SIMILARITY MEASURES OF INTUITIONISTIC TRAPEZOIDAL FUZZY NUMBER USING CENTROIDS OF HORIZONTAL AND VERTICAL AXES AND VALUE AND AMBIGUITY INDICES

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Abstract

The aim of this paper is to develop methods for calculating similarity measures of intuitionistic trapezoidal fuzzy number in fuzzy environment. The Centroids based on horizontal axis and vertical axes of an intuitionistic trapezoidal fuzzy numbers are discussed and the value and ambiguity of the membership and the non-membership of Intuitionistic Trapezoidal Fuzzy Numbers (TrIFNs) are defined. Two distinct procedure have been proposed by using the above discussed notions to find the similarity measures for Intuitionistic Trapezoidal Fuzzy Numbers (TrIFNs). Finally numerical examples are also illustrated for both similarity measures.

1. Introduction

A Similarity Measure is a function which computes the degree of Similarity between a pair of text objects. Since Atanassov [1] proposal intuitionistic fuzzy Sets (IFSs) many different Similarity measures between IFSs have been proposed in literature. Based on a characteristic value of an intuitionistic fuzzy number Chiao [2] proposed a characteristic value for fuzzy number. Li and Cheng [4] discussed some similarity measure on IFSs and then proposal a suitable similarity measure between IFSs, which is the first one to be applied to pattern recognition problems. Hung and Yang [3]
proposed another method to calculate the distance between IFSs based on the Hausdorff distance. Nehi and Maleki [6] introduced intuitionistic trapezoidal fuzzy numbers and some operators for them based on the intuitionistic fuzzy numbers defined by Grzegorzewski. In [7] Shapu Ren introduced the similarity measures based on characteristics value of an intuitionistic trapezoidal fuzzy number. Stephen Dinagar and Fany Helena [8, 9] proposed a similarity measures for generalized trapezoidal intuitionistic fuzzy number based on valued ambiguity and also similarity measures for generalized trapezoidal intuitionistic fuzzy number using centroid ranking are introduced. Based on the above references we proposed similarity measures for intuitionistic trapezoidal fuzzy numbers using centroids of horizontal and vertical axis and another similarity is using value and ambiguity index of intuitionistic trapezoidal fuzzy numbers.

This paper is organized as follows: Section 2 gives the basic definitions of intuitionistic fuzzy number, Intuitionistic trapezoidal fuzzy number and their crisp $\alpha$, $\beta$-cuts. In Section 3 we discussed about the centroids of horizontal and vertical axis of intuitionistic trapezoidal fuzzy number. In section 4 we proposed a method to calculate the similarity Measure for intuitionistic trapezoidal fuzzy numbers using centroids of horizontal and vertical axis and also numerical examples are illustrated. In section 5 we discussed about the value and ambiguity index of intuitionistic trapezoidal fuzzy number. In section 6 we proposed a new similarity measure using value and ambiguity of intuitionistic trapezoidal fuzzy number also numerical example are illustrated for the proposed method. Finally the conclusion is given in section 7.

2. Preliminaries

Definition 2.1. Let $x = \{x\}$ is a collection of objects denoted generally by $x$. Then an intuitionistic fuzzy set $\tilde{A}^i$ in $x$ is a set $\tilde{A}^i = \{(x, \mu_{\tilde{A}^i}(x), v_{\tilde{A}^i}(x))\}$; $x \in X$ where $\mu_{\tilde{A}^i}(x)$ is termed as the grade of membership of $x$ in $A$ and where $v_{\tilde{A}^i}(x)$ termed as the grade of non membership of $x$ in $A$. $\mu_{\tilde{A}^i}(x), v_{\tilde{A}^i}(x) : X \to M$ is a function from $X$ to a space $M$ which are called membership space. When $M$ contains only two points, 0 and 1. Here $0 \leq \mu_{\tilde{A}^i}(x) + v_{\tilde{A}^i}(x) \leq 1, \forall x \in X$. 
Definition 2.2. An intuitionistic fuzzy numbers (IFN) $\tilde{A}^i$ has the following properties:

1. $\tilde{A}^i$ is convex for membership function $\mu_{\tilde{A}^i}$
   i.e., $v_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda)x_2) \geq \min[\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)]$.

