Advances and Applications in Mathematical Sciences
Volume 22, Issue 8, June 2023, Pages 1755-1764
ISSN 0974-6803

A NOTE ON RANK OF TRAPEZOIDAL FUZZY NUMBER MATRICES

U. HARI NARAYANAN[1] and K. KUMARESH[2]

[1,2]PG and Research Department of Mathematics
A. V. C. College (Autonomous)
(Affiliated to Annamalai University, Chidambaram)
Mannampanadal-609305, India
E-mail: hari.narayanan23@gmail.com

Abstract

In this article, some methods are described to find these types of ranks viz, row rank, column rank, fuzzy rank and fuzzy full rank using Trapezoidal Fuzzy Matrices (TrFMs). Also we investigated the relationship between them under the algorithm for Row Reduced Echelon Form (RREF). We studied the cross vector and Schein rank under the relationship is illustrated with suitable example.

1. Introduction

Throughout this paper we deal with fuzzy number matrices that is matrices over fuzzy algebra $\mathbb{F}$. For $A \in \mathbb{F}_{mn}$, $\mathbb{R}(A)$, $\mathbb{C}(A)$, $\rho_r(A)$, $\rho_c(A)$ and $\rho_f(A)$ denote the row space, column space, row rank, column rank, and fuzzy rank under trapezoidal fuzzy matrix respectively. Ismail and Morsi [1] established that fuzzy rank of fuzzy matrix in the product of two fuzzy matrices cannot exceed the fuzzy rank either fuzzy matrix. Also, Latha et al. [2] studied the rank of type 2 triangular fuzzy matrix.

The concept of fuzzy matrix (FM) [3] is one of the recent topics to develop for dealing with the rank of fuzzy number matrices fuzzy matrices defined first time by Thomson [6] and discussed about the convergence of the powers of a fuzzy matrix. In FMs, rows and columns are taken as uncertain. He also

2020 Mathematics Subject Classification: Primary 15B15, Secondary 94D.
Keywords: trapezoidal fuzzy number (TrFN), trapezoidal fuzzy matrix (TrFM), row rank, column rank, fuzzy rank, fuzzy full rank, cross vector, Schein rank.
Received March 23, 2022; Accepted May 4, 2022

investigated different properties of these types of matrices along with application. Shyamal and Pal [4] introduced the concept of triangular fuzzy matrix. Stephen Dinagar and Harinarayanan [5] are studied the rank of fuzzy matrices.

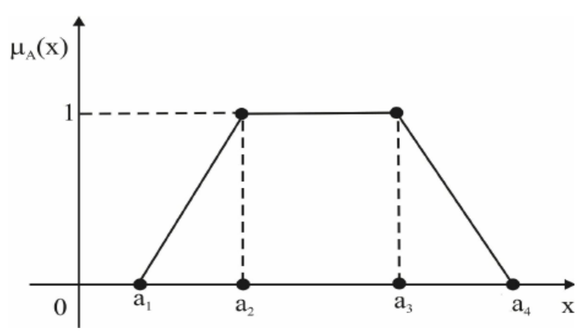
2. Preliminaries

Definition 2.1 (Fuzzy Set). A fuzzy set is defined as $\widetilde{A} = \{x, \mu_A(x) : x \in X\}$, with a membership function $\mu_A(x) : X \to [0, 1]$, where $\mu_A(x)$ denotes the degree of membership of the element x to the set A.

Definition 2.2 (Fuzzy Number). A fuzzy set $\widetilde{A}$, defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics $\widetilde{A}$ is normal $\widetilde{A}$ is convex set. The support of $\widetilde{A}$ is closed and bounded then $\widetilde{A}$ is called fuzzy number.

