) = \mathbf{0}$

$\Rightarrow (\mu^- \wedge \lambda^-)(x) = 0$ and $\Rightarrow (\ \mu^+ \wedge \lambda^+)(x) = 0, (f \wedge g)(x) = 0$ for all $x \in X$,

$\Rightarrow \mu^-(x) \wedge \lambda^-(x) = 0$ and $\mu^+ \wedge (x) \wedge \lambda^+(x) = 0, f(x) \wedge g(x) = 0$ for all $x \in X$,

$\Rightarrow$ either $\mu^-(x) = 0$ or $\lambda^-(x) = 0$ and either $\mu^+(x) = 0$ or $\lambda^+(x) = 0$, either $f(x) = 0$ or $g(x) = 0$ for all $x \in X$

$\Rightarrow$ either $\mu^-(x) \wedge = 1^-(y) = 0$ or $\lambda^-(x) \wedge 1^-(y) = 0$ and either $\mu^+(x) \wedge 1^+(y) = 0$ or $\lambda^+(x) \wedge 1^+(y) = 0$, either $f(x) \wedge \mathbf{1}(y) = 0$ or $g(x) \wedge \mathbf{1}(y) = 0$ for all $x \in X$ and $y \epsilon Y$

$\Rightarrow$ either $(\mu^- * 1)(x, y) = 0$ or $(\lambda^- * 1^-)(x, y) = 0$ and either $(\ \mu^+ * 1^+)(x, y) = 0$ or $(\lambda^+ * 1^+)(x, y) = 0$,

either $(f * \mathbf{1})(x, y) = 0$ or $(g * \mathbf{1})(x, y) = 0$ for all $(x, y) \in X \times Y$

$\Rightarrow ((\mu^- * 1^-) \wedge (\lambda^- * 1^-))(x, y) = 0$ and $((\mu^+ * 1^+) \wedge (\lambda^+ * 1^+))(x, y)$ for all $(x, y) \in X \times Y$

$\Rightarrow \langle (\hat{\mu} * \hat{1}), (f * \mathbf{1}) \rangle = \langle \hat{0}, \mathbf{0} \rangle$ and $\langle (\hat{\lambda} * \hat{1}), (g * \mathbf{1}) \rangle = \langle \hat{0}, \mathbf{0} \rangle$

$\Rightarrow (\boxed{A} * \boxed{1}) = \boxed{0}$ and $(\boxed{B} * \boxed{1}) = \boxed{0}$

$\Rightarrow (\boxed{A} * \boxed{1}) \cap \boxed{B} * \boxed{1} = \boxed{0}$

Hence $(X \times Y, \mathcal{C}_1 \times \mathcal{C}_2)$ is a cubic W-Hausdorff space.

Theorem 3.11. *Arbitrary product of cubic* W-*Hausdorff spaces is a cubic* W-*Hausdorff space in the product topology*.

Proof. Let $\{(X_\lambda \boxed{A_\lambda} / \lambda \in \Lambda)\}$ be a collection of cubic W-Hausdorff spaces.

Consider $X = \prod_{\lambda \in \Lambda} X_\lambda$ in the product topology.

Consider two distinct points $(x_\lambda)_{\lambda \in \Lambda} (y_\lambda)_{\lambda \in \Lambda} \in \prod_{\lambda \in \Lambda} X_\lambda$. Therefore $x_\mu \neq y_\mu$ for some $\mu \in \wedge$.

Therefore there exists two cubic open sets $\boxed{A_\mu} = \langle \hat{\mu}_\mu, f_\mu \rangle \in \mathcal{C}_\mu, \boxed{B_\mu} = \langle \hat{\lambda}_\mu, g_\mu \rangle \in \mathcal{C}_\mu$ such that $\boxed{A_\mu}(x_\mu) = \boxed{1_\mu}, \boxed{B_\mu}(y_\mu) = \boxed{1_\mu}$ and $\boxed{A_\mu} \cap \boxed{A_\mu} = \boxed{0_\mu}$.

