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A RIEMANN SOLVER WITH ARITHMETIC AVERAGING FOR ONE-DIMENSIONAL PROBLEM IN DUSTY GAS

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Abstract

In this paper, we use the Riemann problem for a quasilinear hyperbolic system of equations for unsteady flow of an ideal gas with dust particles in one-dimensional regime. We have applied the various arithmetic procedures on flux variables. Three waves structure with characteristic speed as eigenvalues of Jacobian matrix obtained from the system of equations is studied. Here, some cases discussed and using symbols obtained after using the arithmetic averaging to the vector of conservative variable and flux function and analyzed the behaviour of eigenvectors in dusty gas.

Introduction

For the conservation law, the Riemann problem is the initial value problem with discontinuous initial data. The solution of the Riemann problem with data U_L and U_R be composed of three waves in which the middle wave is always a contact discontinuity and rest of two waves are either shock wave or rarefaction waves. The solution of the Riemann problem has applications in the theoretical and numerical study of the system of conservation laws in reacting flows, magnetogasdynamics, shallow water theory, ideal gas-

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dynamics, non-ideal gas-dynamics. Godunov [4] presented numerical method of integrating system of quasilinear hyperbolic system of equations, applicable to the solution of a class of problems of gas dynamics, aerodynamics and continuum mechanics. Glaister [2] has studied of various of arithmetic averaging for approximate Riemann solver for two-dimensional Euler equations for compressible flow of an ideal gas.

In computational fluid-dynamics (CFD) Upwind methods of Godunov type has a significant collision, Godunov type methods has complexity and computing cost, a required solution of Riemann problem is complex and expensive in computation. Approximate Riemann solver generally less expensive than exact Riemann solver. Toro [15] offered two approaches to find direct approximate solution to the Riemann solver, his presentation resulted in the time-dependent Euler equation but the idea was treated to systems similarly. Glimm [3] found the weak solution to the Riemann problem, these solutions was not differential or continuous in general and has jump discontinuities in the form of shock.

Jena and Sharma [6] used a group theoretical method to establish the entire class of self similar solution to the problem of shock wave entering through the dusty gas. Singh et al. [14] discussed a problem of entering of strong plane and converging shock wave in a mixture of gas and small solid particles with the assumption that solid particles are continuously distributed in the gas, they derived the jump conditions for the plane and converging shock waves using the nonstandard analysis. Miura et al. [8] discussed the passage of shock waves through dusty gas layer, they obtained properties of gas and dusty gas by idealized equilibrium-gas approximation and got criteria for the wave reflexion at the contact surface separating the pure gas from the dusty gas. Pai et al. [12] investigated the similarity solution of a strong shock wave entering in a mixture of a gas and small solid particles, and described similarity solution exists when the shock is very strong and the surrounding medium is of constant density and at rest with negligible counter pressure, with the derivation of the non-dimensional fundamental equations. Nath et al. [10] studied the Riemann problem for a quasilinear hyperbolic system of equations of unsteady flow of an ideal polytropic gas with dust particles in one dimension, and analyzed properties of shock waves, rarefaction waves and contact discontinuity for dusty gas and existence and uniqueness of the solution of Riemann problem in a dusty gas.

Nath et al. [11] assessed that how dust particles in the gas effects the existence of shock or not and also discussed how to variation of mass fraction of the dust particles effects on the growth decay behaviour of shock in cylindrical symmetric and spherically symmetric flows. Manjunatha et al. [7] discussed a two phase model employing on linearly streching cylinder engrossed in a porous media subjected to effect of radiation in dusty fluid. Nath and Sahu [9] studied a self similar model under a rotating atmosphere in a non ideal gas behind a cylindrical shock wave evicted by a piston travelling with time according to an exponential law in one-dimensional unsteady adiabatic flow. Doromin and Larkin [1] have analyzed an one-dimensional initial value problem for two phase flow of a compressible, viscous dusty gas in a channel. Gupta et al. [5] have discussed about a direct approach to one-dimensional Riemann problem for unsteady planner flow of an isentropic, compressible, inviscid fluid for a quasilinear hyperbolic system of equations. Singh [3] has described a problem for an unsteady flow of a radiating gas near the optically thin limit with the assumption that the gas is optically grey, inviscid and in thermodynamic equilibrium using a wave front expansion in one dimension.