Definition 2.3 (Trapezoidal Fuzzy Number). A fuzzy number on $\widetilde{A}_T$ is a trapezoidal fuzzy number denoted by $\widetilde{A}_T = (a_1, a_2, a_3, a_4)$ are the membership function of this fuzzy number will be interpreted as

$$\mu_{\widetilde{A}_T}(x)\begin{cases} 0, & x < a_1 \\ \left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \left(\dfrac{a_4-x}{a_4}\right), & a_4 \le x \le a_5 \\ 0, & x > a_5 \end{cases}$$



Trapezoidal Fuzzy Number

2.4. Arithmetic Operations on Trapezoidal Fuzzy Numbers (TrFNs). Let us consider $\widetilde{A}_T = (a_1, a_2, a_3, a_4)$ and $\widetilde{B}_T = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers then,

(i) Addition: $\widetilde{A}_T(+)\widetilde{B}_T = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$.

(ii) Subtraction: $\widetilde{A}_T(-)\widetilde{B}_T = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4)$.

(iii) Multiplication: $\widetilde{A}_T(\times)\widetilde{B}_T = \{(a_1R(\widetilde{B}_T), (a_2R(\widetilde{B}_T), (a_3R(\widetilde{B}_T), (a_4R(\widetilde{B}_T)\}$.

Where $R(\widetilde{B}_T) = \dfrac{b_1 + b_2 + b_3 + b_4}{4}$

(iv) Division: $\widetilde{A}_T(\div)\widetilde{B}_T = \left(\dfrac{a_1}{R(\widetilde{B}_T)}, \dfrac{a_2}{R(\widetilde{B}_T)}, \dfrac{a_3}{R(\widetilde{B}_T)}, \dfrac{a_4}{R(\widetilde{B}_T)}\right)$.

Where $R(\widetilde{B}_T) = \dfrac{b_1 + b_2 + b_3 + b_4}{4}$.

Definition 2.5 Ranking Function. We define a ranking function $R : F(R) \to R$ which maps each fuzzy numbers to real line $F(R)$ represent the set of all trapezoidal fuzzy numbers. If R be any linear ranking functions.

$$R(\widetilde{A}_T) = \left(\frac{a_1 + a_2 + a_3 + a_4}{4}\right).$$

Definition 2.6 Inverse Trapezoidal Fuzzy Number. If $\widetilde{a}_T$ is trapezoidal fuzzy number and $\widetilde{a}_T \neq \widetilde{0}_T$, then we define $\widetilde{a}_T^{-1} = \dfrac{\widetilde{1}_T}{\widetilde{a}_T}$.

3. Trapezoidal Fuzzy Matrices (TrFMs)

Definition 3.1 (Trapezoidal Fuzzy Matrix). A trapezoidal fuzzy matrix of order $m \times n$ is defined as $\hat{A}_T = (\widetilde{a}_{ij})_{m\times n}$ where $\widetilde{a}_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ is ij^{th} element of $\hat{A}_T$.

3.2. Operations on Trapezoidal Fuzzy Matrices (TrFMs). Let $\hat{A}_T = (\widetilde{a}_{ij})_{m\times n}$ and $\hat{B}_T = (\widetilde{b}_{ij})_{m\times n}$ be two TrFMs of same order. Then we have the following

$$\hat{A}_T + \hat{B}_T = (\tilde{a}_{ij} + \tilde{b}_{ij})$$

$$\hat{A}_T - \hat{B}_T = (\tilde{a}_{ij} - \tilde{b}_{ij})$$

For $\hat{A}_T = (\tilde{a}_{ij})_{m \times n}$ and $\hat{B}_T = (\tilde{b}_{hij})_{m \times k}$ then $\hat{A}_T \hat{B}_T = (\tilde{c}_{ij})_{m \times k}$ where $\tilde{c}_{ij} = \sum_{p=1}^{n} \tilde{a}_{ip} \tilde{b}_{pj}$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, k$.

$\hat{A}_T^T$ or $\hat{A}'_T = (\tilde{a}_{ij})$.

$k\hat{A}_T = (k\tilde{a}_{ij})$, where k is scalar.

Definition 3.3 (Zero Trapezoidal Fuzzy Matrix). A trapezoidal fuzzy matrix (TrFM) is said to be a zero TrFM and all its entries are 0 and it is denoted by $\hat{O}_T$.