Let $\boxed{A} = \prod_{\lambda \in \wedge}(\boxed{A_\lambda})$ where $\boxed{A_\lambda} = \boxed{1_\lambda}$ for $\lambda \neq \mu$ and $\boxed{B} = \prod_{\lambda \in \wedge}(\boxed{B_\lambda})$, where $\boxed{B_\lambda} = \boxed{1_\lambda}$ for $\lambda \neq \mu$. Then $\boxed{A}, \boxed{B} \in \prod_{\lambda \in \wedge} \mathcal{C}_\lambda$

$$\boxed{A} = \prod_{\lambda \in \wedge}(\boxed{A_\lambda}) = \langle \prod_{\lambda \in \wedge}(\hat{\mu}\lambda), \prod_{\lambda \in \wedge} f_\lambda \rangle$$

$$= \langle [\prod_{\lambda \in \wedge} \mu_\lambda^-, \prod_{\lambda \in \wedge} \mu_\lambda^+], \prod_{\lambda \in \wedge} f_\lambda \rangle$$

$$\prod_{\lambda \in \wedge}(\mu_\lambda^-)(x_\lambda) = \min\{(\mu_\lambda^-)(x_\lambda))\} \text{ for all } \lambda \in \wedge$$

$$= (\mu_\mu^-)(x_\mu) \text{ for some } \mu \in \wedge$$

$$= 1$$

$$\prod_{\lambda \in \wedge}(\mu_\lambda^+)(x_\lambda) = \min\{((\mu_\lambda^+)(x_\lambda))\} \text{ for all } \lambda \in \wedge$$

$$= (\mu_\mu^+)(x_\mu) \text{ for some } \mu \in \wedge$$

$$= 1$$

$$\prod_{\lambda \in \wedge} f_\lambda(x_\lambda) = \min\{f_\lambda(x_\lambda)\} \text{ for all } \lambda \in \wedge$$

$$= (f_\mu)(x_\mu) \text{ for some } \mu \in \wedge$$

$$= 1$$

$$\boxed{B} = \prod_{\lambda \in \wedge}(\boxed{B_\lambda}) = \langle \prod_{\lambda \in \wedge}(\hat{\lambda}_\lambda), \prod_{\lambda \in \wedge} g_\lambda \rangle$$

$$= \langle [\prod_{\lambda \in \wedge} \lambda_\lambda^-, \prod_{\lambda \in \wedge} \lambda_\lambda^+], \prod_{\lambda \in \wedge} g_\lambda \rangle$$

$$\prod_{\lambda \in \wedge} \lambda_\lambda^-(y_\lambda) = \min\{(\lambda_\lambda^-(y_\lambda))\}, \text{ for all } \lambda \in \wedge$$

$$= \lambda_\mu^-(y_\mu), \text{ for some } \mu \in \wedge$$

$$= 1$$

$$\prod_{\lambda \in \wedge} \lambda_{\lambda}^{+}(y_{\lambda}) = \min\{(\lambda_{\lambda}^{+}(y_{\lambda}))\}, \text{ for all } \lambda \in \wedge$$

$$= \lambda_{\mu}^{+}(y_{\mu}), \text{ for some } \mu \in \wedge$$

$$= 1$$

$$\prod_{\lambda \in \wedge} g_{\lambda}(x_{\lambda}) = \min\{g_{\lambda}(x_{\lambda})\} \text{ for all } \lambda \in \wedge$$

$$= (g_{\mu})(x_{\mu}) \text{ for some } \mu \in \wedge$$

$$= 1$$

Consider

$$\prod_{\lambda \in \wedge} \boxed{A_{\lambda}} \cap \prod_{\lambda \in \wedge} \boxed{B_{\lambda}} = \langle \prod_{\lambda \in \wedge} (\hat{\mu}\lambda), \prod_{\lambda \in \wedge} f_{\lambda} \rangle \cap \langle \prod_{\lambda \in \wedge} (\hat{\lambda}_{\lambda}), \prod_{\lambda \in \wedge} g_{\lambda} \rangle$$

$$= \langle (\prod_{\lambda \in \wedge} (\hat{\mu}\lambda) \hat{\cap} \prod_{\lambda \in \wedge} (\hat{\lambda}_{\lambda}), (\prod_{\lambda \in \wedge} f_{\lambda} \wedge \prod_{\lambda \in \wedge} g_{\lambda})\rangle$$

$$\prod_{\lambda \in \wedge} \hat{\mu}_{\lambda} \hat{\cap} \prod_{\lambda \in \wedge} (\hat{\lambda}_{\lambda}) = (\prod_{\lambda \in \wedge} \mu_{\lambda}^{-}, \prod_{\lambda \in \wedge} \mu_{\lambda}^{+}) \wedge (\prod_{\lambda \in \wedge} \mu_{\lambda}^{-}, \prod_{\lambda \in \wedge} \mu_{\lambda}^{+})$$

$$= ((\prod_{\lambda \in \wedge} \mu_{\lambda}^{-} \wedge \prod_{\lambda \in \wedge} \lambda_{\lambda}^{-}), (\mu_{\lambda}^{+} \wedge \prod_{\lambda \in \wedge} \lambda_{\lambda}^{+}))$$