2. $\tilde{A}^i$ is concave for non membership function $v_{\tilde{A}^i}$
   i.e., $v_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda)x_2) \leq \max[v_{\tilde{A}^i}(x_1), v_{\tilde{A}^i}(x_2)]$.

3. $\tilde{A}$ is normal.

Definition 2.3. A TrIFN $\tilde{A}^i = [(a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4)]$ is a special IF set on the real number set $R$, whose membership function and nonmembership function are defined as follows:

$$
\mu_{\tilde{A}^i}(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\
1 & \text{if } a_2 \leq x \leq a_3 \\
\frac{a_4 - x}{a_4 - a_3} & \text{if } a_3 \leq x \leq a_4 \\
0 & \text{otherwise}
\end{cases}
$$

Figure 1.
respectively, where \( b_1 \leq a_1, b_2 \leq a_2, a_3 \leq b_3 \) and \( a_4 \leq b_4 \).

**Definition 2.4.** An \((\alpha, \beta)\)-cut set of \( \tilde{a}^i \) is a crisp subset \( R \), which is defined as follows:

\[
\tilde{a}_{\alpha, \beta}^i = \{ x | \mu_{\tilde{a}^i}(x) \geq \alpha, \nu_{\tilde{a}^i}(x) \leq \beta \},
\]

where \( 0 \leq \alpha \leq 1, 0 \leq \beta \leq 1 \), and \( 0 \leq \alpha + \beta \leq 1 \).

**Definition 2.5.** A \( \alpha \)-cut set of \( \tilde{a}^i \) is a crisp subset \( R \), which is defined as follows:

\[
\tilde{a}_{\alpha}^i = \{ x | \mu_{\tilde{a}^i}(x) \geq \alpha \}.
\]

Using \( \tilde{a}_{\alpha, \beta}^i = \{ x | \mu_{\tilde{a}^i}(x) \geq \alpha, \nu_{\tilde{a}^i}(x) \leq \beta \} \) and the above definition, it follows that \( \tilde{a}^i_\alpha = [L^{\alpha}(\tilde{a}^i), R^{\alpha}(\tilde{a}^i)] \), which can be calculated as follows:

\[
[L^{\alpha}(\tilde{a}^i), R^{\alpha}(\tilde{a}^i)] = [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)].
\]

**Definition 2.6.** A \( \beta \)-cut set of \( \tilde{a}^i \) is a crisp subset \( R \), which is defined as follows:

\[
\tilde{a}_{\beta}^i = \{ x | \nu_{\tilde{a}^i}(x) \leq \beta \}.
\]

Using \( \tilde{a}_{\alpha}^i = \{ x | \mu_{\tilde{a}^i}(x) \leq \alpha \} \) and the above definition, it follows that \( \tilde{a}_{\beta}^i \) is closed interval, denoted by \( \tilde{a}_{\beta}^i = [L^{\beta}(\tilde{a}^i), R^{\beta}(\tilde{a}^i)] \), which can be calculated as follows:

\[
[L^{\beta}(\tilde{a}^i), R^{\beta}(\tilde{a}^i)] = [b_2 - \beta(b_2 - b_1), b_3 + \beta(b_4 - b_3)].
\]


Here we consider centroid technique of trapezoidal intuitionistic fuzzy
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numbers TrIFN. This technique uses geometric centre of these fuzzy numbers with centroid values. The centre corresponds to the values $\tilde{x}(A_{TR}^i)$ on the horizontal axis and the values $\tilde{y}(A_{TR}^i)$ on the vertical axis. The centroid points of TrIFN is $(\tilde{x}(A_{TR}^i), \tilde{y}(A_{TR}^i))$.