In this paper we have studied Riemann problem in the conservation form of compressible fluid using arithmetic averaging procedure. We discussed some cases by making jacobian matrices with the help of arithmetic averaging on the flux function and the vector of conservative variables and discussed behaviour of eigenvectors in the dusty gas.

Governing Equations

We have governing equations expressing a planar flow of an ideal polytropic gas with dust particles and the conservation form of Euler system of partial differential equation of compressible fluid flow in a dusty gas in one dimension Nath et al. [9] is:

$$U_t + F_x = 0, \tag{1}$$

where the vector of conserved flow variables U, the flux function $F(U)$ is associated with the planar flow is defined, respectively, as:

$$U = \left(\rho,\ \rho u,\ \rho\left(\frac{u^2}{2} + E\right)\right), \tag{2}$$

$$F(U) = \left(\rho u,\ p + \rho u^2,\ \rho\left(\frac{u^2}{2} + E\right) + pu\right), \tag{3}$$

where the density ρ, velocity u, and the pressure p are the function of spacial co-ordinate x and time t, E is the internal energy per unit mass of the mixture expressed as

$$E = \frac{(1 - Z)p}{(\Gamma - 1)\rho},\ Z = \theta\rho \text{ with } \theta = k_p/\rho_{sp}, \tag{4}$$

with $Z = V_{sp}/V_g$ is the volume fraction with $k_p = m_{sp}/m_g$ is the mass fraction of the solid particles and ρ_{sp} is the mixture where m_{sp} is the total mass and V_{sp} is the volumetric extension of the solid particles respectively, V_g is the total volume and m_g the total mass of the mixture respectively, the Gruneisen coefficient $\Gamma = \gamma(1 + \lambda\omega)/(1 + \lambda\omega\gamma)$ with $\lambda = k_p/(1 - k_p)$, $\omega = c_{sp}/c_p$, $\gamma = c_p/c_v$, where c_{sp} is the specific heat of the solid particles; c_p and c_v are the specific heat of the gas at constant pressure and the specific heat of the gas at constant volume and the equation of state for a polytropic dusty gas respectively.

$$p = ke^{S/c_v}\left(\frac{\rho}{1 - \theta_p}\right)^{\Gamma}, \tag{5}$$

where k, c_v and γ are positive constant.

The Jacobian Matrices and Structure

If the vector of physical variables is denoted by $\vec{u} = (\rho,\ u,\ p)^T$, the two jacobian matrices P and Q which are the derivatives of vector of conserved variables and flux function $F(U)$ respectively is constructed as:

$$P = \frac{\partial U}{\partial \vec{u}} = \begin{bmatrix} 1 & 0 & 0 \\ u & \rho & 0 \\ \frac{u^2}{2} - \frac{\theta p}{(\Gamma - 1)} & \rho u & \frac{(1 - \theta\rho)}{(\Gamma - 1)} \end{bmatrix}, \tag{6}$$

$$Q = \frac{\partial F(U)}{\partial \vec{u}} = \begin{bmatrix} u & \rho & 0 \\ u^2 & 2\rho u & 1 \\ \frac{u^3}{2} - \frac{\theta p u}{(\Gamma - 1)} & \frac{3}{2}\rho u^2 + \frac{(\Gamma - \theta\rho)p}{(\Gamma - 1)} & \frac{(\Gamma - \theta\rho)u}{(\Gamma - 1)} \end{bmatrix}, \tag{7}$$