Definition 3.4 (Unit Trapezoidal Fuzzy Matrix). The square TrFM is said to be a unit TrFM if the diagonal elements are 1 and the rest of elements are 0. It is denoted by $\hat{I}_T$.

4. Rank of Trapezoidal Fuzzy Matrices (TrFMs)

Definition 4.1 (Trapezoidal Fuzzy Vector). A trapezoidal fuzzy vector is an n-tuple of elements from a fuzzy algebra. That is, a trapezoidal fuzzy vector is of the form $(x_1, x_2, \ldots, x_n)$ where each element $x_i \in \mathbb{F}$, $i = 1, 2, \ldots, n$.

Definition 4.2 (Row Rank). The row space $\mathbb{R}(\hat{A}_T)$ of an $m \times n$ trapezoidal fuzzy matrix $\hat{A}_T$ is the subspace of $\mathbb{V}_n$ generated by the rows of $\hat{A}_T$. The row rank $\rho_r(\hat{A}_T)$ of $\hat{A}_T$ is minimum possible size of a spanning set of $\mathbb{R}(\hat{A}_T)$.

Definition 4.3 (Column Rank). The column space $\mathbb{C}(\hat{A}_T)$ of an $m \times n$ trapezoidal fuzzy matrix $\hat{A}_T$ is the subspace of $\mathbb{V}_m$ generated by the rows of

$\hat{A}_T$. The row rank $\rho_r(\hat{A}_T)$ of $\hat{A}_T$ is the smallest possible size of a spanning set of $\mathbb{C}(\hat{A}_T)$.

Definition 4.4 (Fuzzy Rank). A trapezoidal fuzzy matrix $\hat{A}_T$ is said to be a fuzzy rank of rank r, if $\rho_r(\hat{A}_T) = \rho_c(\hat{A}_T) = r$ and it is denoted by $\rho_f(\hat{A}_T)$.

4.1 Elementary Transformation. The elementary transformation of a TrFM $\hat{A}_T = (\tilde{a}_{ij})$ of the following transformation,

1. Interchange of two rows or columns.

2. Multiplication of a row (or column) by an arbitrary trapezoidal fuzzy number not equal to $\tilde{O}_T$.

3. Addition of multiple of one row (or column) by trapezoidal fuzzy number not equal to $\tilde{O}_T$ to another row (column).

4.2 Row Reduced Echelon form of TrFM. Let $\hat{A}_T = (\tilde{a}_{ij})_{m\times n}$ be the trapezoidal fuzzy matrix then the following steps are,

1. If $\tilde{a}_{11} = \tilde{O}_T$ then an interchange of rows and columns will change element in the position $\tilde{a}_{11}$, so that $\tilde{a}_{11} \neq \tilde{O}_T$.

2. Convert the element $\tilde{a}_{11}$ to $\tilde{1}_T$ by multiplying the first row by $1/\tilde{a}_{11}$.

3. Subtract from the i^{th} row, $i > 1$, the first row multiplied by $\tilde{a}_{11}$, whenever $\tilde{a}_{11} \neq \tilde{O}_T$ then the element $\tilde{a}_{i1}(i > 1)$ will be replaced by $\tilde{O}_T$.

4. Subtract from the j^{th} column, $j > 1$, the first column multiplied by $\tilde{a}_j$, whenever $\tilde{a}_j \neq \tilde{O}_T$, then the element $\tilde{a}_j(j > 1)$ will be replaced by $\tilde{O}_T$.

5. Performing the same manipulations (step 1 to step 4) with the submatrix that remains in the lower right corner and so on. We finally after a finite number of manipulations arrive at a diagonal-equivalent TrFM with the same rank as the original TrFM $\tilde{A}_T$.