Then

$$(\prod_{\lambda \in \wedge} \mu_{\lambda}^{-} \wedge \prod_{\lambda \in \wedge} \lambda_{\lambda}^{-})(x_{\lambda}) = (\prod_{\lambda \in \wedge} \mu_{\lambda}^{-}(x_{\lambda})) \wedge (\prod_{\lambda \in \wedge} \lambda_{\lambda}^{-}(x_{\lambda}) \text{ for all } \lambda \in \wedge$$

$$= [\min\{(\mu_{\lambda}^{-})(x_{\lambda})\}] \wedge [\min\{(\lambda_{\lambda}^{-})(x_{\lambda})\}] \text{ for all } \lambda \in \wedge$$

$$= (\mu_{\mu}^{-})(x_{\mu}) \wedge (\lambda_{\mu}^{-})(x_{\mu})$$

$$= (\mu_{\mu}^{-} \wedge \lambda_{\mu}^{-})(x_{\mu}) = 0$$

$$(\prod_{\lambda \in \wedge} \mu_{\lambda}^{+} \wedge \prod_{\lambda \in \wedge} \lambda_{\lambda}^{+})(x_{\lambda}) = (\prod_{\lambda \in \wedge} \mu_{\lambda}^{+}(x_{\lambda})) \wedge (\prod_{\lambda \in \wedge} \lambda_{\lambda}^{+}(x_{\lambda})) \text{ for all } \lambda \in \wedge$$

$$= [\min\{(\mu_{\lambda}^{+})(x_{\lambda})\}] \wedge [\min\{(\lambda_{\lambda}^{+})(x_{\lambda})\}]. \text{ for all } \lambda \in \wedge$$

$$= (\mu_{\mu}^{+})(x_{\mu}) \wedge (\lambda_{\mu}^{+})(x_{\mu})$$

$$= (\mu_{\mu}^{+} \wedge \lambda_{\mu}^{+})(x_{\mu}) = 0$$

$$\left(\prod_{\lambda\in\wedge} f_{\lambda} \wedge \prod_{\lambda\in\wedge} g_{\lambda}\right)(x_{\lambda}) = \left(\prod_{\lambda\in\wedge} f_{\lambda}(x_{\lambda})\right) \wedge \left(\prod_{\lambda\in\wedge} g_{\lambda}(x_{\lambda})\right) \text{ for all } \lambda \in \wedge$$

$$= \min\{f_{\lambda}(x_{\lambda})\} \wedge \min\{g_{\lambda}(x_{\lambda})\} \text{ for all } \lambda \in \wedge$$

$$= (f_{\mu})(x_{\mu}) \wedge (g_{\mu})(x_{\mu})$$

$$= (f_{\mu} \wedge g_{\mu})(x_{\mu})$$

$$= 0.$$

Theorem 3.12. *Subspace of a cubic* K-*Hausdorff space is a cubic* K-*Hausdorff space*.

Proof. Let $(X, \mathcal{C})$ be a cubic K-Hausdorff space. Let Y be a subspace of X. That is $(X, \mathcal{C}/Y)$ is a cubic of $(X, \mathcal{C})$ where $\mathcal{C}/Y = \{(\boxed{A}/Y : \boxed{A} \in \mathcal{C})\}$.

To Prove: $(Y, \mathcal{C}/Y)$ is a cubic K-Hausdorff space.

Consider $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Then $y_1, y_2 \in X$

Since $(X, \mathcal{C})$ is a cubic K-Hausdorff space, there exist $\boxed{A} = \langle \widehat{\mu}, f \rangle \in \mathcal{C} \boxed{B} = \langle \widehat{\lambda}, g \rangle \in \mathcal{C}$ such that $\boxed{A}(y_1) > \boxed{0}, \boxed{B}(y_1) > \boxed{0}$ and $\boxed{A} \cap \boxed{B} = \boxed{0}$. Therefore $(\boxed{A}/Y), (\boxed{B}/Y) \in \mathcal{C}/Y$.