P^{-1} is calculated as

$$P^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{u}{\rho} & \frac{1}{\rho} & 0 \\ \left(\frac{u^2}{2} + \frac{\theta p}{(\Gamma - 1)}\right)\left(\frac{\Gamma - 1}{1 - \theta\rho}\right) & -u\left(\frac{\Gamma - 1}{1 - \theta\rho}\right) & \left(\frac{\Gamma - 1}{1 - \theta\rho}\right) \end{bmatrix}, \tag{8}$$

using (7) and (8), the usual jacobian matrix is

$$A = P^{-1}Q = \left(\frac{\partial U}{\partial \vec{u}}\right)^{-1} \frac{\partial F(U)}{\partial \vec{u}} = \begin{bmatrix} u & \rho & 0 \\ 0 & u & 1/\rho \\ 0 & C^2\rho & u \end{bmatrix}, \tag{9}$$

where

$$C^2 = \frac{\Gamma p}{(1 - \theta\rho)\rho}, \; C \text{ is the velocity} \tag{10}$$

eigenvalues and eigenvectors of the matrix A are

$$\lambda_1 = u - C, \; \lambda_2 = u + C, \; \lambda_3 = u, \tag{11}$$

$$r_1 = \left(-\frac{\rho}{C}, 1, -\rho C\right), \; r_2 = \left(\frac{\rho}{C}, 1, \rho C\right), \; r_3 = (1, 0, 0). \tag{12}$$

The characteristic field corresponding to first and second eigenvalues is genuinely non-linear and corresponding to third eigenvalue it is linearly degenerate, this give rise to a three waves structure among of them two waves are travelling with the speed $u \pm C$ and one moving with the speed u.

Determination of A

Here several arithmetic averaging procedures is applied in determination of A and we find out some identities, we allow a number of degree of freedom for the symbols used in place of physical variables and this process gives a significant meaning to the purpose of finding a simplest form of the matrix A and its eigenvalues. First we will rewrite the component of ΔU in terms of $\vec{u} = (\rho, u, p)^T$, where

$U = \left(\rho, \rho u, \rho\left(\frac{u^2}{2} + E\right)\right)$, using Δ operator defined as $\Delta(q) = (q)_R - (q)_L$

with L and R represent the left and right hand states of non-linear waves namely shock wave, rarefaction wave and contact discontinuity; and averaging operator defined below.

$$\bar{q} = \frac{1}{2}(q_L + q_R), \tag{13}$$

where q is any component of $\vec{u} = (\rho, u, p)^T$, the vector of physical variables.

Third component of ΔU is defined by

$$\begin{aligned} \Delta\left(\frac{\rho u^2}{2} + \rho E\right) &= \Delta\left(\frac{\rho u^2}{2} + \frac{(1 - \theta\rho)p}{\Gamma - 1}\right) \\ &= \Delta\left(\frac{\rho u^2}{2}\right) + \Delta\left(\frac{(1 - \theta\rho)}{\Gamma - 1}\right), \end{aligned} \tag{14}$$

second term of third component of ΔU is defined by

$$\Delta(\theta\rho p) = \xi\Delta\rho + \sigma\Delta\theta + \tau\Delta p, \tag{15}$$

where

$$\xi = \bar{\theta}\,\bar{p}, \sigma = \bar{\rho}\,\bar{p}, \qquad \tau = \bar{\rho}\hat{\theta}, \tag{16}$$

$$\xi = \bar{\theta}\,\bar{p}, \sigma = \bar{\rho}\hat{p}, \tau = \bar{\rho}\bar{\theta}, \tag{17}$$

$$\xi = \overline{\theta_p}, \sigma = \bar{\rho}\,\bar{p}, \tau = \bar{\rho}\,\bar{\theta}, \tag{18}$$

$$\xi = \overline{\theta}\,\overline{p},\ \sigma = \overline{\rho}\,\widetilde{p},\ \tau = \overline{\rho}\,\widetilde{\theta}, \tag{19}$$

$$\Delta\left(\frac{\rho u^2}{2} + \rho E\right) = \left(\frac{\alpha}{2} - \frac{\xi}{\Gamma - 1}\right)\Delta\rho + \beta\Delta u - \frac{\sigma}{\Gamma - 1}\Delta\theta - \frac{\tau}{\Gamma - 1}\Delta p, \tag{20}$$

where α and β are referred to Glaister [6] and here are four choices for $\Delta(\theta\rho p)$, namely (16)-(19) in which either choice can be made in ΔU and hence in P.