Example 4.5. Consider the TrFM,

$$\hat{A}_T = \begin{bmatrix} (-1, 0, 1, 4) (-1, 1, 2, 6) (-1, 0, 1, 4) (-1, 1, 4, 8) \\ (-1, 1, 2, 6) (-1, 2, 5, 10) (-1, 0, 1, 4) (-6, -2, 11) \\ (-1, 1, 4, 8) (-1, 6, 8, 11) (-1, 1, 4, 8) (-12, -10, -8, 2) \end{bmatrix}$$

into RREF.

$$\sim \begin{bmatrix} \left(\frac{-25}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{31}{2}\right) & \left(\frac{-41}{2}, \frac{-8}{2}, \frac{12}{2}, \frac{53}{2}\right) & \left(\frac{-25}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{27}{2}\right) & \left(\frac{-115}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{31}{2}\right) \\ \left(\frac{-23}{2}, \frac{-5}{2}, \frac{5}{2}, \frac{23}{2}\right) & \left(\frac{-41}{2}, \frac{-8}{2}, \frac{10}{2}, \frac{39}{2}\right) & \left(\frac{-19}{2}, \frac{-3}{2}, \frac{7}{2}, \frac{23}{2}\right) & \left(\frac{-44}{2}, \frac{-14}{2}, \frac{5}{2}, \frac{53}{2}\right) \\ \left(\frac{-9}{16}, \frac{-3}{16}, \frac{3}{16}, \frac{9}{16}\right) & \left(\frac{-13}{16}, \frac{-6}{16}, \frac{2}{16}, \frac{17}{16}\right) & \left(\frac{-9}{16}, \frac{-3}{16}, \frac{3}{16}, \frac{9}{16}\right) & \left(\frac{-5}{16}, \frac{11}{16}, \frac{22}{16}, \frac{36}{16}\right) \end{bmatrix}$$

$$c = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbb{C}(\hat{A}_T) = \{c_1, c_2, c_3, c_4\} \text{ and } \mathbb{R}(\hat{A}_T) = \{r_1, r_2, r_3\}$$

Clearly c_1, c_3 and c_4 are linearly independent,

$$\text{The basis of } \mathbb{C}(\hat{A}_T) = \left\{ \begin{bmatrix} (-1, 0, 1, 4) \\ (-1, 1, 2, 6) \\ (-1, 1, 4, 8) \end{bmatrix} \begin{bmatrix} (-1, 0, 1, 4) \\ (-1, 0, 1, 4) \\ (-1, 1, 4, 8) \end{bmatrix} \begin{bmatrix} (-1, 1, 4, 8) \\ (-6, -2, -1, 1) \\ (-12, -10, -8, 2) \end{bmatrix} \right\}$$

The basis of $\mathbb{R}(\hat{A}_T) =$

$$\sim \begin{bmatrix} \left(\frac{-25}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{31}{2}\right) & \left(\frac{-41}{2}, \frac{-8}{2}, \frac{12}{2}, \frac{53}{2}\right) & \left(\frac{-25}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{27}{2}\right) & \left(\frac{-115}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{31}{2}\right) \\ \left(\frac{-23}{2}, \frac{-5}{2}, \frac{5}{2}, \frac{23}{2}\right) & \left(\frac{-41}{2}, \frac{-8}{2}, \frac{10}{2}, \frac{39}{2}\right) & \left(\frac{-19}{2}, \frac{-3}{2}, \frac{7}{2}, \frac{23}{2}\right) & \left(\frac{-44}{2}, \frac{-14}{2}, \frac{5}{2}, \frac{53}{2}\right) \\ \left(\frac{-9}{16}, \frac{-3}{16}, \frac{3}{16}, \frac{9}{16}\right) & \left(\frac{-13}{16}, \frac{-6}{16}, \frac{2}{16}, \frac{17}{16}\right) & \left(\frac{-9}{16}, \frac{-3}{16}, \frac{3}{16}, \frac{9}{16}\right) & \left(\frac{-5}{16}, \frac{11}{16}, \frac{22}{16}, \frac{36}{16}\right) \end{bmatrix}$$

$$\dim(\mathbb{C}(\hat{A}_T)) = \rho_c(\hat{A}_T) = 3, \ \dim(\mathbb{R}(\hat{A}_T)) = \rho_r(\hat{A}_T) = 3$$

The common value of row rank and column rank is fuzzy rank,

(i.e.), $\rho_r(\hat{A}_T) = \rho_c(\hat{A}_T) = \rho_f(\hat{A}_T) = 3$.