Also

$$(\boxed{A}/Y)(y_1) = \boxed{A}(y_1) \text{ if } y_1 \in Y$$

$$> \boxed{0}.$$

$$(\boxed{B}/Y)(y_2) = \boxed{B}(y_2) \text{ if } y_2 \in Y$$

$$> \boxed{0}.$$

$$((\boxed{A}/Y) \cap (\boxed{B}/Y))y = ((\boxed{A}/Y)(y) \cap (\boxed{B}/Y)(y) \text{ for every } y \in Y \subseteq X$$

$$= (\boxed{A}/Y)(y) \cap (\boxed{B}/Y)(y) \text{ for every } y \in Y \subseteq X$$

$$= \boxed{A}(y) \cap \boxed{B}(y) \text{ for every } y \in Y \subseteq X$$

$$= (\boxed{A} \cap \boxed{B})(y) \text{ for every } y \in Y \subseteq X$$

$$= \boxed{0}(y) \text{ for every } y \in Y \subseteq X$$

$(\boxed{A}/Y) \cap (\boxed{B}/Y)\boxed{0}$.

Hence $(X, \mathcal{C}/Y)$ is a cubic K-Hausdorff space.

Theorem 3.13. *Product of two cubic* K-*Hausdorff spaces is a cubic* K-*Hausdorff space in the product topology*.

Proof. Let $(X, \mathcal{C}_1)$ and $(Y, \mathcal{C}_2)$ be two cubic K-Hausdorff spaces.

To Prove: $(X \times Y, \mathcal{C}_1 \times \mathcal{C}_2)$ is a cubic K-Hausdorff space.

Consider two distinct points $(x_1, y_1), (x_2, y_2) \in X \times Y$.

Either $x_1 \neq x_2$ or $y_1 \neq y_2$.

Assume $x_1 \neq x_2$. Therefore there exists two cubic open sets $\boxed{A} = \langle \hat{\mu}, f \rangle \epsilon \mathcal{C}, \boxed{B} = \langle \hat{\lambda}, g \rangle \in \mathcal{C}$, such that $\boxed{A}(x_1)) > \boxed{0}, \boxed{B}(x_2) > \boxed{0}$ and $\boxed{A} \cap \boxed{B} = \boxed{0}$.

Therefore $\boxed{A} * \boxed{1} \in \mathcal{C}_1 \times \mathcal{C}_2$, since $\boxed{A} \in \mathcal{C}_1, \boxed{1} \in \mathcal{C}_2$ and

$\boxed{B} * \boxed{1} \in \mathcal{C}_1 \times \mathcal{C}_2$, since $\boxed{B} \in \mathcal{C}_1, \boxed{1} \in \mathcal{C}_2$, where

$\boxed{A} * \boxed{1} = \langle (\hat{\mu} * \hat{1}), (f * 1) \rangle$ and $\boxed{B} * \boxed{1} = \langle (\hat{\lambda} * \hat{1}), (g * 1) \rangle$

Consider

$$(\boxed{A} * \boxed{1})(x_1, y_1) = \langle (\hat{\mu} * \hat{1}), (f * \mathbf{1}) \rangle (x_1, y_2) \text{ for all } (x_1, y_1) \in X \times Y$$

$$= (\hat{\mu} * 1^-)(x_1, y_1), (f * \mathbf{1}) > (x_1, y_1) \text{ for all } (x_1, y_1) \in X \times Y$$

$$= [(\mu^- * 1^-)(x_1, y_1), (\mu^+ * 1^+)(x_1, y_1)], (f * \mathbf{1})(x_1, y_1) \text{ for all } (x_1, y_1) \in X \times Y$$

$= [\min(\mu^-(x_1,), 1^-(y_1)), \min(\mu^+(x_1,), 1^+(y_1))], \min(f(x_1), \mathbf{1}(y_1)$ for all $(x_1, y_1) \in X \times Y$

$> \boxed{0}.$

$(\boxed{B} * \boxed{1})(x_2, y_2) = \langle (\hat{\lambda} * \hat{1}), (g * \mathbf{1}) \rangle (x_2, y_2)$ for all $(x_2, y_2) \in X \times Y$