Using arithmetic averaging on equation (6) and (8) we get

$$\widetilde{P} = \frac{\partial U}{\partial \vec{u}} = \begin{bmatrix} 1 & 0 & 0 \\ \overline{u} & \overline{\rho} & 0 \\ \frac{\alpha}{2} - \frac{\xi}{(\Gamma - 1)} & \beta & \frac{(1-\xi)}{(\Gamma - 1)} \end{bmatrix}, \tag{21}$$

$$\widetilde{P}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{\overline{u}}{\rho} & 0 & 0 \\ \left(\frac{\alpha}{2} + \frac{\xi}{(\Gamma - 1)}\right)\left(\frac{\Gamma - 1}{1 - \xi}\right) & -\overline{u}\left(\frac{\Gamma - 1}{1 - \xi}\right) & \left(\frac{\Gamma - 1}{1 - \xi}\right) \end{bmatrix}. \tag{22}$$

We have vector $F(U) = \left(\rho u,\ p + \rho u^2,\ \rho u\left(\frac{u^2}{2} + E\right) + pu\right)$, then ΔF can be written in terms of $\Delta\vec{u}$ and applying arithmetic averaging on ΔF.

$$\Delta F(U) = \Delta\left(\rho u,\ p + \rho u^2,\ \rho u\left(\frac{u^2}{2} + E\right) + pu\right), \tag{23}$$

third component of ΔF is

$$\Delta\left(\rho u\left(\frac{u^2}{2} + E\right) + pu\right) = \Delta\left(\frac{\rho u^3}{2} + \frac{\Gamma pu}{\Gamma - 1} - \frac{\theta\rho pu}{\Gamma - 1}\right), \tag{24}$$

$$\Delta(\rho u^3) = \delta\Delta\rho + 3\varepsilon\Delta u, \tag{25}$$

where

$$\delta = \overline{u}^3,\ \varepsilon = \overline{\rho}\,\widetilde{\hat{u}}^2, \tag{26}$$

$$\delta = \overline{u^3}, \; \varepsilon = \overline{\rho}\widetilde{\hat{u}}^2, \tag{27}$$

in the expression denoted by equation (24) , the term ρu^3 give rise to two alternatives for $\Delta(\rho u^3)$ given in the expression in the equations (25) and the term $\theta\rho pu$ in the equation (24) can be express in terms of arithmetic averaging as

$$\Delta(\theta\rho pu) = \overline{pu}\,\overline{\theta}\Delta\rho + \overline{\theta\rho}\,\overline{p}\,\Delta u + \overline{\theta\rho}\,\overline{u}\,\Delta p + \overline{pu}\,\overline{\rho}\Delta\theta, \tag{28}$$

using this expression in equation (24) we get

$$\Delta\left(\rho u\left(\frac{u^2}{2} + E\right) + pu\right) = \frac{1}{2}(\delta\Delta\rho + \varepsilon\Delta u) + \frac{\Gamma}{\Gamma - 1}(\overline{p}\,\Delta u + \overline{u}\,\Delta p)$$

$$- \frac{1}{\Gamma - 1}(\chi\overline{\theta}\,\Delta\rho + \tau\overline{p}\Delta u + \tau\overline{u}\,\Delta p + \chi\overline{\rho}\Delta\theta), \tag{29}$$

where $\chi = \overline{pu}, \; \tau = \overline{\theta\rho},$ using equations (25) and (28) in equation (7) and in equation (9), we get the jacobian matrix Q and the jacobian flux matrix A as:

$$\widetilde{Q} = \frac{\partial F(U)}{\partial \vec{u}} = \begin{bmatrix} \overline{u} & \overline{\rho} & 0 \\ \alpha & 2\beta & 1 \\ \frac{\delta}{2} - \frac{\xi\overline{u}}{(\Gamma - 1)} & \frac{3}{2}\varepsilon + \frac{(\Gamma - \tau)\overline{p}}{(\Gamma - 1)} & \frac{(\Gamma - \tau)\overline{u}}{(\Gamma - 1)} \end{bmatrix}, \tag{30}$$

$$\widetilde{A} = \begin{bmatrix} \overline{u} & \overline{\rho} & 0 \\ \frac{(\alpha - \overline{u}^2)}{\overline{\rho}} & \frac{2\beta}{\overline{\rho}} - \overline{u} & 1/\overline{\rho} \\ \frac{(\Gamma - 1)}{(1 - \tau)}\left(\frac{\delta}{2} - \frac{\alpha}{2}\overline{u}\right) & \frac{(\Gamma - 1)}{(1 - \tau)}\left(\left(\frac{\alpha}{2} + \frac{\xi}{(\Gamma - 1)}\right)\overline{\rho} - 2\beta\overline{u} + \frac{3}{2}\varepsilon\right) + \frac{(\Gamma - \tau)}{(1 - \tau)}\overline{p} & \overline{u} \end{bmatrix}. \tag{31}$$

There are four choices for ξ, σ and τ in (16)-(19), two alternatives for ε and δ in (26) and (27), we have (24) total possible alternatives to form (24) possible jacobian flux matrix A and hence eigenvalues. As all (24) possible jacobian matrices cannot be discussed here so we have described three cases and studied behaviour of the eigenvalues and associated eigenvectors.

Case 1: $\overline{u^2}, \; \beta = \overline{\rho}\overline{u}, \; \delta = \overline{u}\,\overline{u^2}$ rest choices for ξ, ε and τ are referred to Glaister [6], we get a simplest form of usual jacobian matrix (31)

$$\tilde{A} = \begin{bmatrix} \bar{u} & \bar{\rho} & 0 \\ \frac{1}{4}\frac{(\Delta u)^2}{\bar{\rho}} & \bar{u} & 1/\bar{\rho} \\ 0 & \frac{(\Gamma - 1)}{(1-\bar{\theta}\bar{\rho})}\left(\frac{1}{4}(\Delta u)^2\,\bar{\rho} - \bar{\rho}\bar{u}(\hat{u} - \bar{u})\right) + \bar{\rho}C^2 & \bar{u} \end{bmatrix}, \tag{32}$$

whose eigenvalues and eigenvectors are

$$\lambda_1 = \bar{u} - c_f,\ \lambda_2 = \bar{u} + c_f,\ \lambda_3 = \bar{u}, \tag{33}$$

where $c_f^2 = \frac{(\Gamma - \bar{\theta}\bar{\rho})}{(1-\bar{\theta}\bar{\rho})}\frac{1}{4}(\Delta u)^2 - \frac{(\Gamma - 1)}{(\Gamma - \bar{\theta}\bar{\rho})}\bar{u}(\hat{u} - \bar{u}) + C^2,$

$$r_1 = \left(1, -\frac{c_f}{\bar{\rho}}, c_f^2 - \frac{1}{4}(\Delta u)^2\right), \tag{34}$$

$$r_2 = \left(1, \frac{c_f}{\bar{\rho}}, c_f^2 - \frac{1}{4}(\Delta u)^2\right), \tag{35}$$

$$r_3 = \left(1, 0, -\frac{1}{4}(\Delta u)^2\right). \tag{36}$$

The wave moving with the speed u does not carry a jump in velocity as the second entry in the corresponding eigenvector is zero.