Hence the rank of trapezoidal fuzzy matrix $\hat{A}_T$ is 3.

Property 4.6. Let $\hat{A}_T \in \mathbb{F}_{mn}$ with $\rho_c(\hat{A}_T) = r$. Then there exist matrices $\hat{B}_T \in \mathbb{F}_{mr}$ and $\hat{C}_T \in \mathbb{F}_{rn}$ such that $\rho_r(\hat{A}_T) = \rho_r(\hat{C}_T) = r$ and $\hat{A}_T = \hat{B}_T\hat{C}_T$.

Property 4.7. Let $\hat{A}_T \in \mathbb{F}_{mn}$ with $\rho_c(\hat{A}_T) = s$. Then there exist matrices $\hat{B}_T \in \mathbb{F}_{ms}$ and $\hat{C}_T \in \mathbb{F}_{sn}$ such that $\rho_c(\hat{A}_T) = \rho_c(\hat{A}_T) = s$ and $\hat{A}_T = \hat{B}_T\hat{C}_T$.

Definition 4.8. Let $\hat{A}_T$ and $\hat{B}_T$ be the two TrFMs such that $\hat{A}_T\hat{B}_T$ is defined by $\rho(\hat{A}_T\hat{B}_T) \leq \min\{\rho(\hat{A}_T), \rho(\hat{B}_T)\}$.

Definition 4.9. Let $\hat{A}_T = (\tilde{a}_{ij})_{m\times n} \in \mathbb{F}_{mn}$, then $\hat{A}_T$ is called row full rank if $\rho_r(\hat{A}_T) = m$ and $\hat{A}_T$ is called column full rank if $\rho_c(\hat{A}_T) = n$ and let $\hat{A}_T \in \mathbb{F}_{mn}$ then $\hat{A}_T$ is said to be fuzzy full rank if $\rho_f(\hat{A}_T) = \rho_r(\hat{A}_T) = \rho_c(\hat{A}_T) = n$.

Example 4.10. Consider a Trapezoidal fuzzy matrix,

$$\hat{A}_T = \begin{pmatrix} (-2, 0, 2, 4) & (-1, 3, 4, 6) & (-1, 6, 7, 8) \\ (-1, 0, 1, 8) & (4, 6, 8, 10) & (6, 10, 11, 13) \\ (-2, 0, 2, 4) & (-1, 3, 4, 6) & (4, 5, 7, 8) \end{pmatrix} \text{ into RREF is}$$

$$\sim \begin{pmatrix} (-2, 0, 2, 4) & (-7, -1, 1, 7) & (-9, -1, 1, 9) \\ (-9, -1, 1, 9) & (-20, 3, 8, 13) & (-34, 5, 11, 18) \\ (-6, -2, 2, 6) & (-13, -3, 4, 12) & (-16, -5, 7, 18) \end{pmatrix}$$

$$c = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbb{C}(\hat{A}_T) = \{c_1, c_2, c_3\} \text{ and } \mathbb{R}(\hat{A}_T) = \{r_1, r_2, r_3\}$$

Clearly c_1, c_2 and c_3 are linearly independent.

$$\text{The basis of } \mathbb{C}(\hat{A}_T) = \begin{Bmatrix} (-2, 0, 2, 4) & (-1, 3, 4, 6) & (-1, 6, 7, 8) \\ (-1, 0, 1, 8) & (4, 6, 8, 10) & (6, 10, 11, 13) \\ (-2, 0, 2, 4) & (-1, 3, 4, 6) & (4, 5, 7, 8) \end{Bmatrix}$$