\[
\tilde{x}_\mu(A_{TR}^i) = \frac{1}{3} \left[ \frac{a_3^2 + a_2^2 - a_1^2 - a_2^2 - a_1 a_2 + a_3 a_4}{a_4 + a_3 - a_2 - a_1} \right],
\]

\[
\tilde{x}_\nu(A_{TR}^i) = \frac{1}{3} \left[ \frac{2b_1^2 - 2b_2^2 + 2b_2^2 + 2b_3^2 + b_1 b_2 - b_3 b_4}{b_4 + b_3 - b_1 - b_2} \right],
\]

\[
\tilde{y}_\mu(A_{TR}^i) = \frac{1}{3} \left[ \frac{a_1 + 2a_2 - 2a_3 - a_4}{a_1 + a_2 - a_3 - a_4} \right]; \quad \tilde{y}_\nu(A_{TR}^i) = \frac{1}{3} \left[ \frac{2b_1 + b_2 - b_3 - 2b_4}{b_1 + b_2 - b_3 - b_4} \right].
\]

Then

\[
\tilde{x}(A_{TR}^i) = \frac{\tilde{x}_\mu(A_{TR}^i) + \tilde{x}_\nu(A_{TR}^i)}{2}.
\]

Similarly

\[
\tilde{y}(A_{TR}^i) = \frac{\tilde{y}_\mu(A_{TR}^i) + \tilde{y}_\nu(A_{TR}^i)}{2}.
\]

Likewise for $B$

\[
\tilde{x}(B_{TR}^i) = \frac{\tilde{x}_\mu(B_{TR}^i) + \tilde{x}_\nu(B_{TR}^i)}{2}
\]

\[
\tilde{y}(B_{TR}^i) = \frac{\tilde{y}_\mu(B_{TR}^i) + \tilde{y}_\nu(B_{TR}^i)}{2}.
\]

4. Proposed Method of Similarity Measure Using Centroids of horizontal and Vertical Axis of Trapezoidal Intuitionistic Fuzzy Number

The similarity Measures of two trapezoidal intuitionistic fuzzy can be calculated by the below proposed formula based on the centroids of horizontal and vertical axis is,
The similarity measures for the above intuitionistic trapezoidal fuzzy numbers on the horizontal and vertical axis of the two TRIFNs. It can be calculated as

\[
S(\tilde{A}_i, \tilde{B}_i) = \left[ 1 - \frac{\sum_{i=1}^{4} [\mu_{\tilde{A}_i}(x) + v_{\tilde{A}_i}(x)] + \sum_{i=1}^{4} [\mu_{\tilde{B}_i}(x) + v_{\tilde{B}_i}(x)]}{4 \times 2} \right]
\]

Where \(\tilde{x}(\tilde{A}_i), \tilde{y}(\tilde{A}_i), \tilde{x}(\tilde{B}_i), \tilde{y}(\tilde{B}_i)\) are the centre corresponds to the values on the horizontal and vertical axis of the two TRIFNs. It can be calculated as

\[
\tilde{x}(\tilde{A}_i) = \frac{\tilde{x}_{\mu}(\tilde{A}_i) + \tilde{x}_{\nu}(\tilde{A}_i)}{2}; \quad \tilde{y}(\tilde{A}_i) = \frac{\tilde{y}_{\mu}(\tilde{A}_i) + \tilde{y}_{\nu}(\tilde{A}_i)}{2}
\]

\[
\tilde{x}(\tilde{B}_i) = \frac{\tilde{x}_{\mu}(\tilde{B}_i) + \tilde{x}_{\nu}(\tilde{B}_i)}{2}; \quad \tilde{y}(\tilde{B}_i) = \frac{\tilde{y}_{\mu}(\tilde{B}_i) + \tilde{y}_{\nu}(\tilde{B}_i)}{2}
\]