$= [(\lambda^- * 1^-)(x_2, y_2), (\lambda^+ * 1^+)(x_2, y_2)$ for all $(x_2, y_2) \in X \times Y$

$= [\min(\lambda^-(x_2), 1^-(y_2)), \min(\lambda^+(x_2), 1^+(y_2))], \min(g(x_2), \mathbf{1}(y_2))$ for all $(x_2, y_2) \in X \times Y$

$> \boxed{0}.$

Also

$\boxed{A} \cap \boxed{B} = \boxed{0} \Rightarrow \langle \hat{\mu}, f \rangle \cap \langle \hat{\lambda}, g \rangle = \langle \hat{0}, \mathbf{0} \rangle$

$\Rightarrow \langle \hat{\mu} \hat{\cap} \hat{\lambda}, f \wedge g \rangle = \langle \hat{0}, \mathbf{0} \rangle$

$\Rightarrow (\mu^- \wedge \lambda^-, \mu^+ \wedge \lambda^+) = [0, 0], (f \wedge g) = \mathbf{0}$

$\Rightarrow (\mu^- \wedge \lambda^-)(x) = 0$ and $(\mu^+ \wedge \lambda^+)(x) = 0, (f \wedge g)(x) = 0$ for all $x \in X$,

$\Rightarrow \mu^-(x) \wedge \lambda^-(x) = 0$ and $\mu^+(x) \wedge \lambda^+(x) = 0, f(x) \wedge g(x) = 0$ for all $x \in X$,

$\Rightarrow$ either $\mu^-(x) = 0$ or $\lambda^-(x) = 0$ and either $\mu^+(x) = 0$ or $\lambda^+(x) = 0$, either $f(x) = 0$ or $g(x) = 0$ for all $x \in X$

$\Rightarrow$ either $\mu^-(x) \wedge 1^-(y) = 0$ or $\lambda^-(x) \wedge 1^-(y) = 0$ and either $\mu^+(x) \wedge 1^+(y) = 0$ or $\lambda^+(x) \wedge 1^+(y) = 0$, either $f(x) \wedge \mathbf{1}(y) = 0$ or $g(x) \wedge \mathbf{1}(y) = 0$ for all $x \in X$ and $y \in Y$

$\Rightarrow$ either $(\mu^- * 1^-)(x, y) = 0$ or $(\lambda^- * 1^-)(x, y) = 0$ and either $(\mu^+ * 1^+)(x, y) = 0$ or $(\lambda^+ * 1^+)(x, y) = 0$,

either $(f * \mathbf{1})(x, y) = 0$ or $(g * 1)(x, y) = 0$ for all $(x, y) \in X \times Y$

$\Rightarrow ((\mu^- * 1^-) \wedge (\lambda^- * 1^-))(x, y) = 0$ and $((\mu^+ * 1^+) \wedge (\lambda^+ * 1^+))(x, y) = 0((f * 1) \wedge (g * 1))(x, y) = 0$ for all $(x, y) \in X \times Y$

$\Rightarrow \langle(\hat{\mu} * \hat{1}), (f * \mathbf{1})\rangle = \langle\hat{0}, \mathbf{0}\rangle$ and $\langle(\hat{\lambda} * \hat{1}), (g * \mathbf{1})\rangle = \langle\hat{0}, \mathbf{0}\rangle$

$\Rightarrow (\boxed{A} * \boxed{1}) = \boxed{0}$ and $(\boxed{B} * \boxed{1}) = \boxed{0}$

$\Rightarrow (\boxed{A} * \boxed{1} \cap (\boxed{B} * \boxed{1} = \boxed{0})$

Hence $(X \times Y, \mathcal{C}_1 \times \mathcal{C}_2)$ is a cubic K-Hausdorff space.

Theorem 3.14. *Arbitrary product of cubic* K-*Hausdorff spaces is a cubic* K-*Hausdorff space in the product topology*.

Proof. Let $\{X_\lambda \boxed{A_\lambda} /_{\lambda \in \Lambda})\}$ be a collection of cubic K-Hausdorff spaces.

Consider $X = \prod_{\lambda \in \Lambda} X_\lambda$ in the product topology.

Consider two distinct points $(x_\lambda)_{\lambda \in \Lambda}, (y_\lambda)_{\lambda \in \Lambda} \in \prod_{\lambda \in \Lambda} X_\lambda$

Therefore $x_\mu \neq y_\mu$ for some $\mu \in \wedge$.