The basis of

$$\mathbb{R}(\hat{A}_T) = \begin{Bmatrix} (-2, 0, 2, 4) & (-7, -1, 1, 7) & (-9, -1, 1, 9) \\ (-9, -1, 1, 9) & (-20, 3, 8, 10) & (-34, 5, 11, 18) \\ (-6, -2, 2, 6) & (-13, -3, 4, 12) & (-16, -5, 7, 18) \end{Bmatrix}$$

$$\dim(\mathbb{C}(\hat{A}_T)) = \rho_c(\hat{A}_T) = 3, \ \dim(\mathbb{R}(\hat{A}_T)) = \rho_r(\hat{A}_T) = 3$$

The common value of row rank and column rank order 3 is same for the fuzzy full rank is also 3

(i.e.), $\rho_r(\hat{A}_T) = \rho_c(\hat{A}_T) = \rho_f(\hat{A}_T) = 3.$

5. Cross Vector and Schein Rank

Cross vector play an important role in fuzzy as well as trapezoidal fuzzy matrix theory to evaluate the rank of matrices.

Definition 5.1. For vectors u, v the cross vector (u, v) is the TrFM $\hat{A}_T = (\tilde{a}_{ij}) = u^t v$ such that $\tilde{a}_{ij} = u_i v_i$, where $\tilde{a}_{ij}, u_i, v_i \in \mathbb{F}$ in fuzzy algebra.

Example 5.2. We consider two vectors $u = [(-1, 0, 1, 4), (-1, 0, 3, 6), (-2, 0, 6, 8)]$ and $v = [(-2, 0, 6, 8), (2, 6, 7, 9), (0, 2, 6, 8)]$, then the cross vector of these two vectors u and v is

$$u^t v = \begin{pmatrix} (-1, 0, 1, 4) \\ (-1, 0, 3, 6) \\ (-2, 0, 6, 8) \end{pmatrix} ((-2, 0, 6, 8), (2, 6, 7, 9), (0, 2, 6, 8))$$

$$= \begin{pmatrix} (-3, 0, 3, 12) & (-6, 0, 6, 24) & (-4, 0, 4, 16) \\ (-3, 0, 9, 18) & (-6, 0, 18, 36) & (-4, 0, 12, 24) \\ (-6, 0, 18, 24) & (-12, 0, 24, 32) & (-8, 0, 24, 32) \end{pmatrix}$$

Here we see that the row vectors of the cross vectors are linear combination of the vector $[(-2, 0, 6, 8), (2, 6, 7, 9), (0, 2, 6, 8)]$ and the column vectors are the linear combination of the vector $[(-1, 0, 1, 4), (-1, 0, 3, 6), (-2, 0, 6, 8)]^t$, which shows that the row vectors and column vectors of the cross vector are linearly dependent and row rank = column rank = fuzzy rank = 1.

Example 5.3. Let us consider $u_1 = \{(-2, 0, 2, 4), (-1, 0, 1, 8)\} \in \mathbb{V}_2$, $u_2 = [(-1, 3, 4, 6), (-1, 1, 5, 11)] \in \mathbb{V}_2$,

$v_1 = [(-1, 3, 4, 6), (4, 5, 7, 8), (-2, 0, 2, 4)] \in \mathbb{V}_3$ and

$v_2 = [(-1, 1, 5, 11), (4, 6, 8, 10), (-1, 0, 1, 8)] \in \mathbb{V}_3$. Now the cross vectors

$$\hat{A}_{1T} = u_1^t v_1 = \begin{pmatrix} (-6, 0, 6, 12) & (-12, 0, 12, 24) & (-2, 0, 2, 4) \\ (-3, 0, 3, 24) & (-6, 0, 6, 32) & (-1, 0, 1, 8) \end{pmatrix}$$

$$\hat{A}_{2T} = u_2^t v_2 = \begin{pmatrix} (-4, 12, 16, 24) & (-7, 21, 28, 42) & (-2, 6, 8, 12) \\ (-4, 4, 20, 44) & (-7, 7, 35, 77) & (-2, 2, 10, 22) \end{pmatrix}$$

$$\hat{A}_T = \begin{pmatrix} (-10, 12, 22, 36) & (-19, 21, 40, 66) & (-4, 6, 10, 16) \\ (-7, 4, 23, 68) & (-13, 7, 41, 109) & (-3, 2, 11, 30) \end{pmatrix}$$