4.1. Illustrations

Example 1. The two TrIFNs are \(\tilde{A}_i = [(0.4, 0.49, 0.74, 0.78)], \tilde{B}_i = [(0.56, 0.58, 0.86, 0.89)]\). The similarity measures for the above intuitionistic trapezoidal fuzzy numbers is given by

\[
S(\tilde{A}_i, \tilde{B}_i) = \left[ 1 - \frac{\sum_{i=1}^{4} [\mu_{\tilde{A}_i}(x) + v_{\tilde{A}_i}(x)] + \sum_{i=1}^{4} [\mu_{\tilde{B}_i}(x) + v_{\tilde{B}_i}(x)]}{4 \times 2} \right]
\]

\[
\frac{\min [\tilde{x}(\tilde{A}_i), \tilde{x}(\tilde{B}_i)] + \min [\tilde{y}(\tilde{A}_i), \tilde{y}(\tilde{B}_i)]}{\max [\tilde{x}(\tilde{A}_i), \tilde{x}(\tilde{B}_i)] + \max [\tilde{y}(\tilde{A}_i), \tilde{y}(\tilde{B}_i)]} = 0.7292.
\]
**Example 2.** The two TrIFNs are \( \tilde{A}^i = [0.64, 0.78, 0.79, 0.9] \)
(0.56, 0.68, 0.79, 0.98]) and \( \tilde{B}^i = [0.34, 0.58, 0.59, 0.73)(0.23, 0.5, 0.6, 0.8] \).
The similarity measures for the above intuitionistic trapezoidal fuzzy numbers is given by

\[
S(\tilde{A}^i, \tilde{B}^i) = \left[ 1 - \frac{2}{4} \sum_{i=1}^{4} \left[ \mu_{\tilde{A}^i}(x) + v_{\tilde{A}^i}(x) \right] \right] \times \left[ \frac{\min [\tilde{x}(\tilde{A}^i_T), \tilde{x}(\tilde{B}^i_T)] + \min [\tilde{y}(\tilde{A}^i_T), \tilde{y}(\tilde{B}^i_T)]}{\max [\tilde{x}(\tilde{A}^i_T), \tilde{x}(\tilde{B}^i_T)] + \max [\tilde{y}(\tilde{A}^i_T), \tilde{y}(\tilde{B}^i_T)]} \right] = 0.5337.
\]

5. Value and Ambiguity of Trapezoidal Intuitionistic Trapezoidal Fuzzy Number [9]

**Definition 5.1.** Let \( \tilde{a}^i_{\alpha} \) and \( \tilde{a}^i_{\beta} \) be any \( \alpha \)-cut and \( \beta \)-cut set of an TrFN \( \tilde{a}^i \), respectively. The value of the membership function \( \mu_{\tilde{a}^i}(x) \) and nonmembership function \( v_{\tilde{a}^i}(x) \) for the TrIFN \( \tilde{a}^i \) is defined as follows:

\[
V_{\mu} = \int_{0}^{1} \left( L_{\tilde{a}^i}(\alpha) + R_{\tilde{a}^i}(\alpha) \right) f(\alpha) d\alpha,
\]

\[
V_{v} = \int_{0}^{1} \left( L_{\tilde{a}^i}(\beta) + R_{\tilde{a}^i}(\beta) \right) g(\beta) d\beta,
\]

respectively.