Case 2: $\alpha = \overline{u^2},\ \beta = \bar{\rho}\tilde{u},\ \delta = \bar{u}\,\overline{u^2}$

$$\widetilde{A} = \begin{bmatrix} \bar{u} & \bar{\rho} & 0 \\ \frac{1}{4}\frac{(\Delta u)^2}{\bar{\rho}} & \hat{u} & 1/\bar{\rho} \\ 0 & \frac{(\Gamma - 1)}{(1-\bar{\theta}\bar{\rho})}\frac{1}{4}(\Delta u)^2\bar{\rho} + \bar{\rho}C^2 & \bar{u} \end{bmatrix}, \tag{37}$$

whose eigenvalues and eigenvectors are

$$\lambda_1 = \tilde{u} - c_g,\ \lambda_2 = \tilde{u} + c_g,\ \lambda_3 = \bar{u}, \tag{38}$$

where

$$c_g^2 = \frac{(\Delta\rho\delta u)^2}{64\bar{\rho}^2} + \frac{(\Gamma - \bar{\theta}\bar{\rho})}{(1 - \bar{\theta}\bar{\rho})} \frac{1}{4} (\Delta u)^2 + C^2,$$

$$r_1 = \left(1, \frac{\tilde{u} - \bar{u} - c_f}{\bar{\rho}}, \frac{(\Gamma - 1)}{(1 - \bar{\theta}\bar{\rho})} \frac{1}{4} (\Delta u)^2 + C^2\right), \tag{39}$$

$$r_2 = \left(1, \frac{\tilde{u} - \bar{u} + c_f}{\bar{\rho}}, \frac{(\Gamma - 1)}{(1 - \bar{\theta}\bar{\rho})} \frac{1}{4} (\Delta u)^2 + C^2\right), \tag{40}$$

$$r_3 = \left(1, 0 - \frac{1}{4}(\Delta u)^2\right). \tag{41}$$

The wave rising with the speed λ_3 does not show a jump in velocity when passes through a dusty gas.

Case 3: $\alpha = \bar{u}^2, \beta = \rho\bar{u}, \delta = \bar{u}^3$

$$\tilde{A} = \begin{bmatrix} \bar{u} & \bar{\rho} & 0 \\ 0 & \bar{u} & 1/\bar{\rho} \\ 0 & \frac{(\Gamma - 1)}{(1 - \bar{\theta}\bar{\rho})}\left(\frac{1}{8}(\Delta u)^2 + \frac{\bar{u}(\Delta\rho)(\Delta u)}{4\bar{\rho}} - \bar{u}^2\right)\bar{\rho} + C^2 & \bar{u} \end{bmatrix}, \tag{42}$$

whose eigenvalues and eigenvectors are

$$\lambda_1 = \bar{u} - c_h, \lambda_2 = \bar{u} + c_h, \lambda_3 = \bar{u}, \tag{43}$$

where

$$c_h^2 = \frac{C^2}{\bar{\rho}},$$

$$r_1 = \left(1, -\frac{c_f}{\bar{\rho}}, c_f^2 - \frac{1}{4}(\Delta u)^2\right), \tag{44}$$

$$r_2 = \left(1, \frac{c_h}{\bar{\rho}}, c_h^2\right), \tag{45}$$

$$r_3 = (1, 0, 0). \tag{46}$$

Here are changes in all the physical variables of the wave moving with speed λ_1 and λ_2; the wave moving with the speed λ_3 has a jump in density.

Results and Discussion

We have applied here averaging procedures on the vector of conservative variables and flux function in one-dimensional dusty gas. We have studied the behaviour of eigenvectors associated to characteristic speed of elementary waves. We have discussed three cases using symbols obtained after applying arithmetic averaging on the jacobian matrices and studied behaviour of eigenvectors of jacobian matrices. These eigenvectors associated to the characteristic speed of elementary waves showing the changes in the density, pressure and velocity of the elementary waves. it is noteworthy that after applying the averaging operator there is no change in the physical variables at the contact discontinuity and changes take place in the shock waves and rarefaction waves when passes through dusty gas.

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