Therefore there exist two cubic open sets $\boxed{A_\mu} = \langle\hat{\mu}_\mu, f_\mu\rangle \in \mathcal{C}_\mu \boxed{B_\mu}$ $= \langle\hat{\lambda}_\mu, g_\mu\rangle \in \mathcal{C}_\mu$

Such that $\boxed{A_\mu}(x_\mu) > \boxed{0_\mu}, \boxed{B_\mu}(y_\mu) > \boxed{0_\mu}$ and $\boxed{A_\mu} \cap \boxed{B_\mu} = \boxed{0_\mu}$

Let $\boxed{A} = \prod_{\lambda \in \Lambda}(\boxed{A_\lambda})$ where $\boxed{A_\lambda} = \boxed{1_\lambda}$ for $\lambda \neq \mu$ and $\boxed{B} = \prod_{\lambda \in \Lambda}(\boxed{B_\lambda})$, where $\boxed{B_\lambda} > \boxed{1_\lambda}$ for $\lambda \neq \mu$

Then $\boxed{A}, \boxed{B} \in \prod_{\lambda \in \wedge} \mathcal{C}_\lambda$

$$\boxed{A} = \prod_{\lambda \in \wedge} \boxed{A_\lambda} = \langle \prod_{\lambda \in \wedge} (\hat{\mu}\, \lambda), \prod_{\lambda \in \wedge} f_\lambda \rangle$$

$$= \langle [\prod_{\lambda \in \wedge} \mu_\lambda^-, \prod_{\lambda \in \wedge} \mu_\lambda^+], \prod_{\lambda \in \wedge} f_\lambda \rangle$$

$$\prod_{\lambda \in \wedge} (\mu_\lambda^-)(x_\lambda) = \min\{((\mu_\lambda^-)(x_\lambda))\} \text{ for all } \lambda \in \wedge$$

$$= (\mu_\mu^-)(x_\mu) \text{ for some } \mu \in \wedge$$

$$> 0.$$

$$\prod_{\lambda\in\wedge}(\mu_\lambda^+)(x_\lambda) = \min\{((\mu_\lambda^+)(x_\lambda))\} \text{ for all } \lambda\in\wedge$$

$$= (\mu_\mu^+)(x_\mu) \text{ for some } \mu\epsilon\wedge$$

$$> 0$$

$$\prod_{\lambda\in\wedge} f_\lambda(x_\lambda) = \min\{f_\lambda(x_\lambda)\} \text{ for all } \lambda\in\wedge$$

$$= (f_\mu)(x_\mu) \text{ for some } \mu\in\wedge$$

$$> 0$$

$$\boxed{B} = \prod_{\lambda\in\wedge}\boxed{B_\lambda} = \langle \prod_{\lambda\in\wedge}(\hat{\lambda}_\lambda), \prod_{\lambda\in\wedge} g_\lambda\rangle$$

$$= \langle [\prod_{\lambda\in\wedge}\lambda_\lambda^-, \prod_{\lambda\in\wedge}\lambda_\lambda^+], \prod_{\lambda\in\wedge} g_\lambda\rangle$$

$$\prod_{\lambda\in\wedge}\lambda_\lambda^-(y_\lambda) = \min\{(\lambda_\lambda^-(y_\lambda))\}, \text{ for all } \lambda\in\wedge$$

$$= \lambda_\mu^-(y_\mu), \text{ for some } \mu\in\wedge$$

$$> 0$$

$$\prod_{\lambda\in\wedge}\lambda_\lambda^+(y_\lambda) = \min\{(\lambda_\lambda^+(y_\lambda))\}, \text{ for all } \lambda\in\wedge$$

$$= \lambda_\mu^+(y_\mu), \text{ for some } \mu\in\wedge$$

$$> 0$$

$$\prod_{\lambda\in\wedge} g_\lambda(x_\lambda) = \min\{g_\lambda(x_\lambda)\}, \text{ for all } \lambda\in\wedge$$

$$= (g_\mu)(x_\mu) \text{ for some } \mu\in\wedge$$

$$> 0$$

Consider

$$\prod_{\lambda\in\wedge}\boxed{A_\lambda}\cap\prod_{\lambda\in\wedge}\boxed{B_\lambda} = \langle \prod_{\lambda\in\wedge}(\hat{\mu}\lambda), \prod_{\lambda\in\wedge} f_\lambda\rangle \cap \langle \prod_{\lambda\in\wedge}(\hat{\lambda}_\lambda), \prod_{\lambda\in\wedge} g_\lambda\rangle$$