Where \( f(\alpha) = \alpha; g(\beta) = 1 - \beta. \) The value of the membership function of a TrIFN \( \tilde{a}^i \) is calculated as follows:

\[
V_{\mu} = \int_{0}^{1} \left( L_{\tilde{a}^i}(\alpha) + R_{\tilde{a}^i}(\alpha) \right) f(\alpha) d\alpha = \frac{1}{6}(\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4).
\]
In a similar way, the value of the non membership function of a TrIFN $\tilde{a}^i$ is calculated as follows:

$$V_\gamma = \int_0^1 (L_{\tilde{a}^i}(\beta) + R_{\tilde{a}^i}(\beta))g(\beta)d\beta = \frac{1}{6} (b_1 + 2b_2 + 2b_3 + b_4).$$

**Definition 5.2.** Let $\tilde{a}_\alpha^i$ and $\tilde{a}_\beta^i$ be any $\alpha$-cut and $\beta$-cut set of an TrIFN $\tilde{a}^i$, respectively. The ambiguity of the membership function $\mu_{\tilde{a}^i}(x)$ and nonmembership function $v_{\tilde{a}^i}(x)$ for the TrIFN $\tilde{a}^i$ is defined as follows:

$$A_\mu = \int_0^1 (R_{\tilde{a}^i}(\alpha) - L_{\tilde{a}^i}(\alpha))f(\alpha)d\alpha,$$

$$A_v = \int_0^1 (R_{\tilde{a}^i}(\beta) - L_{\tilde{a}^i}(\beta))g(\beta)d\beta,$$

respectively.

Where $f(\alpha) = \alpha; g(\beta) = 1 - \beta$. The ambiguity of the membership function of a TrIFN $\tilde{a}^i$ is calculated as follows:

$$A_\mu = \int_0^1 (R_{\tilde{a}^i}(\alpha) - L_{\tilde{a}^i}(\alpha))f(\alpha)d\alpha = \frac{1}{6} (-a_1 - 2a_2 + 2a_3 + a_4).$$

Likewise, the ambiguity of the non membership function of a TrIFN $\tilde{a}^i$ is calculated as follows:

$$A_v = \int_0^1 (R_{\tilde{a}^i}(\beta) - L_{\tilde{a}^i}(\beta))g(\beta)d\beta = \frac{1}{6} (-b_1 - 2b_2 + 2b_3 + b_4).$$

6. **New Similarity Measure Using Value and Ambiguity of Intuitionistic Trapezoidal Fuzzy Number**

Let $\tilde{A}_{TrIFN} = [(a_1, a_2, a_3, a_4)(b_1, b_2, b_3, b_4)]$ and $\tilde{B}_{TrIFN} = [(c_1, c_2, c_3, c_4)(d_1, d_2, d_3, d_4)]$ are two trapezoidal intuitionistic fuzzy numbers. The similarity measures using values and ambiguities of TrIFN has been proposed that is
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\[
S_{VA}(\tilde{A}^i, \tilde{B}^i) = \left[ 1 - \frac{\sum_{i=1}^{4} [\mu_{\tilde{A}^i}(x) + v_{\tilde{A}^i}(x)]}{4} \right] \times \left[ \frac{\min [V(\tilde{A}^i), V(\tilde{B}^i)] + \min [A(\tilde{A}^i), A(\tilde{B}^i)]}{\max [V(\tilde{A}^i), V(\tilde{B}^i)] + \max [A(\tilde{A}^i), A(\tilde{B}^i)]]} \right]
\]

Where

\[
V(\tilde{A}^i) = \frac{V_{\mu}(\tilde{A}^i) + V_{v}(\tilde{A}^i)}{2}, \quad V(\tilde{B}^i) = \frac{V_{\mu}(\tilde{B}^i) + V_{v}(\tilde{B}^i)}{2},
\]

\[
A(\tilde{A}^i) = \frac{A_{\mu}(\tilde{A}^i) + A_{v}(\tilde{A}^i)}{2}, \quad A(\tilde{B}^i) = \frac{A_{\mu}(\tilde{B}^i) + A_{v}(\tilde{B}^i)}{2}.
\]

Calculating the values of \( V_{\mu}(\tilde{A}^i), V_{v}(\tilde{A}^i) \) using (i.e.)