Also,

$$\hat{U}_T = \begin{pmatrix} (-2, 0, 2, 4) & (-1, 3, 4, 6) \\ (-1, 0, 1, 8) & (-1, 1, 5, 11) \end{pmatrix},$$

$$\hat{V}_T = \begin{pmatrix} (-1, 3, 4, 6) & (-4, 5, 7, 8) & (-2, 0, 2, 4) \\ (-1, 1, 5, 11) & (-4, 6, 8, 10) & (-1, 0, 1, 8) \end{pmatrix}$$

$$\hat{U}_T \hat{V}_T = \begin{pmatrix} (-10, 12, 22, 36) & (-19, 21, 40, 66) & (-4, 6, 10, 16) \\ (-7, 4, 23, 68) & (-13, 7, 41, 109) & (-3, 2, 11, 30) \end{pmatrix} = \hat{A}_{Tr}$$

$$\therefore \hat{U}_T \hat{V}_T = \hat{A}_T.$$

Definition 5.4 (Schein Rank). The Schein rank $\rho_s(\hat{A}_T)$ of a trapezoidal fuzzy matrix $\hat{A}_T$ is the least number of matrices of rank 1 whose sum is $\hat{A}_{Tr}$.

Example 5.5. The Schein ranks of the trapezoidal fuzzy matrix $\hat{A}_T = \begin{bmatrix} (-2, 0, 2, 4) & (0, 0, 0, 0) & (-2, 0, 2, 4) \\ (-2, 0, 2, 4) & (-2, 0, 2, 4) & (-1, 0, 1, 8) \\ (0, 0, 0, 0) & (-2, 0, 2, 4) & (-2, 0, 2, 4) \end{bmatrix}$ are 3, because $\hat{A}_T = \hat{A}_{1T} + \hat{A}_{2T} + \hat{A}_{3T}$.

Where $\hat{A}_{1T} = \begin{pmatrix} (-2, 0, 2, 4) & (0, 0, 0, 0) & (0, 0, 0, 0) \\ (-2, 0, 2, 4) & (0, 0, 0, 0) & (0, 0, 0, 0) \\ (0, 0, 0, 0) & (0, 0, 0, 0) & (0, 0, 0, 0) \end{pmatrix}$,

$$\hat{A}_{2T} = \begin{pmatrix} (0, 0, 0, 0) & (0, 0, 0, 0) & (0, 0, 0, 0) \\ (0, 0, 0, 0) & (-2, 0, 2, 4) & (0, 0, 0, 0) \\ (0, 0, 0, 0) & (-2, 0, 2, 4) & (0, 0, 0, 0) \end{pmatrix},$$

$$\hat{A}_{3T} = \begin{pmatrix} (0, 0, 0, 0) & (0, 0, 0, 0) & (-2, 0, 2, 4) \\ (0, 0, 0, 0) & (0, 0, 0, 0) & (-1, 0, 1, 8) \\ (0, 0, 0, 0) & (0, 0, 0, 0) & (-2, 0, 2, 8) \end{pmatrix}$$

With all $\hat{A}_i$'s are of rank 1.

6. Conclusion

In this paper, we find out the different types of ranks using trapezoidal fuzzy matrices such as fuzzy rank, fuzzy full rank, cross vector and Schein rank. The evaluation of rank in topics of trapezoidal fuzzy matrices in fuzzy algebra $\mathbb{F}$ have been attempted. Our future investigation of study may be in the domain of fuzzy linear space of fuzzy linear system under fuzzy rank method.

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