$$= \langle (\prod_{\lambda\in\wedge}(\hat{\mu}\lambda) \mathbin{\hat{\cap}} \prod_{\lambda\in\wedge}(\hat{\lambda}_\lambda), (\prod_{\lambda\in\wedge} f_\lambda \wedge \prod_{\lambda\in\wedge} g_\lambda)\rangle$$

$$\prod_{\lambda\in\wedge}\hat{\mu}_{\lambda}\,\hat{\cap}\prod_{\lambda\in\wedge}(\hat{\lambda}_{\lambda})=(\prod_{\lambda\in\wedge}\mu_{\lambda}^{-}\prod_{\lambda\in\wedge}\mu_{\lambda}^{+})\wedge(\prod_{\lambda\in\wedge}\lambda_{\lambda}^{-},\prod_{\lambda\in\wedge}\lambda_{\lambda}^{+})$$

$$=((\prod_{\lambda\in\wedge}\mu_{\lambda}^{-}\wedge\prod_{\lambda\in\wedge}\lambda_{\lambda}^{-}),(\mu_{\lambda}^{+}\wedge\prod_{\lambda\in\wedge}\lambda_{\lambda}^{+})).$$

Then

$$\prod_{\lambda\in\wedge}\mu_{\lambda}^{-}\wedge\prod_{\lambda\in\wedge}\lambda_{\lambda}^{-})(x_{\lambda})=(\prod_{\lambda\in\wedge}\mu_{\lambda}^{-}(x_{\lambda}))\wedge(\prod_{\lambda\in\wedge}\lambda_{\lambda}^{-}(x_{\lambda}))\text{ for all }\lambda\in\wedge$$

$$=[\min\{(\mu_{\mu}^{-})(x_{\lambda})\}]\wedge[\min\{(\lambda_{\lambda}^{-})(x_{\lambda})\}]\text{ for all }\lambda\in\wedge$$

$$=(\mu_{\mu}^{-})(x_{\mu})\wedge(\lambda_{\mu}^{-})(x_{\mu})$$

$$=(\mu_{\mu}^{-}\wedge\lambda_{\mu}^{-})(x_{\mu})=0$$

$$(\prod_{\lambda\in\wedge}\mu_{\lambda}^{+}\wedge\prod_{\lambda\in\wedge}\lambda_{\lambda}^{+}(x_{\lambda})=(\prod_{\lambda\in\wedge}\mu_{\lambda}^{+}(x_{\lambda}))\wedge(\prod_{\lambda\in\wedge}\lambda_{\lambda}^{+}(x_{\lambda}))\text{ for all }\lambda\in\wedge$$

$$=[\min\{(\mu_{\mu}^{+})(x_{\lambda})\}]\wedge[\min\{(\lambda_{\lambda}^{+})(x_{\lambda})\}]\text{ for all }\lambda\in\wedge$$

$$=(\mu_{\mu}^{+})(x_{\mu})\wedge(\lambda_{\mu}^{+})(x_{\mu})$$

$$=(\mu_{\mu}^{+}\wedge\lambda_{\mu}^{+})(x_{\mu})=0$$

$$(\prod_{\lambda\in\wedge}f_{\lambda}\wedge\prod_{\lambda\in\wedge}g_{\lambda})(x_{\lambda})=(\prod_{\lambda\in\wedge}f_{\lambda}(x_{\lambda}))\wedge(\prod_{\lambda\in\wedge}g_{\lambda}(x_{\lambda}))\text{ for all }\lambda\in\wedge$$

$$=\min\{f_{\lambda}(x_{\lambda})\}\wedge\min\{g_{\lambda}(x_{\lambda})\}\text{ for all }\lambda\in\wedge$$

$$=(f_{\mu})(x_{\mu})\wedge(g_{\mu})(x_{\mu})$$

$$=(f_{\mu}\wedge g_{\mu})(x_{\mu})$$

$$=0.$$

IV. Conclusion

In this paper the separation axioms cubic W-T_0, cubic W-T_1, cubic W-Hausdorff, cubic K-T_0, cubic K-T_1, cubic K-Hausdorff are introduced and some basic properties of these axioms are proved.

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