\[
V_{\mu}(\tilde{A}^i) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}; \quad V_{v}(\tilde{A}^i) = \frac{b_1 + 2b_2 + 2b_3 + a_4}{6}.
\]

Similarly for \( V_{\mu}(\tilde{B}^i), V_{v}(\tilde{B}^i) \) using (i.e.) \( V_{\mu}(\tilde{B}^i) = \frac{c_1 + 2c_2 + 2c_3 + c_4}{6}; \)

\[
V_{v}(\tilde{B}^i) = \frac{d_1 + 2d_2 + 2d_3 + d_4}{6}.
\]

And for calculating the ambiguities \( A_{\mu}(\tilde{A}^i), A_{v}(\tilde{A}^i) \) using (i.e.)

\[
A_{\mu}(\tilde{A}^i) = \frac{-a_1 - 2a_2 + 2a_3 + a_4}{6}; \quad A_{v}(\tilde{A}^i) = \frac{-b_1 - 2b_2 + 2b_3 + a_4}{6}.
\]

Similarly for \( A_{\mu}(\tilde{B}^i), A_{v}(\tilde{B}^i) \) using (i.e.) \( A_{\mu}(\tilde{B}^i) = \frac{-c_1 - 2c_2 + 2c_3 + c_4}{6}; \)

\[
A_{v}(\tilde{B}^i) = \frac{-d_1 - 2d_2 + 2d_3 + d_4}{6}.
\]

6.1. Illustrations

Example 1. Consider \( \tilde{A}^i = [(0.6, 0.75, 0.8, 0.84)(0.4, 0.68, 0.8, 0.9)] \) and

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The similarity measure for the above trapezoidal intuitionistic fuzzy numbers is given by

\[
S_{VA}(\tilde{A}^i, \tilde{B}^i) = \left[1 - \frac{\sum_{i=1}^{4} [\mu_{\tilde{A}^i}(x) + v_{\tilde{A}^i}(x)]}{2} - \frac{\sum_{i=1}^{4} [\mu_{\tilde{B}^i}(x) + v_{\tilde{B}^i}(x)]}{2}\right] \times \frac{[\min [V(\tilde{A}^i), V(\tilde{B}^i)] + \min [A(\tilde{A}^i), A(\tilde{B}^i)]]}{[\max [V(\tilde{A}^i), V(\tilde{B}^i)] + \max [A(\tilde{A}^i), A(\tilde{B}^i)]]} = 0.9231.
\]

**Example 2.** Consider \(\tilde{A}^i = [(0.8, 0.84, 0.89, 0.9)(0.7, 0.82, 0.9, 0.95)]\) and \(\tilde{B}^i = [(0.76, 0.8, 0.84, 0.89)(0.7, 0.78, 0.89, 0.94)]\). The similarity measure for the above trapezoidal intuitionistic fuzzy numbers is given by

\[
S_{VA}(\tilde{A}^i, \tilde{B}^i) = \left[1 - \frac{\sum_{i=1}^{4} [\mu_{\tilde{A}^i}(x) + v_{\tilde{A}^i}(x)]}{2} - \frac{\sum_{i=1}^{4} [\mu_{\tilde{B}^i}(x) + v_{\tilde{B}^i}(x)]}{2}\right] \times \frac{[\min [V(\tilde{A}^i), V(\tilde{B}^i)] + \min [A(\tilde{A}^i), A(\tilde{B}^i)]]}{[\max [A(\tilde{A}^i), A(\tilde{B}^i)] + \max [A(\tilde{A}^i), A(\tilde{B}^i)]]} = 0.9385.
\]

**7. Conclusion**

In this paper, we defined the centroids based on horizontal and vertical axis value. Using these two values we proposed similarity measure for TrIFNs. Furthermore we defined value and ambiguity which are used to define the value index and ambiguity index of the TrIFNs then we calculated another new similarity measure using the value and ambiguity index.
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References